chapter test

Section 4.1

Verify that the given values of x solve the corresponding polynomial equations.

1. $4x^3 - 5x^2 = -3x + 18;\ x = 2$

2. $x^2 - 6x = -13;\ x = 3 + 2i$

3. $x^3 + x = 6x^2 - 164;\ x = 5 - 4i$

4. $x^3 + (1+4i)x = (7-2i)x^2 - 2i + 36;\ x = -2i$

For each of the following polynomial functions, describe the behavior of its graph as $x \to \pm\infty$ and identify the x- and y-intercepts. Use this information to then sketch the graph of each polynomial.

5. $f(x) = (x+1)^2(x-2)$ **6.** $g(x) = (x+4)(x-3)(x+6)$ **7.** $s(x) = x^3 + 5x^2 + 6x$

Solve the following polynomial inequalities.

8. $2x^2 + 15 \le 11x$

9. $(x-3)^2(x+1)^2 > 0$

10. $(x-4)(x+2)(x^2-1) \le 0$

Section 4.2

Use polynomial long division to rewrite each of the following fractions in the form $q(x) + \dfrac{r(x)}{d(x)}$, where $d(x)$ is the denominator of the original fraction, $q(x)$ is the quotient, and $r(x)$ is the remainder.

11. $\dfrac{8x^4 - 6x^3 + 2x^2 + 3x + 4}{2x^2 - 1}$

12. $\dfrac{11x^2 + 2x - 5}{x - 3}$

13. $\dfrac{x^4 - 3x^2 + x - 8}{x^2 + 3x + 2}$

Use synthetic division to determine if the given value for k is a zero of the corresponding polynomial. If not, determine $p(k)$.

14. $p(x) = 6x^5 - 23x^4 - 95x^3 + 70x^2 + 204x - 72;\ k = 1$

15. $p(x) = 48x^4 + 10x^3 - 51x^2 - 10x + 3;\ k = \dfrac{1}{6}$

16. $p(x) = 18x^5 - 87x^4 + 110x^3 - 28x^2 - 16x + 3;\ k = \dfrac{2}{3}$

Construct a polynomial that has the given properties.

17. Third-degree; zeros of –1, 4 and –5; and goes to $-\infty$ as $x \to \infty$

18. Second-degree; zeros of 3 and –2; and y-intercept of –6

Section 4.3

List all of the potential rational zeros of the following polynomials. Then use polynomial division and the quadratic formula, if necessary, to identify the actual zeros.

19. $f(x) = x^4 + 3x^3 - 3x^2 - 11x - 6$ **20.** $g(x) = 2x^3 - 11x^2 + 18x - 9$

Use Descartes' Rule of Signs to determine the possible number of positive and negative real zeros of each of the following polynomials.

21. $f(x) = 2x^4 - 3x^3 - x^2 + 3x + 10$ **22.** $g(x) = x^6 - 4x^5 - 2x^4 + x^3 - 6x^2 - 11x + 6$

Use synthetic division to identify integer upper and lower bounds of the real zeros of the following polynomials.

23. $f(x) = 2x^3 - 11x^2 + 3x + 36$ **24.** $g(x) = 4x^3 - 16x^2 - 79x - 35$

Use the Intermediate Value Theorem to show that each of the following polynomials has a real zero between the indicated values.

25. $f(x) = x^3 + 3x^2 - 4x - 10$; 1 and 2 **26.** $f(x) = -x^4 + 5x^3 - x^2 - 10x - 2$; –2 and –1

Section 4.4

Use all available methods (e.g. the Rational Zero Theorem, Descartes' Rule of Signs, polynomial division, etc.) to solve each polynomial equation. Use the Linear Factor Theorem to make sure you find the appropriate number of solutions, counting multiplicity.

27. $3x^5 + x^4 + 5x^3 = x^2 + 28x + 20$ **28.** $8x^5 + 12x^4 - 18x^3 - 35x^2 = 18x + 3$

Use all available methods (in particular, the Conjugate Roots Theorem, if applicable) to factor each of the following polynomials completely, making use of the given zero.

29. $f(x) = 14x^4 - 109x^3 + 296x^2 - 321x + 70$; $2+i$ is a zero.

30. $f(x) = x^4 - 5x^3 + 19x^2 - 125x - 150$; $-5i$ is a zero.

Construct polynomial functions with the stated properties.

31. Fourth degree, only real coefficients, $\frac{1}{2}$ and $1+2i$ are two of the zeros, y-intercept is -30, leading coefficient is 2.

32. Fifth degree, only real coefficients, -1 is a zero of multiplicity 3, $\sqrt{6}$ is a zero, y-intercept is -6, leading coefficient is 1.

Section 4.5

Find equations for the vertical asymptotes, if any, for each of the following rational functions.

33. $f(x) = \frac{4}{2x-5}$

34. $f(x) = \frac{x^2 - 3x + 2}{x - 1}$

Find equations for the horizontal or oblique asymptotes, if any, for each of the following rational functions.

35. $f(x) = \frac{2x^3 + 5x^2 - 1}{x^2 - 2x}$

36. $f(x) = \frac{x^2 - x + 8}{3x^2 - 7}$

Sketch the graphs of the following rational functions.

37. $f(x) = \frac{x^2 + 5}{x + 5}$

38. $f(x) = \frac{x + 2}{x^2 + 7x}$

Solve the following rational inequalities.

39. $\frac{7}{x+3} \geq \frac{2x}{x+3}$

40. $\frac{x}{x^2 - 5x + 6} < \frac{3}{x^2 - 5x + 6}$

chapter test

Section 5.1

Sketch the graphs of the following functions.

1. $f(x) = \left(\frac{1}{2}\right)^{x-1} + 3$

2. $r(x) = 2^{-x+4} - 2$

Solve the following elementary exponential equations.

3. $3^{3x-5} = 81$

4. $\left(\frac{2}{5}\right)^{-4x} = \left(\frac{25}{4}\right)^{x-1}$

5. $10{,}000^{x} = 10^{-2x-12}$

Section 5.2

Solve the following problems involving exponential functions.

6. Melissa has recently inherited \$15,000 which she wants to deposit into a savings account for 10 years. She has determined that her two best bets are an account that compounds annually at a rate of 3.95% and an account that compounds continuously at an annual rate of 3.85%. Which account would pay Melissa more interest?

7. Bill has come upon a 37 gram sample of iodine-131. He isolates the sample and waits for 2 weeks. After this time period, only 11 grams of iodine-131 remains. Determine the equation modeling the radioactive decay of this substance.

8. Katherine is working in a lab testing bacteria populations. Starting out with a population of 870 bacteria, she notices that the population doubles every 22 minutes. Find **(a)** the equation for the population P in terms of time t in minutes, and **(b)** the population after 68 minutes.

Section 5.3

Sketch the graphs of the following functions.

9. $f(x) = \log_4(3 - x) + 2$

10. $g(x) = 1 - \log_5(2x)$

Evaluate the following logarithmic expressions without the use of a calculator.

11. $\log_{27} 9^{\log 1000}$

12. $\log_{1/3} 9$

13. $\log_4\left(\frac{1}{64}\right)$

Use the elementary properties of logarithms to solve the following equations.

14. $\log_6 6^{\log_5 x} = 3$ **15.** $\log_9 x^{1/2} = \frac{3}{4}$ **16.** $\log_x\left(\log_{1/2}\frac{1}{16}\right) = 2$

Solve the following equations, using a calculator if necessary to evaluate the logarithms.

17. $\log 27 = 5x$ **18.** $\ln(2x - 1) = 3$

Section 5.4

Use properties of logarithms to expand the following expressions as much as possible. Simplify any numerical expressions that can be evaluated without a calculator.

19. $\log\sqrt{\frac{x^3}{4\pi^5}}$ **20.** $\ln\left(\frac{\sqrt{a^5}mn^2}{e^5}\right)$

Use properties of logarithms to condense the following expressions as much as possible, writing each answer as a single term with a coefficient of 1.

21. $\frac{1}{3}\left(\log_2\left(a^5\right) - \log_2\left(bc^3\right)\right)$ **22.** $\ln 4 - \ln x^2 - 7\ln y$

Evaluate the following logarithmic expressions.

23. $\log_7 18$ **24.** $3\log_{1/4} 6$

Use your knowledge of logarithmic functions to answer the following application questions.

25. Cameron had front row tickets to a recent rock concert. The noise intensity in the front row was 10^{-1} W/m^2. Given that $I_0 = 10^{-12}$ W/m^2, what was the decibel level Cameron experienced the night of the concert?

26. The largest earthquake since 1900 occurred May 22, 1960 in Chile, measuring 9.5 on the Richter Scale. What was the intensity, relative to a 0-level earthquake?

Section 5.5

Solve the following exponential and logarithmic equations. When appropriate, write the answer as both an exact expression and as a decimal approximation.

27. $e^{8-5x} = 16$

28. $10^{6/x} = 321$

29. $\ln(x+1) + \ln(x-1) = \ln(x+5)$

30. $\log_2(x+3) + \log_2(x+4) = \log_2(3x+8)$

Use your knowledge of exponential and logarithmic functions to answer the following questions.

31. Rick puts \$6500 in a high interest money market account at 4.36% annual interest compounded monthly. Assuming he makes no deposits or withdrawals, how long will it take for his investment to grow to \$7000?

32. Sodium-24 has a half-life of approximately 15 hours. How long would it take for 350 grams of sodium-24 to decay to 12 grams?