Guidelines

- You are to work independently. This is not a team assignment. Feel free to help your fellow classmates understand principles and concepts, but please do not directly share significant portions of your MATLAB code.
- For this assignment, you will create a PowerPoint presentation, save it as a PDF, and upload it to Blackboard.
- All submitted plots should be easy to see and well-labeled.

Title Slide

- **Slide 1:** Title slide with your name, student ID number, date, lab name, class number/title.

Circular Coax

- This part doesn't involve MATLAB. Simply compute the capacitance per-unit-length C_ℓ of a symmetrical, circular coaxial cable with the dimensions shown in Figure 1. Present your computations and value in **Slide 2** (if you used an online capacitance calculator, please give the name and link to the calculator). Assume that air separates the two conductors. If your computations are correct, your answer will be between 5 and 25 pF/m.

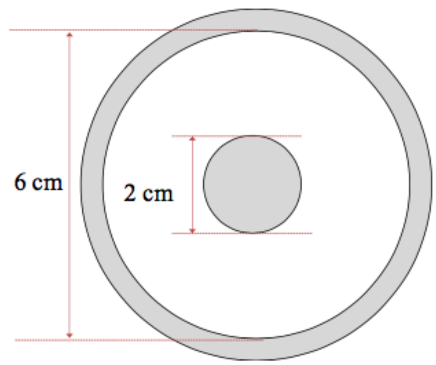

Figure 1. Cross section of circular coax.

Rectangular Coax

- The finite difference technique (FDT) is easiest to apply to rectangular geometries, and you will use the FDT to investigate the rectangular coax shown in Figure 2. This rectangular coax is similar in size to the circular coax analyzed earlier, and so we should expect both to have similar capacitances (here "similar" means on the same order of magnitude).

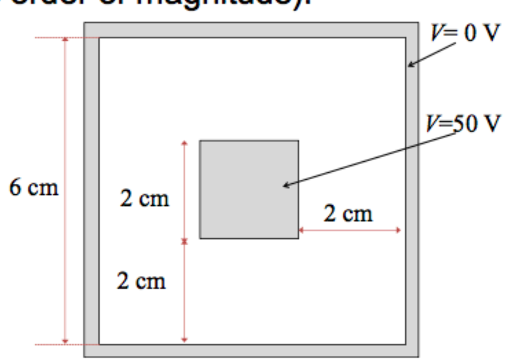

Figure 2. Cross section of rectangular coax. For this particular example, the inner conductor is at 50 V and the outer conductor is at 0 V.

- For this part, you will create a MATLAB script that computes and plots the electrostatic potential using the FDT for the rectangular coaxial geometry. Assume the inner conductor is held at 50 V and the outer conductor is held at 0 V, as shown in Figure 2. Please adhere to the following guidelines.
  - Assume the geometry shown in Figure 2 is a cross section in the *x-y* plane. The rectangular coax is uniform in the *z*-direction, i.e.

$$\frac{\partial^2 V}{\partial z^2} = 0$$

  - Make sure your grid for the numerical FDT is dense enough to accurately capture the geometry involved. Choose the grid density to be 2 mm by 2 mm.
  - Your code should include a method to determine the convergence of your solution.
  - Turn in at least two well-labeled, clear, plots of the electrostatic potential viewed from different angles. Present your plots in **Slides 3 and 4**. Your plots should be clearly labeled and titled. The titles of both plots should include your name in them.
  - In **slide 5** clearly explain your convergence method, and comment on how many iterations needed for convergence.
  - Turn in your code, with comments. Your code should be copied and pasted into **Slides 5**, **6**, **7**, and **8**.

Rectangular Coax: Net charge per-unit-length

- Expand your MATLAB script so that it numerically calculates the total net charge per-unit-length Q_ℓ in C/m on the inner surface of the outer square conductor. Do the same for the outer surface of the inner rectangular conductor. Theoretically, these charges (per-unit-length) should be equal and opposite. Given Q_ℓ, one can find the total charge Q in a conductor of length *d* by taking the product $Q = d \times Q_\ell$. Again, the inner and outer conductors should have equal and opposite charges. Present your results in **slide 8**.
- **Hint for calculating surface charge:**

  After numerically solving for the potential (you calculated the potential in the previous part using the FDT), you can then estimate the electric field **E** at a position along the surface of the conductor by taking the difference between two adjacent potential values at that position, as illustrated in Figure 3. Recall that the electric field is normal (perpendicular) to the surface of a conductor.

  Once the E field is determined at each position along the surface of a conductor, the surface charge density in C/m^2 at each position along the surface can be calculated numerically, as illustrated in Figure 4. Recall that for any conductor in electrostatic equilibrium, all the excess charge resides on the surface (i.e. the interior of the conductor is overall neutral). Therefore, the total net charge per-unit-length Q_ℓ in C/m on a conductor is found by integrating (i.e. continuously summing) the surface charge density along the surface. This integration can be computed numerically by taking a discrete sum of the surface charge density along the surface of the conductor in a given cross-section, i.e.

$$Q_\ell\left[\frac{C}{m}\right] = \oint_{\substack{\text{Conductor}\\ \text{surface}\\ \text{in x-y}\\ \text{cross-section}}} \rho_s dl \approx \sum_{\substack{i,k\\ \text{Conductor}\\ \text{surface}\\ \text{in x-y}\\ \text{cross-section}}} \rho_{s(i,k)} h,$$

where h is the grid spacing for the FDT.

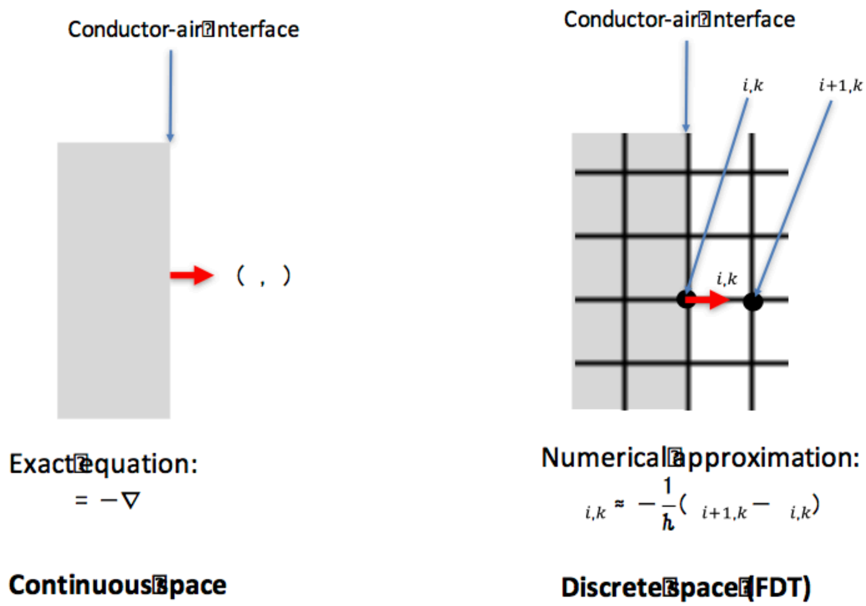

Figure 3. Numerically calculating the electric field at the surface of a conductor. Note that, for the FDT, h is the grid spacing for the FDT.

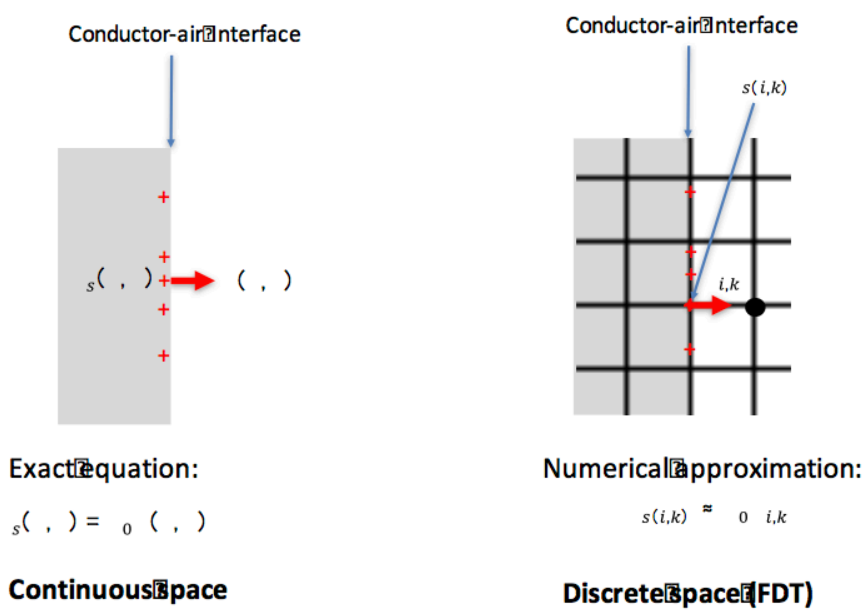

Figure 4. Numerically calculating the surface charge density at the surface of a conductor.

- Expand your MATLAB script so that calculates the capacitance per-unit-length C_ℓ in C/m of the rectangular coaxial cable. If you completed the previous sections correctly, this part is easy. All you need to do at this point is use the definition for the capacitance per-unit-length: $C_\ell = Q_\ell/V$, where Q_ℓ is the charge per-unit-length along one of the conductors (you calculated this in the previous part), and V is the voltage difference between the outer and inner conductors (i.e. 50 V in this case). Present your calculations and result in **slide 9**.
- Check your result above for the capacitance per-unit-length, by comparing your numerical result with the known capacitance per-unit-length of a similarly sized circular coaxial cable shown in Figure 1. These two capacitances should be on the same order of magnitude. Present a comparison and calculated percent difference in **slide 10**. If these two results do not agree at all (i.e. they are not on the same order of magnitude), then you likely made a mistake in your MATLAB code, and should go back and correct.

Lightning Rod

- Figure 4 shows a very crude model of a cloud over a grounded lightning rod. Again, assume no variation in the z-direction:

$$\frac{\partial^2 V}{\partial z^2} = 0$$

- Here you are free to choose the size of the cloud and the enclosure as you see fit (the proportions should roughly look like that in Figure 5). Turn in a picture of your geometry with dimensions in **slide 11**.
- Create a MATLAB script that computes and plots the electrostatic potential using the FDT for your chosen geometry. Turn in at least two well-labeled, clear, plots of the electrostatic potential from different angles. Present your plots in **Slides 12 and 13**. Your plots should be clearly labeled and titled. The titles of both plots should include your name in them.
- Expand your MATLAB script so that computes the surface charge density along the bottom edge of the cloud, and present a plot of this surface charge density in **slide 14**. Figure 6 gives an example of the type of plot I'm looking for. Note that in this example plot, the surface charge density magnitude is highest in the middle of the cloud (right above the lightning rod). Your results may be different. Describe the location(s) of the surface charge density maxima for your geometry. Comment on the reasons for these maxima in slide 14 (i.e. why would excess charge accumulate in these certain areas?).
- In **sides 14 – 17,** present your MATLAB code for this lightning rod example.

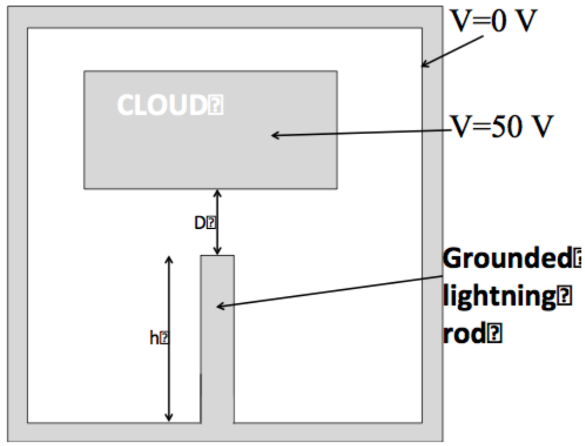

Figure 5 Cloud over grounded lightning rod model

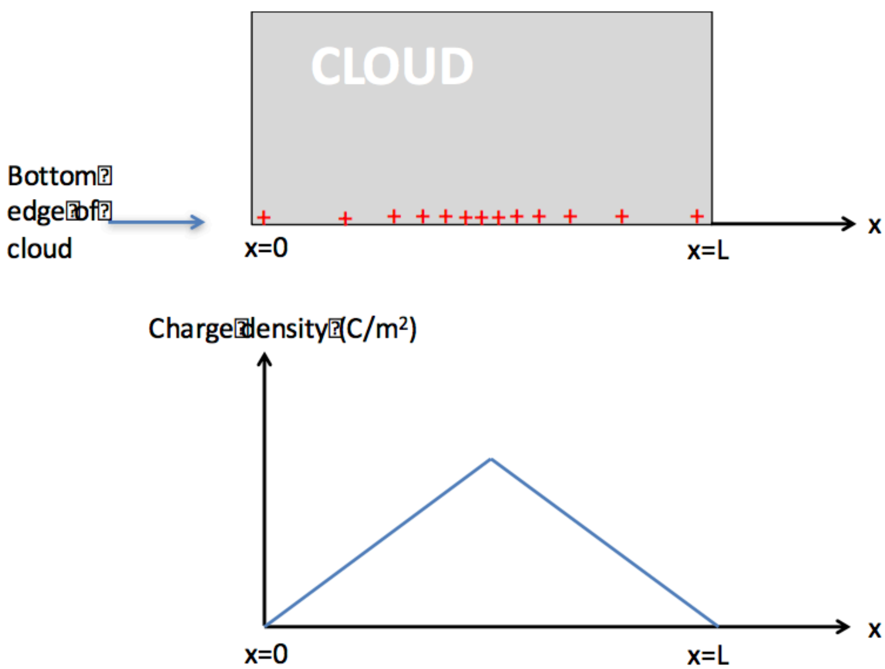

Figure 6 Charge density along bottom edge of cloud (this is an illustrative example, your results by vary).