M 172 Quiz 13/ Quiz 8 (Due Monday July 31st) Name:________________

Instructions: For each problem below, identify the error or state the test/result that is being applied correctly. You do not need to correct the error, but **must** identify why it is an error. The mistakes are all logical errors as opposed to mathmatical errors.

1. Determine if $\sum_{n=1}^{\infty} \frac{1}{n^2-2n+4}$ converges or diverges.

$$\lim_{n\to\infty}\left(\frac{1}{n^2-2n+4}\right) = 0 \implies \sum_{n=1}^{\infty} \frac{1}{n^2-2n+4} \text{ converges}$$

2. Determine if $\sum_{n=1}^{\infty} \frac{n+1}{n^3}$ converges or diverges.

$$0 \le \frac{1}{n^4} \le \frac{n+1}{n^3} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n^4} \quad \text{converges} \implies \sum_{n=1}^{\infty} \frac{n+1}{n^3} \quad \text{converges}$$

3. Find $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$ or state that it diverges.

$$a_n = \frac{1}{n} - \frac{1}{n+1} \implies f(x) = \frac{1}{x} - \frac{1}{x+1}$$

Positive: $\frac{1}{x} - \frac{1}{x+1} \geq 0$ for $x \geq 1$

Continuous: $\frac{1}{x} - \frac{1}{x+1}$ is continuous for $x \geq 1$

Decreasing : $\frac{d}{dx}\left(\frac{1}{x} - \frac{1}{x+1}\right) = -\frac{1}{x^2} + \frac{1}{(x+1)^2} < 0 \implies \frac{1}{x} - \frac{1}{x+1}$ decreasing for $x \geq 1$

$$\int_1^\infty \frac{1}{x} - \frac{1}{x+1} dx = \left(\ln(x) - \ln(x+1)\right)\Big|_1^\infty = \ln\left(\frac{x}{x+1}\right)\Big|_1^\infty = \lim_{x\to\infty}\left(\ln\left(\frac{x}{x+1}\right)\right) - \ln\left(\frac{1}{2}\right)$$

$$= \ln(1) - \left(\ln(1) - \ln(2)\right) = 0 - 0 + \cancel{2 = 2}$$

$\ln(2) = \ln(2)$

$$\implies \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+1}\right) = \cancel{2}\ln(2)$$

4. Determine if $\sum_{n=1}^{\infty} \frac{1}{(\frac{n}{2}-1)^3}$ converges or diverges.

$$a_n = \frac{1}{(\frac{n}{2}-1)^3} \implies \text{choose} \quad b_n = \frac{1}{n^3}$$

$$\lim_{n\to\infty} \left(\frac{b_n}{a_n}\right) = \lim_{n\to\infty} \left(\frac{(\frac{n}{2}-1)^3}{n^3}\right) = \left(\lim_{n\to\infty} \left(\frac{\frac{n}{2}-1}{n}\right)\right)^3 = (2)^3 = 8$$

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \quad \text{converges } (p=3) \implies \sum_{n=1}^{\infty} \frac{1}{(\frac{n}{2}-1)^3} \quad \text{converges}$$

5. Find $\sum_{n=1}^{\infty} \frac{n-1}{n}$ or state that it diverges.

$$\lim_{n\to\infty} \left(\frac{n-1}{n}\right) = 1 \implies \sum_{n=1}^{\infty} \frac{n-1}{n} = 1$$

6. Find $\sum_{n=2}^{\infty} \frac{3^n + 4^n}{\pi^n}$ or state that it diverges.

$$\sum_{n=2}^{\infty} \frac{3^n + 4^n}{\pi^n} = \sum_{n=2}^{\infty} \left(\frac{3}{\pi}\right)^n + \sum_{n=2}^{\infty} \left(\frac{4}{\pi}\right)^n = \frac{\left(\frac{3}{\pi}\right)^2}{1 - \frac{3}{\pi}} + \frac{\left(\frac{4}{\pi}\right)^2}{1 - \frac{4}{\pi}}$$

7. Determine if $\sum_{n=1}^{\infty} \frac{\overset{\cos}{\cancel{\sin}}(n\pi)}{n^5}$ converges or diverges.

$$\frac{\overset{\cos}{\cancel{\sin}}(n\pi)}{n^5} \leq \frac{1}{n^5} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{n^5} \quad \text{converges } (p = 5) \implies \sum_{n=1}^{\infty} \frac{\overset{\cos}{\cancel{\sin}}(n\pi)}{n^5} \quad \text{converges}$$