To receive full credit, you must show your work and use proper notation. Keep everything neat and organized. It could help to box your solutions. Write clearly. Some parts may rely on previous ones.

1. Consider the linear system

$$\begin{aligned} x_1 - 2x_2 - x_3 &= 4 \\ -2x_1 + 4x_2 - 5x_3 &= 6 \end{aligned} \tag{1}$$

(a) Write system (1) as an augmented matrix.

(b) Row reduce the matrix from (a) to reduced echelon form.

(c) Find the solution set to system (1), and write it as a parametric vector equation.

(d) Give a geometric description of the solution set.

(e) Let $\mathbf{u} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$. Find $\mathbf{u} - 3\mathbf{v}$.

(f) Does $\mathbb{R}^2 = \text{Span}\{\mathbf{u}, \mathbf{v}\}$? Justify your answer.

Bonus: Use part (c) to write $\mathbf{b} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$ as a linear combination of $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$.

2. The equation $A\mathbf{x} = \mathbf{b}$ is represented by the augmented matrix

$$\left[\begin{array}{cc|c} 1 & -1 & 8 \\ 0 & 3 & 2 \\ 3 & 0 & 1 \end{array}\right]$$

(a) Write $A\mathbf{x} = \mathbf{b}$ as a vector equation.

(b) Does $\mathbf{b}$ belong to the plane in $\mathbb{R}^3$ spanned by the columns of A? Show why or why not.

(c) Define $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$. Is T an onto mapping? Explain. If not, then give a vector in $\mathbb{R}^3$ that does not belong to the range of T.

3. Determine *by inspection* if the vectors are linearly independent. Give a reason for each answer.

(a) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

4. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = 3x + 2$. Is f a linear transformation? Show why or why not.

5. Let $T \colon \mathbb{R}^3 \to \mathbb{R}^2$ be a linear mapping defined by

$$T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad T(\mathbf{e}_3) = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Find the image of $\mathbf{x} = (x_1, x_2, x_3)$ under T.

Bonus: Let $\mathbf{b} \in \mathbb{R}^2$. Is there a unique solution to $T(\mathbf{x}) = \mathbf{b}$? Justify your answer.

6. Compute $\begin{bmatrix} 1 & 3 & -4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

7. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear mapping that first reflects points through the horizontal x_1-axis, and then reflects points through the line $x_2 = x_1$.

(a) Find the standard matrix of T.

(b) The mapping T is a rotation about the origin. Determine the angle φ of the rotation in radians.

8. Let $A = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$ and $AB = \begin{bmatrix} -3 & -1 \\ 11 & 4 \end{bmatrix}$.

(a) Calculate $\det A$.

(b) Find the inverse of A.

(c) Use A^{-1} to determine the second column of B. (*Hint:* $AB = [A\mathbf{b}_1 \ A\mathbf{b}_2]$)

9. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ -4 & -7 & 3 \\ -2 & -6 & 5 \end{bmatrix}$.