31–34 True–False Determine whether the statement is true or false. Explain your answer.

31. Sequences are functions.

32. If $\{a_n\}$ and $\{b_n\}$ are sequences such that $\{a_n + b_n\}$ converges, then $\{a_n\}$ and $\{b_n\}$ converge.

33. If $\{a_n\}$ diverges, then $a_n \to +\infty$ or $a_n \to -\infty$.

34. If the graph of $y = f(x)$ has a horizontal asymptote as $x \to +\infty$, then the sequence $\{f(n)\}$ converges.

35–36 Use numerical evidence to make a conjecture about the limit of the sequence, and then use the Squeezing Theorem for Sequences (Theorem 9.1.5) to confirm that your conjecture is correct.

35. $\lim\limits_{n\to+\infty} \dfrac{\sin^2 n}{n}$ 36. $\lim\limits_{n\to+\infty} \left(\dfrac{1+n}{2n}\right)^n$

FOCUS ON CONCEPTS

37. Give two examples of sequences, all of whose terms are between -10 and 10, that do not converge. Use graphs of your sequences to explain their properties.

38. (a) Suppose that f satisfies $\lim_{x\to 0^+} f(x) = +\infty$. Is it possible that the sequence $\{f(1/n)\}$ converges? Explain.
(b) Find a function f such that $\lim_{x\to 0^+} f(x)$ does not exist but the sequence $\{f(1/n)\}$ converges.

39. (a) Starting with $n = 1$, write out the first six terms of the sequence $\{a_n\}$, where

$$a_n = \begin{cases} 1, & \text{if } n \text{ is odd} \\ n, & \text{if } n \text{ is even} \end{cases}$$

(b) Starting with $n = 1$, and considering the even and odd terms separately, find a formula for the general term of the sequence

$$1, \frac{1}{2^2}, 3, \frac{1}{2^4}, 5, \frac{1}{2^6}, \ldots$$

(c) Starting with $n = 1$, and considering the even and odd terms separately, find a formula for the general term of the sequence

$$1, \frac{1}{3}, \frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{1}{7}, \frac{1}{7}, \frac{1}{9}, \frac{1}{9}, \ldots$$

(d) Determine whether the sequences in parts (a), (b), and (c) converge. For those that do, find the limit.

40. For what positive values of b does the sequence $b, 0, b^2, 0, b^3, 0, b^4, \ldots$ converge? Justify your answer.

41. Assuming that the sequence given in Formula (2) of this section converges, use the method of Example 10 to show that the limit of this sequence is $\sqrt{a}$.

42. Consider the sequence

$$a_1 = \sqrt{6}$$
$$a_2 = \sqrt{6+\sqrt{6}}$$
$$a_3 = \sqrt{6+\sqrt{6+\sqrt{6}}}$$
$$a_4 = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$$
$$\vdots$$

(a) Find a recursion formula for a_{n+1}.
(b) Assuming that the sequence converges, use the method of Example 10 to find the limit.

43. Each year on his birthday Tim's grandmother deposits \$20 into a "birthday account" that earns an annual interest rate of 10%. The year's interest is added to the account at the same time as that year's deposit.
(a) Let a_n denote the amount in the savings account after n years with $a_0 = 20$ (dollars) the very first deposit on the day of Tim's birth. Find a recursion formula for a_{n+1}. (Assume that no money is ever withdrawn from the account.)
(b) Use the formula from part (a) to determine how much money will be in the account when Tim is 40 years old.

44. Prove that the sequence a_n defined recursively in the preceding exercise is given explicitly by the formula $a_n = 20(11 \cdot (1.1)^n - 10), n \geq 0$.

45. (a) Suppose that a study of coho salmon suggests the population model

$$p_{n+1} = \frac{2.08p_n}{1 + 0.00018p_n}$$

where p_0 =10,000 is the number of salmon at the start of the study and p_n is the population n years later. Based upon this model, what will the population of salmon be 10 years after the start of the study?
(b) Find a value of an initial population $p_0 > 0$ of coho salmon such that, with the recursion formula in part (a), the population remains fixed at p_0 from year to year.

46. (a) Prove that the sequence p_n defined recursively in part (a) of the preceding exercise is given explicitly by

$$p_n = \frac{60{,}000 \cdot (2.08)^n}{10 \cdot (2.08)^n - 4}, \quad n \geq 0$$

(b) Use the result of part (a) to determine $\lim_{n\to+\infty} p_n$.

47. (a) Use a graphing utility to generate the graph of the equation $y = (2^x + 3^x)^{1/x}$, and then use the graph to make a conjecture about the limit of the sequence

$$\{(2^n + 3^n)^{1/n}\}_{n=1}^{+\infty}$$

(b) Confirm your conjecture by calculating the limit.