Homework3: Problem 26

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(1 pt) Find the Laplace transform of

$$f(t) = \begin{cases} 0, & t < 9 \\ (t-9)^5, & t \geq 9 \end{cases}$$

$F(s) =$ ______ .

Homework3: Problem 25

Prev Up Next

(1 pt) Find a solution to the initial value problem

$$y' + \sin(t)y = g(t), \quad y(0) = 6,$$

that is continuous on the interval $[0, 2\pi]$ where

$$g(t) = \begin{cases} \sin(t) & \text{if } 0 \le t \le \pi, \\ -\sin(t) & \text{if } \pi < t \le 2\pi. \end{cases}$$

$$y(t) = \begin{cases} \quad & \text{if } 0 \le t \le \pi, \\ \quad & \text{if } \pi < t \le 2\pi. \end{cases}$$

Homework3: Problem 24

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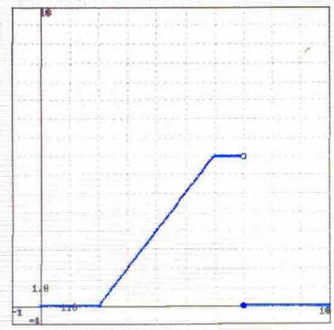
(1 pt) The graph of $f(t)$ is given below:



(Click on graph to enlarge)

a. Represent $f(t)$ using a combination of Heaviside step functions. Use $h(t-a)$ for the Heaviside function shifted a units horizontally.

$f(t) =$ help (formulas)

b. Find the Laplace transform $F(s) = \mathcal{L}\{f(t)\}$ for $s \neq 0$.

$F(s) = \mathcal{L}\{f(t)\} =$ help (formulas)

Homework3: Problem 23

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(1 pt) Find the Laplace transform of

$$f(t) = -2u_5(t) - 3u_8(t) + 2u_9(t)$$

$F(s) =$

Homework3: Problem 22

Prev Up Next

(1 pt) Find the inverse Laplace transform of

$$F(s) = \frac{e^{-8s}}{s^2 + 2s - 8}$$

$f(t) =$. (Use step(t-c) for $u_c(t)$.)

Homework3: Problem 21

Prev Up Next

(1 pt) Find the Laplace transform of

$$f(t) = \begin{cases} 0, & t < 5 \\ t^2 - 10t + 31, & t \geq 5 \end{cases}$$

$F(s) =$ ______ .

Homework3: Problem 20

Prev Up Next

(1 pt) In an integro-differential equation, the unknown dependent variable y appears within an integral, and its derivative dy/dt also appears. Consider the following initial value problem, defined for $t \geq 0$:

$$\frac{dy}{dt} + 25 \int_0^t y(t-w)\, e^{-10w}\, dw = 5, \qquad y(0) = 0.$$

a. Use convolution and Laplace transforms to find the Laplace transform of the solution.

$Y(s) = \mathcal{L}\{y(t)\} =$

b. Obtain the solution $y(t)$.

$y(t) =$

Homework3: Problem 19

Prev Up Next

(1 pt) Consider the following integral equation, so called because the unknown dependent variable, y, appears within an integral:

$$\int_0^t \sin(5(t-w))\, y(w)\, dw = 2t^2.$$

This equation is defined for $t \geq 0$.

a. Use convolution and Laplace transforms to find the Laplace transform of the solution.

$Y(s) = \mathcal{L}\{y(t)\} =$

b. Obtain the solution $y(t)$.

$y(t) =$

Homework3: Problem 18

Prev Up Next

(1 pt) Consider the initial value problem

$$y' + 3y = 45t, \qquad y(0) = 2.$$

a. Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

= help (formulas)

b. Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$

c. Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$

Homework3: Problem 17

Prev Up Next

(1 pt) Consider the initial value problem

$$y'' + 9y = 18t, \qquad y(0) = 7, \quad y'(0) = 5.$$

a. Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

= help (formulas)

b. Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$

c. Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$

Homework3: Problem 16

Prev Up Next

(1 pt) Consider the initial value problem

$$y'' + y = g(t), \qquad y(0) = 0, \quad y'(0) = 0,$$

where $g(t) = \begin{cases} t & \text{if } 0 \le t < 2 \\ 0 & \text{if } 2 \le t < \infty. \end{cases}$

a. Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

= help (formulas)

b. Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$

c. Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$

Homework3: Problem 15

Prev Up Next

(1 pt) Consider the initial value problem

$$y'' + 25y = \cos(5t), \qquad y(0) = 5, \quad y'(0) = 8.$$

a. Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of $y(t)$ by $Y(s)$. Do not move any terms from one side of the equation to the other (until you get to part (b) below).

= help (formulas)

b. Solve your equation for $Y(s)$.

$Y(s) = \mathcal{L}\{y(t)\} =$

c. Take the inverse Laplace transform of both sides of the previous equation to solve for $y(t)$.

$y(t) =$

Homework3: Problem 14

Prev Up Next

(1 pt) Take the Laplace transform of the following initial value and solve for $Y(s) = \mathcal{L}\{y(t)\}$:

$$y'' + 9y = \begin{cases} \sin(\pi t), & 0 \le t < 1 \\ 0, & 1 \le t \end{cases} \qquad y(0) = 0,\ y'(0) = 0$$

$Y(s) =$. Hint: write the right hand side in terms of the Heaviside function.

Now find the inverse transform to find $y(t) =$. (Use step(t-c) for $u_c(t)$.) Note:

$$\frac{\pi}{(s^2+\pi^2)(s^2+9)} = \frac{\pi}{\pi^2-9}\left(\frac{1}{s^2+9} - \frac{1}{s^2+\pi^2}\right)$$

Homework3: Problem 13

Prev Up Next

(1 pt) Use the Laplace transform to solve the following initial value problem:

$$y'' + 8y' = 0 \qquad y(0) = -4, \; y'(0) = 3$$

First, using Y for the Laplace transform of $y(t)$, i.e., $Y = \mathcal{L}\{y(t)\}$,
find the equation you get by taking the Laplace transform of the differential equation

$= 0$

Now solve for $Y(s) =$

and write the above answer in its partial fraction decomposition, $Y(s) = \frac{A}{s+a} + \frac{B}{s+b}$ where $a < b$

$Y(s) =$ +

Now by inverting the transform, find $y(t) =$

Homework3: Problem 12

Prev Up Next

(1 pt) Use the Laplace transform to solve the following initial value problem:

$$y'' - 10y' + 26y = 0 \qquad y(0) = 0,\ y'(0) = 1$$

First, using Y for the Laplace transform of $y(t)$, i.e., $Y = \mathcal{L}\{y(t)\}$,
find the equation you get by taking the Laplace transform of the differential equation

$= 0$

Now solve for $Y(s) =$

By completing the square in the denominator and inverting the transform, find

$y(t) =$

Homework3: Problem 11

Prev Up Next

(1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = \dfrac{8s-27}{s^2-6s+10}$

$$f(t) = \mathcal{L}^{-1}\left\{\frac{8s-27}{s^2-6s+10}\right\} =$$

help (formulas)

Homework3: Problem 10

Prev Up Next

(1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = -\frac{e^{-2s}(5s+8)}{s^2+9}$.

$f(t) = \mathcal{L}^{-1}\left\{-\frac{e^{-2s}(5s+8)}{s^2+9}\right\} =$

help (formulas)

Homework3: Problem 9

Prev Up Next

(1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = -\left(\frac{5}{s^2} + \frac{4}{s+3}\right)$.

$f(t) = \mathcal{L}^{-1}\left\{-\left(\frac{5}{s^2} + \frac{4}{s+3}\right)\right\} =$ help (formulas)

Homework3: Problem 8

Prev Up Next

(1 pt) Find the inverse Laplace transform $f(t) = \mathcal{L}^{-1}\{F(s)\}$ of the function $F(s) = -\frac{5s+2}{s^2+16}$.

$f(t) = \mathcal{L}^{-1}\left\{-\frac{5s+2}{s^2+16}\right\} =$ help (formulas)

Homework3: Problem 7

Prev Up Next

(1 pt) Consider the function $F(s) = \dfrac{2s+7}{s^3 - 3s^2 + 3s - 1}$.

a. Find the partial fraction decomposition of $F(s)$:

$$\frac{2s+7}{s^3 - 3s^2 + 3s - 1} = \boxed{} + \boxed{}$$

b. Find the inverse Laplace transform of $F(s)$.

$f(t) = \mathcal{L}^{-1}\{F(s)\} = \boxed{}$ help (formulas)

(1 pt) Consider the rational function

$$F(s) = \frac{s^3 - 1}{(s^2 + 5)^2 (s + 11)^2}.$$

Select ALL terms below that occur in the general form of the complete partial fraction decomposition of $F(s)$. The capital letters A, B, C, . . . , L denote constants.

- [ ] A. $\frac{A}{s^2 + 5}$
- [ ] B. $\frac{Bs + C}{s^2 + 5}$
- [ ] C. $\frac{Hs + I}{s + 11}$
- [ ] D. $\frac{G}{s + 11}$
- [ ] E. $\frac{J}{(s + 11)^2}$
- [ ] F. $\frac{Es + F}{(s^2 + 5)^2}$
- [ ] G. $\frac{Ks + L}{(s + 11)^2}$
- [ ] H. $\frac{D}{(s^2 + 5)^2}$

Homework3: Problem 5

Prev Up Next

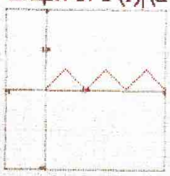
(1 pt) Take the Laplace transform of the following initial value problem and solve for $Y(s) = \mathcal{L}\{y(t)\}$:

$$y'' + 12y' + 16y = T(t) \qquad y(0) = 0,\ y'(0) = 0$$

Where $T(t) = \begin{cases} t, & 0 \le t < 1/2 \\ 1 - t, & 1/2 \le t < 1 \end{cases}, \quad T(t+1) = T(t)$

$Y(s) =$

Graph of $T(t)$ (a triangular wave function):



Homework3: Problem 4

Prev Up Next

(1 pt) Consider the following initial value problem:

$$y'' + 4y = \begin{cases} 4t, & 0 \le t \le 5 \\ 20, & t > 5 \end{cases} \qquad y(0) = 0,\ y'(0) = 0$$

Using Y for the Laplace transform of $y(t)$, i.e., $Y = \mathcal{L}\{y(t)\}$,
find the equation you get by taking the Laplace transform of the differential equation and solve for

$Y(s) =$

Homework3: Problem 3

Prev Up Next

(1 pt) Find the inverse Laplace transform of

$$\frac{5s+7}{s^2+19} \qquad s > 0$$

$y(t) =$ ______ .

Homework3: Problem 2

Prev Up Next

(1 pt) Consider the following initial value problem:

$$y'' - 2y' - 24y = \sin(8t) \qquad y(0) = 6, \; y'(0) = 6$$

Using Y for the Laplace transform of $y(t)$, i.e., $Y = \mathcal{L}\{y(t)\}$,
find the equation you get by taking the Laplace transform of the differential equation and solve for

$Y(s) =$

Homework3: Problem 1

Prev Up Next

(1 pt) Given that

$$\mathcal{L}\{\frac{\cos(4t)}{\sqrt{\pi t}}\} = \frac{e^{-4/s}}{\sqrt{s}}$$

find the Laplace transform of $\sqrt{\frac{t}{\pi}}\cos(4t)$.

$\mathcal{L}\{\sqrt{\frac{t}{\pi}}\cos(4t)\} =$ ______.