MatLab Project Two: due Wednesday, April 26th, 2017 at 7:30 p.m.

MATLAB MODULE FOR NEWTON'S METHOD

Introduction:

Newton's Method, also called the Newton-Raphson Method, is a root-finding algorithm that uses the first few terms of the Taylor Series of a function f(x) in the vicinity of a suspected root. Newton's method is sometimes also known as Newton's Iteration, although in this work the latter term is reserved to the application of Newton's Method for computing square roots.

Note that this material is covered in Section 4.8 of your textbook. We will not cover this material in class directly. You should be able to do this assignment directly, but if you need more background material on this topic, please read section 4.8.

Algorithm: Let f be a differentiable function. Choose a point x_1 near a root of f. Define recursively

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

If the point x_1 is chosen sufficiently close to the root, then the x_n's are successively better approximations of the root. In this module, we are going to explore this algorithm by using MATLAB.

Preliminary notes:

- **Remember that the percent sign % is used to tell MATLAB that these are comments only, and that it should ignore anything after the %. They are included for your benefit, to explain what happens in each line of code.**

- **As with the first assignment, use the % sign to add a comment in your code with your name, course number and date.**

- **Each time you start a MATLAB session, you must declare your symbolic characters (variables). If you only need one, the command is**
  x = sym ('x');

- **If you want to define a function f(x), you cannot use the parentheses in the symbolic character. You can call it just f, or use fx for f(x).**

Applications of Newton's method involve numerical computations.
What follows is an example of Newton's method. The actual exercises that you are to do are at the end.

1. **Finding the** Square Root of 2.

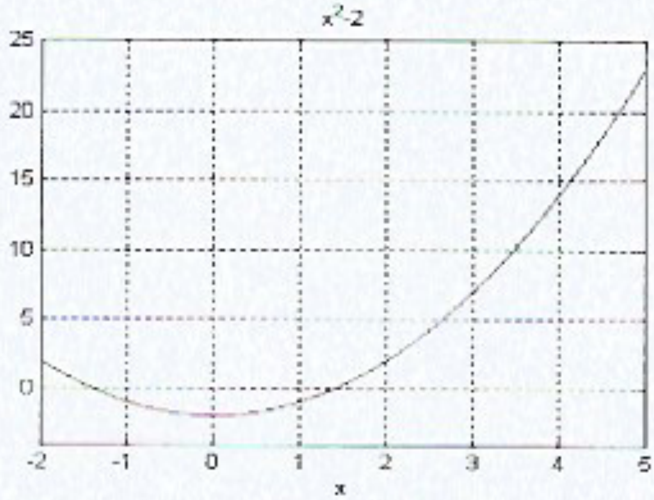
Find the positive root of the equation $g(x) = x^2 - 2 = 0$

```
>> x=sym('x')            % declare our symbolic character

>> f x=x^2-2             % define your function

>> ezplot(fx,[-2,5])     % graphs f(x) like below[-2,5] is x-min and x-max

                         % Notice graph crossing x-axis between 1 and 1.5
```



>> fdx=diff(fx,x)	**% finds derivative for fx** finds f'(x)
>> x1=1	**% x1 holds the value 1**
>> fin=inline(vectorize(fx));	**% creates vectorize function**
>> f1=fin(x1)	**% evaluates function f(x) at x1 value that is f'(1)**
>> din=inline(vectorize(fdx))	**% vectorize derivative**
>> fd1=din(x1)	**% evaluates derivative of f(x) at** x1
>> ntn=inline(vectorize(x-(fx/fdx)))	**% Newton's approximation**
>> x2=ntn(x1)	**% replaces x1 value in Newton's approximation yields x2 value**
>> x3=ntn(x2)	**% replaces x2 value in Newton's approximation yields x3 value**
>> x4=ntn(x3)	**% replaces x3 value in Newton's approximation yields x4 value**
>> x5=ntn(x4)	**% replaces x4 value in Newton's approximation yields x5 value**
>> x5=ntn(x4)	**% replaces x4 value in Newton's approximation yields x5 value**
>> x6=ntn(x5)	**% replaces x5 and yields x6 as you notice it converges to 1.412**

Note: Calculators use this method to compute roots

2. Find the x-coordinate of the point where the curve $y = x^3 - x$ **crosses the horizontal line** $y = 1$.
The curve crosses the line when $x^3-x-1= 0$ or $f(x) = x^3-x-1$ **equal to zero**

>> x=sym ('x')	**% we always start with defining symbolic character**
>> fx = x^3-x-1	**% defines fx function**
>> ezplot(fx,[1 2]);grid on;	**% This graphs fx function with x-min1 and x-max2 in grid as below**

Notice that f (1) = –1 and f (2) = 5 so by intermediate value theorem there exists a value between (1, 2) such that f(x) = 0 so we can start approximation as x1=1

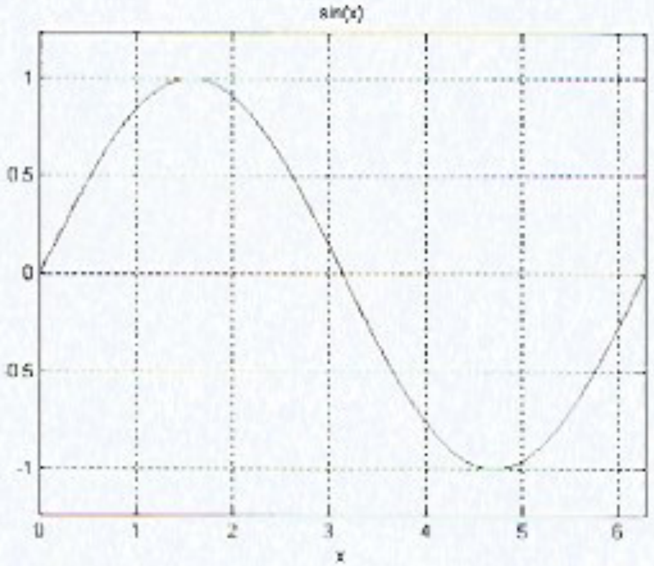
>> fdx=diff(fx,x)	**% computes derivative**
>> x1=1	**% x1 is assigned a value 1**
>> fin=inline(vectorize(fx))	**% vectorize function f(x)**
>> f1=fin(x1)	**% the components of f are evaluated at each in x1**
>> din=inline(vectorize(fdx))	
>> fd1=din(x1)	**% evaluates derivative of f(x) at** x1
>> ntn=inline (vectorize(x-(fx/fdx)))	**% Newton's approximation**
>> x2=ntn(x1)	**% continue process until it converges**

```
>> x3=ntn(x2)
>> x4=ntn(x3)
>> x5=ntn(x4)
>> x6=ntn(x5)                         % you should observe that it converges (approximately) to 1.3247
```

3. Estimating π

We will solve the equation sin (x) = 0 by Newton's method.

```
>> x=sym('x')
>> fx=sin(x)
>> fdx=diff(fx,x)
>> ezplot(fx,[0,2*pi]);grid on;       % you must include the * signs for multiplication
```



As you see in the graph, it crosses the x-axis close to 3. So you can start x1 = 2.5 and proceed with the steps for Newton's method.

```
>> x1=2.5                             % notice these are in radian measure
>> fin=inline(vectorize(fx))          % define fin to vectorize fx
>> f1=fin(x1)
>> din=inline(vectorize(fdx))
>> fd1=din(x1)
>> ntn=inline(vectorize(x-(fx/fdx)))
>> x2=ntn(x1)
>> x3=ntn(x2)
>> x4=ntn(x3)
>> x5=ntn(x4)                         % you will see that it converges at 3.1416.
```

Actual Assignment:

Q1. **Locating a planet**:
To calculate a planet's space coordinates, we have to solve equations like: $x = 1 + 0.5 \sin x$
Print out the graph and commands and results that are relevant.
a. Graph the function $f(x)=x-1-0.5\sin x$ (use ezplot)
b. Identify a value x1 close to the root as the start value.
c. Use Newton's method to estimate the root.

Q2. **Use Intermediate Value Theorem** to show that $f(x) = x^3+2x-4$ has a root between x=1 and x=2. (Use ezplot in MATLAB to plot the graph).
Then use Newton's method to find the root.

Q3. **Approximate $\sqrt{3}$ by Newton's method**.

Q4. **Estimate the value of "pi" by using Newton's method** by solving the equation $\tan(x) = 0$ and x1 = 3.

What to turn in: four separate pages, one for each part, and each includes your name as a comment. You will probably have two additional pages, six in total, including the graphs from Q1 and Q2. Clearly label your work with comments. Make sure that your work concise and clear. Assignments that are difficult to read or follow will receive lower grades.