• X_t *denotes the observed time series,* • n *is the sample size;*
• Z_t *denotes the white noise generating ARMA(p,q) model,*
• $\hat{W}_t$ *denotes the* **rescaled residuals** *obtained after the model is fitted. Recall:*
(i) asymptotically, $\hat{W}_t$ *follow* $WN(0, 1/n)$;
(ii) asymptotically, their acf $\hat{\rho}^2_{\hat{W}}(k) \sim N(0, 1/n)$;
(iii) asymptotically, the χ^2*-statistic (also called Box-Pierce statistic)* $Q_W = n\sum_{j=1}^{h} \hat{\rho}^2_{\hat{W}}(j)$ *follows* $\chi^2(h-p-q)$ *distribution.*

1. The statistical consultant analyzed a time series of 50 observations and proposed a model of the form: $X_t - \mu = Z_t - \theta Z_{t-1}$. The consultant believes that $\theta > 0.9$. Is the following statement compatible with the proposed model?

$$\sum_{k=1}^{5} \hat{\rho}^2_{\hat{W}}(k) = 0.005$$

2. You fit 100 observations, using ARMA$(1,2)$ model. For the 100 residuals, you have determined the first seven autocorrelation coefficients:

k	1	2	3	4	5	6	7
$\hat{\rho}_{\hat{W}}(k)$	.20	.15	-.18	.16	.08	.07	.09

Calculate the value of the χ^2 statistics and determine the number of degrees of freedom.

4. In modeling the weekly sales of a certain commodity over the past six months, the time series model $X_t - \phi_1 X_{t-1} = Z_t + \theta_1 Z_{t-1}$ was thought to be appropriate. Suppose the model was fitted and the autocorrelations of the residuals were:

k	1	2	3	4	5	6	7	8
$\hat{\rho}_{\hat{W}}(k)$	.50	-.04	.03	-.01	.01	.02	.03	-.01
st. dev $\hat{\rho}_{\hat{W}}(k)$	.08	.10	.11	.11	.11	.11	.11	.11

Is the assumed model really appropriate? If not, how would you modify the model? Explain.