EXPERIMENT 6

Conservation of Momentum in Collisions

Equipment

- Air table apparatus
- Pair of Velcro rings
- 2 large plain paper sheets
- 2 identical clear plastic triangles
- Clear plastic protractor
- Clear plastic ruler

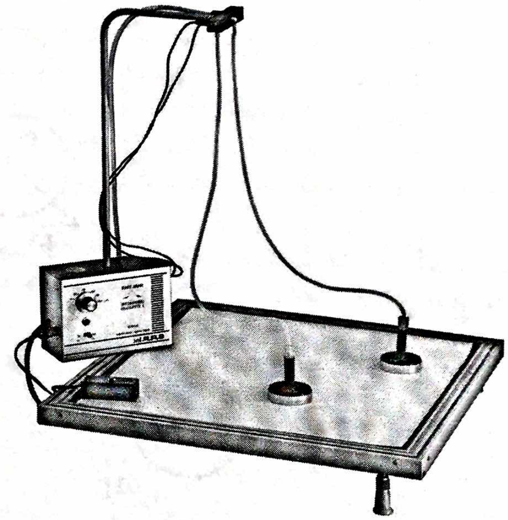

Figure 6.1 – Experimental Apparatus

Objectives

1. To measure, by graphical means, the changes of velocity in elastic and inelastic collisions.
2. To trace the motion of the center of mass of a two-body system.

Introduction

According to Newton's third law, forces come in pairs of equal magnitude and opposite direction. When a tennis racket pushes on a tennis ball, the ball pushes back on the racket with a force of equal magnitude at every instant that they are in contact. The third law has logical consequences that can be tested easily. Consider two masses m_1 and m_2 that are colliding with each other on a frictionless table (Fig. 6.2). Since the only forces on the masses are the forces they exert on each other and since these forces have, at any instant, the same magnitude and opposite direction, we have that at every instant

$$m_1\vec{a}_1 = -m_2\vec{a}_2 \tag{6.1}$$

or

$$m_1\frac{\Delta\vec{v}_1}{\Delta t} = -m_2\frac{\Delta\vec{v}_2}{\Delta t} \tag{6.2}$$

where Δt is a short time interval and $\Delta\vec{v}_i$ is the change in velocity of mass m_i during that interval. Thus for any short time interval during the collision we have

$$m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2 \tag{6.3}$$

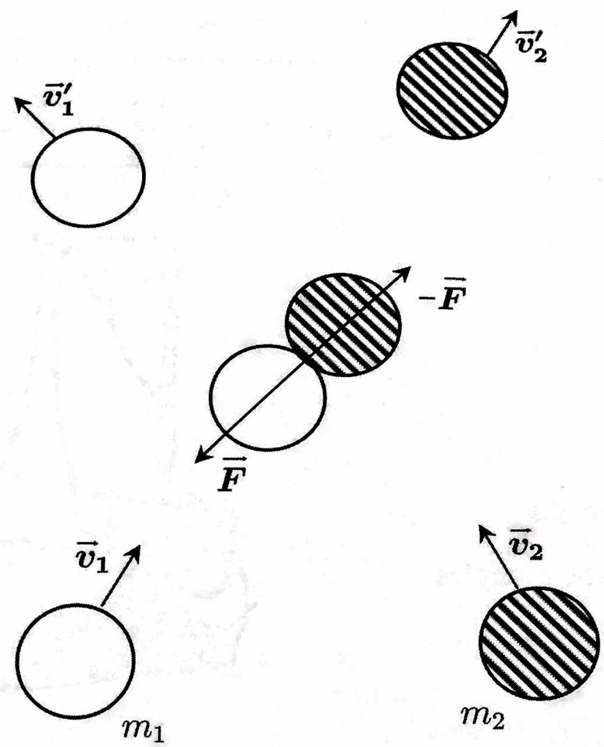

Figure 6.2 – When two masses collide, they exert forces of equal magnitude and opposite direction on each other.

which can be written

$$m_1\left(\vec{v}_1(t+\Delta t)-\vec{v}_1(t)\right)=-m_2\left(\vec{v}_2(t+\Delta t)-\vec{v}_2(t)\right) \tag{6.4}$$

or, rearranging terms,

$$m_1\vec{v}_1'+m_2\vec{v}_2'=m_1\vec{v}_1+m_2\vec{v}_2 \tag{6.5}$$

where the primed velocities ($\vec{v}_1'$ and $\vec{v}_2'$) are a short time later than the unprimed velocities ($\vec{v}_1$ and $\vec{v}_2$).

The **momentum** of a mass m is the product of mass m and velocity $\vec{v}$

$$\vec{p}=m\vec{v}\,. \tag{6.6}$$

In terms of momenta, Eq. (6.5) can be written

$$\vec{p}_1'+\vec{p}_2'=\vec{p}_1+\vec{p}_2\,. \tag{6.7}$$

This says that the sum of the momenta of two masses at time t is equal to the sum at time $t+\Delta t$. Up to now we have assumed that Δt is very small. But if the sum of the momenta at t is equal to the sum of the momenta at $t+\Delta t$, then the sum of the momenta at $t=t+\Delta t$ is equal to the sum of the momenta at $t+2\Delta t$, so the sum of the momenta at t is equal to the sum of the momenta at $t+2\Delta t$.

Continuing this argument, the sum of the momenta does not change; at any one time the sum is equal to the sum at any other time. This important result, which is a deduction from Newton's third law, is called the **conservation of momentum**.

To repeat: if the vector sum of all external forces on a system of masses add up to zero (i.e. there is no net external force), the total momentum of the system of masses does not change. Through collisions, the momenta of the individual masses of the system may change, but always in such a way as to keep the total momentum of the system of masses unchanged.

Collisions have consequences not only for the momenta of the system of masses involved, but also for their energies. During collisions, the kinetic energy of the system may stay constant (elastic collisions), or it may change (inelastic collisions, during which part of the kinetic energy may be converted to heat). The system of particles behaves differently depending on whether a collision is elastic or inelastic. This experiment will explore different situations of pucks colliding with each other, or moving while coupled together.

The air table apparatus has been described previously (in Experiments 3 and 4). Just a reminder: **don't touch the apparatus while the spark timer remote is pressed down, or you may receive a nasty shock.**

Investigation 1

Elastic Collision

1. Your air table will have a sheet of black, carbonized paper on top, which conducts electricity very well. Put a large sheet of plain white paper over the carbonized paper, then place the pucks on top of the white paper face.

How does this apparatus work? Take a look at the bottom of the pucks. Note the sharp points of the central electrodes, next to the air holes. The points are connected to thin conducting chains inside the air hoses that carry electricity from the spark generator to the pucks. When the spark generator is activated, high voltage sparks periodically close a circuit from the pucks through the air gap and the white paper to the conducting carbon paper. The sparks sputter carbon marks onto the underside of the white paper, producing traces of the puck motion.

2. Turn the air on, so that the pucks ride on air cushions, and level the air table. The spark time should be turned off and the Velcro rings around the pucks should be removed. Measure the masses m_1 and m_2 of the two pucks and record the values in your spreadsheet.

3. With the two pucks in *adjacent* corners of the air table, practice launching them so that they collide near the middle of the table and travel toward the other two corners. This will take some time. A good launch will result in traces that are easy to analyze.

Do not touch the pucks or the air table when the remote button is pressed.

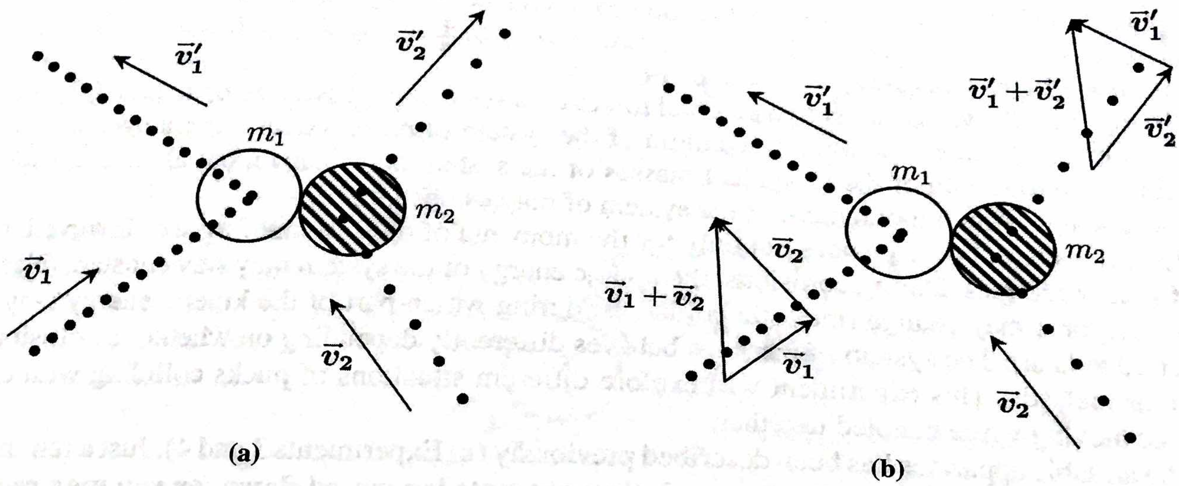

Figure 6.3 – (a) Tracks of two colliding pucks. The velocities of the two pucks before and after the collision are also shown. (b) Same picture as (a) except the vector addition of the velocities of the pucks before and after the collision is included. For a perfectly elastic collision, $\boldsymbol{v_1} + \boldsymbol{v_2}$ should be the same as $\boldsymbol{v'_1} + \boldsymbol{v'_2}$ in both magnitude and direction.

4. For your data run, turn the spark timer on and set it to $30\,ms$ or $30\,Hz$ (there is a difference!), depending on the model. Press the remote button immediately after launching the pucks and release it just before the pucks reach the opposite corners.

5. Remove the white paper and lay it on the lab bench – the side which shows the sparked dots most clearly should face up. Identify the tracks of the motion of the pucks before and after the collision, as shown in Fig. 6.3a (it may help to label these tracks as 1, 2, 1′ and 2′, where the *prime* indicates the motion *after* the collision). Label the individual sparks along the four tracks, starting with 0 at the beginning of each track. For efficiency, label every fourth spark only (0, 4, 8,... etc.).

6. On track 1, use a ruler to draw the velocity vector $\vec{v}_1$ between, say, points 4 and 12. Similarly, draw vectors $\vec{v}_2$, $\vec{v}'_1$ and $\vec{v}'_2$ on the remaining three tracks. You **must** draw each velocity vector across the same number of sparks, that is, over the same length of time. Measure and record the magnitudes of the velocities by dividing the lengths of the corresponding tracks by the time taken between the points. (Remember the time interval between adjacent dots is $30\,ms$, or $33.33\,ms$ if you used $30\,Hz$.)

7. The masses of the two pucks are equal, so Eq. (6.5) becomes

$$\vec{v}'_1 + \vec{v}'_2 = \vec{v}_1 + \vec{v}_2 \,. \tag{6.8}$$

Find $\vec{v}_1 + \vec{v}_2$ and $\vec{v}'_1 + \vec{v}'_2$ by graphically adding the velocities directly on the tracking paper (Fig. 6.3b). Among your equipment should be two plastic triangles. Their purpose is to move lines in parallel from one place to another on the paper. Use the triangles to construct the vector sums shown in the figure. Estimate the errors in these vectors.

8. According to Eq. (6.8), the two sums should be equal and the *difference vector* $(\vec{v}_1' + \vec{v}_2') - (\vec{v}_1 + \vec{v}_2)$ should be zero. Because of friction, the force exerted by the rubber hoses and the fact that no collision can be made perfectly elastic, the measured difference vector will not be zero, but it will be small. Find the magnitude and direction of the difference vector graphically. Divide the magnitude of the difference vector by the magnitude of the initial vector sum $\vec{v}_1 + \vec{v}_2$. For example, if $\vec{v}_1 + \vec{v}_2$ is a vector 12.3 cm long and $(\vec{v}_1' + \vec{v}_2') - (\vec{v}_1 + \vec{v}_2)$ is a vector 0.8 cm long, the ratio 0.8/12.3 = 0.065 tells us that the experiment found a 6.5% difference between the initial and final momentum. This is called the *percent change* in momentum.

In a perfectly elastic collision, the sum of the kinetic energies $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ is the same before and after the collision. Therefore, both energy and momentum are conserved. Although collisions of solid objects are not perfectly elastic, collisions of hard objects are approximately elastic.

9. Calculate the sum of the kinetic energies *before* the collision, K, and calculate its error. Calculate the sum of the kinetic energies *after* the collision, K', and calculate its error. Is kinetic energy conserved within your uncertainty?

10. Calculate the percent change in kinetic energy,

$$\%\ change = \frac{K' - K}{K} \times 100\ .$$

Investigation 2

Inelastic Collision

In an inelastic collision, some of the energy is used up by friction, deformation of colliding objects, etc. Energy is not conserved in inelastic collisions! However, and this is very important, **momentum is still conserved.**

1. Be sure the air table is level.

2. Place the white paper back on the air table, with your first run face up. Your second run will be recorded on the other side. Wrap Velcro strips around the two pucks and repeat the procedure of Investigation 1. (Be sure to take a couple of practice runs before taking a data run.) In this case, because of the Velcro, the two pucks stick together after they collide. Collisions in which the two masses stick together are called completely inelastic. These collisions have the greatest loss of kinetic energy.

3. Analyze your data as you did in Investigation 1, finding the percent changes in total momentum and total kinetic energy in this collision. Is the kinetic energy conserved within your uncertainty? How much energy is dissipated during the collision?

Investigation 3

Motion of the Center of Mass

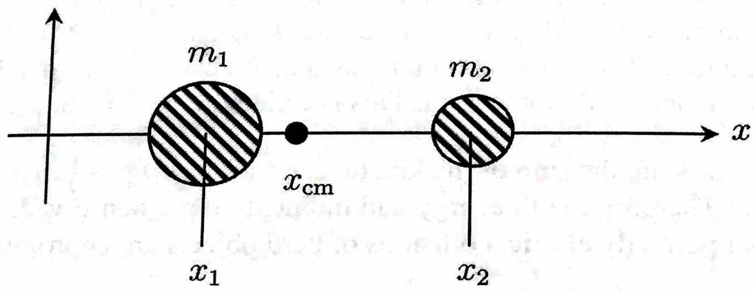

Figure 6.4 – The position x_{cm} of the center of mass of two masses m_1 and m_2 at points x_1 and x_2 on the x-axis. If the two masses are equal, the point x_{cm} will be halfway between x_1 and x_2.

The center of mass of two masses m_1 and m_2 at points x_1 and x_2 on the x-axis (Fig. 6.4) is a mathematical point x_{cm} given by

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{M} \tag{6.9}$$

where $M = m_1 + m_2$. The two masses m_1 and m_2 are not necessarily equal. (In this experiment, of course, they are.) By differentiating this expression with respect to time, one finds that the point x_{cm} moves with velocity v_{cm} given by

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{M} \tag{6.10}$$

or

$$M v_{cm} = m_1 v_1 + m_2 v_2 \tag{6.11}$$

where v_1 and v_2 are the velocities of m_1 and m_2 respectively. For masses moving in a 2D plane, this last expression becomes the vector relation

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{p}_1 + \vec{p}_2 \tag{6.12}$$

which says that the total momentum of the two masses is equal to the momentum of the total mass M moving with the velocity of the center of mass.

This is just a mathematical rewriting of the definitions of velocity and momentum. However, it has important applications – in situations where there are no external forces, the total momentum is conserved and $\vec{v}_{cm}$ is a constant. This means that the center of mass moves along serenely, undisturbed by interactions between the pucks. The forces between the pucks are forces internal to the system, not external. Using the two pucks as the masses, you will do a simple experiment to demonstrate this point.

1. Place a fresh sheet of paper over the carbonized paper on the air table. Stick the two pucks together with Velcro and practice launching them with a spin, so that they make approximately one complete revolution around each other as they slide across the table. Be sure the air table is level. Set the spark timer to 50 ms or 20 Hz and take a data run of the rotating pucks.

2. There should be a complex pattern of dots on the underside of the white paper, because each puck is moving in a sliding circle. However, because there is no external force on the system, the center of mass should move in a straight line at constant speed. For equal masses ($m_1 = m_2$) Eq. (6.9) shows that

$$x_{\text{cm}} = \frac{x_1 + x_2}{2} \tag{6.13}$$

that is, the center of mass is midway between the two masses.

3. Number the sparks made by each puck as you did earlier, starting with 0 for the first spark made on each track. Connect the initial 0 points of each track with a straight line and mark the midpoint of this line. The midpoint is the location of the center of mass of the pucks at this time. Do this for every fourth point (4, 8, 12,... etc.).

Note: *The speed at which your pucks were moving across the paper will influence the numbers of points made by the spark timer. If you get less than 7 center of mass points, you may want to mark and connect every third pair of sparks, instead of every fourth.*

4. Draw a best-fit straight line through the center of mass points obtained in step 3. Measure and record the angle that each line connecting corresponding sparks makes with the center of mass line. (Note: the angle measurements should either increase or decrease steadily with time. If this is not evident in your data, check that you are consistently measuring the *same* angle each time!) Plot these angles against time, putting the first line at $t = 0$. Add a best-fit line and find the slope.

5. Describe and discuss the results of this investigation. What is the physical meaning of the slope of your plot? What quantitative information does the slope give about the rotation of the two puck system? What did you find about the motion of the center of mass?

Questions

1. Compare the energy loss in the completely inelastic case (Investigation 2) to the approximately elastic case (Investigation 1)? Which collision demonstrated a greater energy loss? Do your results agree with theory?

2. Do the centers of mass in Investigation 3 lie on a straight line? Explain *why* they do or do not. Are the distances between adjacent points constant? Explain!

3. Do the points on your plot of angle vs. time in Investigation 3 lie along a straight line? Explain.

4. If there were no external forces acting on the two pucks, their complex motion could be described as the combination of the uniform linear motion of the center of mass and a uniform circular motion of the pucks about the center of mass. Describe how well your results agree with this expectation, and explain any deviations that you observe from the predicted behavior.

5. In Investigation 3, are the momenta of each puck conserved? Explain.

Additional Questions for Honors Sections

1. Imagine that the Acela Train moving with velocity $v = 100\ mph$ collides with a stationary basketball of mass $m = 600\ g$. Calculate how much energy (in Joules) will be converted into heat if the collision is completely inelastic. What is the velocity of the ball after the impact if the collision is fully elastic?

2. Suppose that in your 1st Investigation the puck moving with v_1 hits a stationary puck $v_2 = 0$ off-center. Calculate the angle between velocity vectors v_1' and v_2'. (Hints: i) the angle is always the same; ii) consider momentum conservation together with energy conservation laws and apply the Pythagoras Theorem.)

3. Imagine that in your 2nd Investigation the Velcro bands are removed and the pucks are instead connected with an elastic thread. Will the collision remain inelastic? Would such replacement change the results of your 3rd Investigation?

4. An object (a star or a fireworks petard) explodes with a tremendous energy that exceeds its kinetic energy and breaks into many pieces. Is it possible that, after the explosion, all pieces will be flying away from the observer? Imagine that the same object breaks into just two pieces and one is much lighter than the other. Which of the two will have a much higher velocity? Which one will have a much higher kinetic energy?