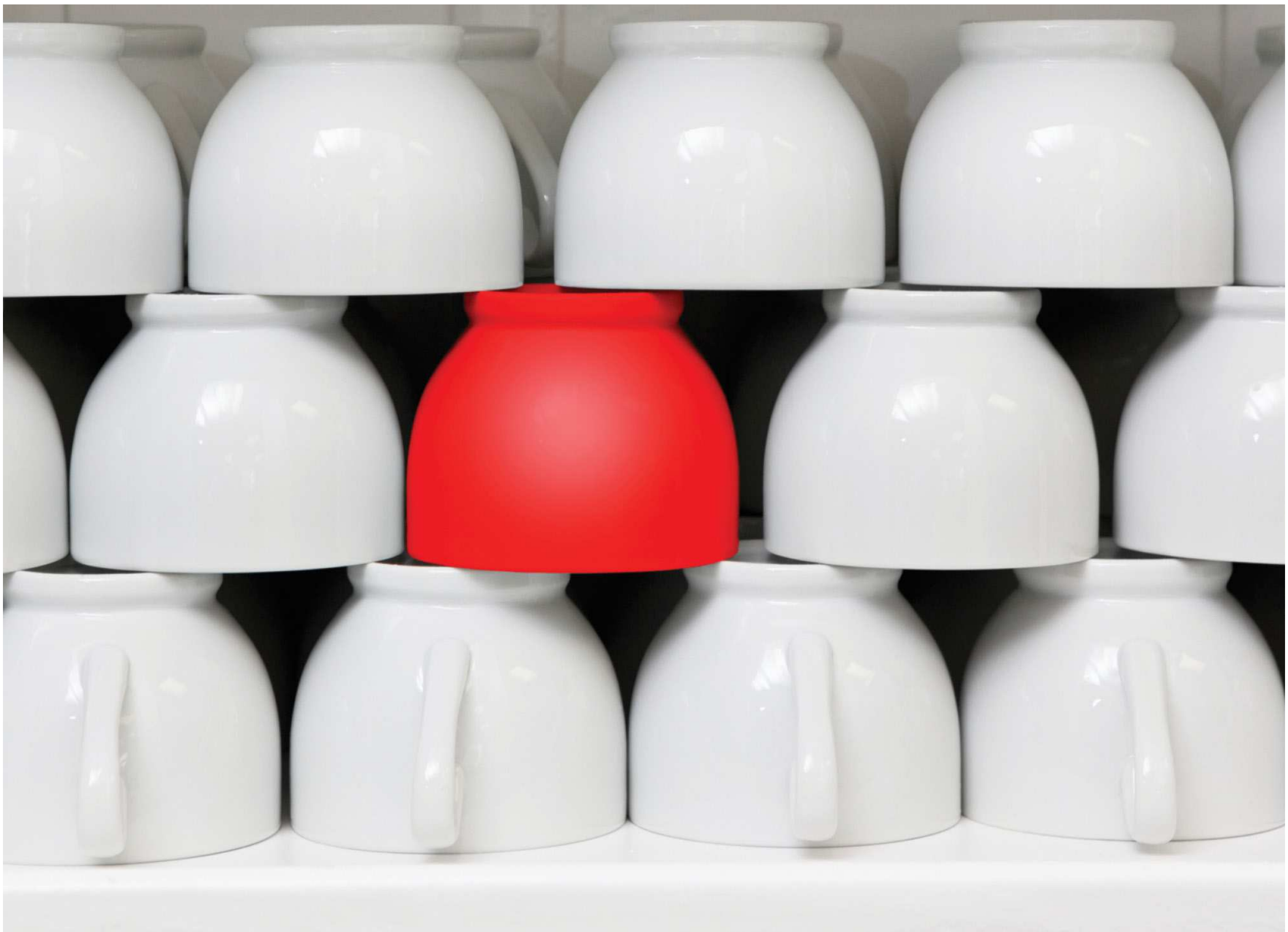
Deductive Reasoning 3



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Learning Objectives

After reading this chapter, you should be able to:

1. Define basic key terms and concepts within deductive reasoning.
2. Use variables to represent an argument's logical form.
3. Use the counterexample method to evaluate an argument's validity.
4. Categorize different types of deductive arguments.
5. Analyze the various statements—and the relationships between them—in categorical arguments.
6. Evaluate categorical syllogisms using the rules of the syllogism and Venn diagrams.
7. Differentiate between sorites and enthymemes.

By now you should be familiar with how the field of logic views arguments: An argument is just a collection of sentences, one of which is the conclusion and the rest of which, the premises, provide support for the conclusion. You have also learned that not every collection of sentences is an argument. Stories, explanations, questions, and debates are not arguments, for example. The essential feature of an argument is that the premises *support*, *prove*, or *give evidence for* the conclusion. This relationship of support is what makes a collection of sentences an argument and is the special concern of logic. For the next four chapters, we will be taking a closer look at the ways in which premises might support a conclusion. This chapter discusses deductive reasoning, with a specific focus on categorical logic.

3.1 Basic Concepts in Deductive Reasoning

As noted in Chapter 2, at the broadest level there are two types of arguments: deductive and inductive. The difference between these types is largely a matter of the strength of the connection between premises and conclusion. Inductive arguments are defined and discussed in Chapter 5; this chapter focuses on deductive arguments. In this section we will learn about three central concepts: validity, soundness, and deduction.

Validity

Deductive arguments aim to achieve *validity*, which is an extremely strong connection between the premises and the conclusion. In logic, the word *valid* is only applied to arguments; therefore, when the concept of validity is discussed in this text, it is solely in reference to arguments, and not to claims, points, or positions. Those expressions may have other uses in other fields, but in logic, validity is a strict notion that has to do with the strength of the connection between an argument's premises and conclusion.

To reiterate, an argument is a collection of sentences, one of which (the conclusion) is supposed to follow from the others (the premises). A **valid** argument is one in which the truth of the premises absolutely guarantees the truth of the conclusion; in other words, it is an argument in which it is *impossible* for the premises to be true while the conclusion is false. Notice that the definition of *valid* does not say anything about whether the premises are actually true, just whether the conclusion could be false *if* the premises were true. As an example, here is a silly but valid argument:

> Everything made of cheese is tasty.
> The moon is made of cheese.
> Therefore, the moon is tasty.

No one, we hope, actually thinks that the moon is made of cheese. You may or may not agree that everything made of cheese is tasty. But you can see that *if* everything made of cheese were tasty, and *if* the moon were made of cheese, then the moon would *have to* be tasty. The truth of that conclusion simply logically follows from the truth of the premises.

Here is another way to better understand the strictness of the concept of validity: You have probably seen some far-fetched movies or read some bizarre books at some point. Books and movies have magic, weird science fiction, hallucinations, and dream sequences—almost anything can happen. Imagine that you were writing a weird, bizarre novel, a novel as far removed from reality as possible. You certainly could write a novel in which the moon was made of cheese. You could write a novel in which everything made of cheese was tasty. But you could *not* write a novel in which both of these premises were true, but in which the moon turned out not to be tasty. If the moon were made of cheese but was not tasty, then there would be at least one thing that was made of cheese and was not tasty, making the first premise false.

Therefore, if we assume, even hypothetically, that the premises are true (even in strange hypothetical scenarios), it logically follows that the conclusion must be as well. Therefore, the argument is *valid*. So when thinking about whether an argument is valid, think about whether it would be possible to have a movie in which all the premises were true but the conclusion was false. If it is not possible, then the argument is valid.

> Here is another, more realistic, example:
>
> All whales are mammals.
> All mammals breathe air.
> Therefore, all whales breathe air.

Is it possible for the premises to be true and the conclusion false? Well, imagine that the conclusion is false. In that case there must be at least one whale that does not breathe air. Let us call that whale Fred. Is Fred a mammal? If he is, then there is at least one mammal that does not breathe air, so the second premise would be false. If he isn't, then there is at least one whale that is *not* a mammal, so the first premise would be false. Again, we see that it is impossible for the conclusion to be false and still have all the premises be true. Therefore, the argument is valid.

> Here is an example of an invalid argument:
>
> All whales are mammals.
> No whales live on land.
> Therefore, no mammals live on land.

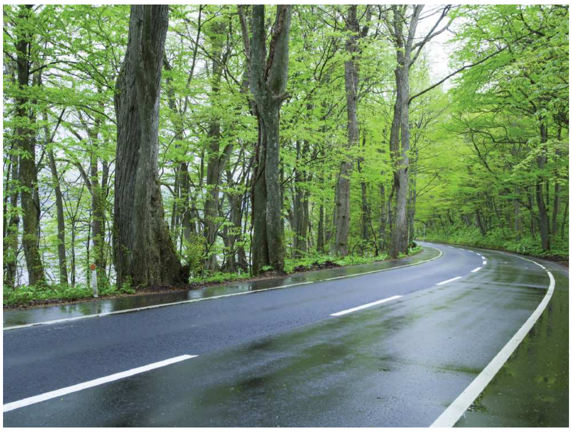
In this case we can tell that the truth of the conclusion is not guaranteed by the premises because the premises are actually true and the conclusion is actually false. Because a valid argument means that it is impossible for the premises to be true and the conclusion false, we can be sure that an argument in which the premises are *actually* true and the conclusion is *actually* false must be invalid. Here is a trickier example of the same principle:

All whales are mammals.
Some mammals live in the water.
Therefore, some whales live in the water.

This one is trickier because both premises are true, and the conclusion is true as well, so many people may be tempted to call it valid. However, what is important is not whether the premises and conclusion are *actually* true but whether the premises *guarantee* that the conclusion is true. Think about making a movie: Could you make a movie that made this argument's premises true and the conclusion false?

Suppose you make a movie that is set in a future in which whales move back onto land. It would be weird, but not any weirder than other ideas movies have presented. If seals still lived in the water in this movie, then both premises would be true, but the conclusion would be false, because all the whales would live on land.

Because we can create a scenario in which the premises are true and the conclusion is false, it follows that the argument is invalid. So even though the conclusion isn't actually false, it's enough that it is possible for it to be false in some situation that would make the premises true. This mere possibility means the argument is invalid.



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Consider the following argument: "If it is raining, then the streets are wet. The streets are wet. Therefore, it is raining." Is this a valid argument? Could there be another reason why the road is wet?

Soundness

Once you understand what *valid* means in logic, it is very easy to understand the concept of *soundness*. A **sound** argument is just a valid argument in which all the premises are true. In defining *validity*, we saw two examples of valid arguments; one of them was sound and the other was not. Since both examples were valid, the one with true premises was the one that was sound.

We also saw two examples of invalid arguments. Both of those are unsound simply because they are invalid. Sound arguments have to be valid *and* have all true premises. Notice that since only arguments can be valid, only arguments can be sound. In logic, the concept of soundness is not applied to principles, observations, or anything else. The word *sound* in logic is only applied to arguments.

Here is an example of a sound argument, similar to one you may recall seeing in Chapter 2:

All men are mortal.
Bill Gates is a man.
Therefore, Bill Gates is mortal.

There is no question about the argument's validity. Therefore, as long as these premises are true, it follows that the conclusion must be true as well. Since the premises are, in fact, true, we can reason the conclusion is too.

It is important to note that having a true conclusion is not part of the definition of soundness. If we were required to know that the conclusion was true before deciding whether the argument is sound, then we could never use a sound argument to discover the truth of the conclusion; we would already have to know that the conclusion was true before we could judge it to be sound. The magic of how deductive reasoning works is that we can judge whether the reasoning is valid independent of whether we know that the premises or conclusion are actually true. If we also notice that the premises are all true, then we may infer, by the power of pure reasoning, the truth of the conclusion.

Therefore, knowledge of the truth of the premises and the ability to reason validly enable us to arrive at some new information: that the conclusion is true as well. This is the main way that logic can add to our bank of knowledge.

Although soundness is central in considering whether to accept an argument's conclusion, we will not spend much time worrying about it in this book. This is because logic really deals with the connections between sentences rather than the truth of the sentences themselves. If someone presents you with an argument about biology, a logician can help you see whether the argument is valid—but you will need a biologist to tell you whether the premises are true. The truth of the premises themselves, therefore, is not usually a matter of logic. Because the premises can come from any field, there would be no way for logic alone to determine whether such premises are true or false. The role of logic—specifically, deductive reasoning—is to determine whether the reasoning used is valid.

Deduction

You have likely heard the term *deduction* used in other contexts: As Chapter 2 noted, the detective Sherlock Holmes (and others) uses *deduction* to refer to any process by which we infer a conclusion from pieces of evidence. In rhetoric classes and other places, you may hear *deduction* used to refer to the process of reasoning from general principles to a specific conclusion. These are all acceptable uses of the term in their respective contexts, but they do not reflect how the concept is defined in logic.

In logic, deduction is a technical term. Whatever other meanings the word may have in other contexts, in logic, it has only one meaning: A **deductive argument** is one that is presented as being valid. In other words, a deductive argument is one that is *trying* to be valid. If an argument is presented as though it is supposed to be valid, then we may infer it is deductive. If an argument is deductive, then the argument can be evaluated in part on whether it is, in fact, valid. A deductive argument that is not found to be valid has failed in its purpose of demonstrating its conclusion to be true.

In Chapters 5 and 6, we will look at arguments that are *not* trying to be valid. Those are inductive arguments. As noted in Chapter 2, inductive arguments simply attempt to establish their conclusion as probable—not as absolutely guaranteed. Thus, it is not important to assess whether inductive arguments are valid, since validity is not the goal. However, if a deductive argument is not valid, then it has failed in its goal; therefore, for deductive reasoning, validity is a primary concern.

Consider someone arguing as follows:

> All donuts have added sugar.
> All donuts are bad for you.
> Therefore, everything with added sugar is bad for you.

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Interpreting the intention of the person making an argument is a key step in determining whether the argument is deductive.

Even though the argument is invalid—exactly why this is so will be clearer in the next section—it seems clear that the person thinks it is valid. She is not merely suggesting that maybe things with added sugar might be bad for you. Rather, she is presenting the reasoning as though the premises guarantee the truth of the conclusion. Therefore, it appears to be an attempt at deductive reasoning, even though this one happens to be invalid.

Because our definition of validity depends on understanding the author's intention, this means that deciding whether something is a deductive argument requires a bit of interpretation—we have to figure out what the person giving the argument is trying to do. As noted briefly in Chapter 2, we ought to seek to provide the most favorable possible interpretation of the author's intended reasoning. Once we know that an argument is deductive, the next question in evaluating it is whether it is valid. If it is deductive but not valid, we really do not need to consider anything further; the argument fails to demonstrate the truth of its conclusion in the intended sense.

Practice Problems 3.1

Examine the following arguments. Then determine whether they are deductive arguments or not. Click here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems3.1.pdf) to check your answers.

1. Charles is hard to work with, since he always interrupts others. Therefore, I do not want to work with Charles in the development committee.
2. No physical object can travel faster than light. An electron is a physical object. So an electron cannot travel faster than light.
3. The study of philosophy makes your soul more slender, healthy, and beautiful. You should study philosophy.

4. We should go to the beach today. It's sunny. The dolphins are out, and I have a bottle of fine wine.
5. Triangle A is congruent to triangle B. Triangle A is an equilateral triangle. Therefore, triangle B is an equilateral triangle.
6. The farmers in Poland have produced more than 500 bushels of wheat a year on average for the past 10 years. This year they will produce more than 500 bushels of wheat.
7. No dogs are fish. Some guppies are fish. Therefore, some guppies are not dogs.
8. Paying people to mow your lawn is not a good policy. When people mow their own lawns, they create self-discipline. In addition, they are able to save a lot of money over time.
9. If Mount Roosevelt was completed in 1940, then it's only 73 years old. Mount Roosevelt is not 73 years old. Therefore, Mount Roosevelt was not completed in 1940.
10. You're either with me, or you're against me. You're not with me. Therefore, you're against me.
11. The worldwide use of oil is projected to increase by 33% over the next 5 years. However, reserves of oil are dwindling at a rapid rate. That means that the price of oil will drastically increase over the next 5 years.
12. A nation is only as great as its people. The people are reliant on their leaders. Leaders create the laws in which all people can flourish. If those laws are not created well, the people will suffer. This is why the people of the United States are currently suffering.
13. If we save up money for a house, then we will have a place to stay with our children. However, we haven't saved up any money for a house. Therefore, we won't have a place to stay with our children.
14. We have to focus all of our efforts on marketing because right now; we don't have any idea of who our customers are.
15. Walking is great exercise. When people exercise they are happier and they feel better about themselves. I'm going to start walking 4 miles every day.
16. Because all libertarians believe in more individual freedom, all people who believe in individual freedom are libertarians.
17. Our dogs are extremely sick. I have to work every day this week, and our house is a mess. There's no way I'm having my family over for Festivus.
18. Pigs are smarter than dogs. Animals that are easier to train are smarter than other animals. Pigs are easier to train than dogs.
19. Seventy percent of the students at this university come from upper class families. The school budget has taken a hit since the economic downturn. We need funding for the three new buildings on campus. I think it's time for us to start a phone campaign to raise funds so that we don't plunge into bankruptcy.
20. If she wanted me to buy her a drink, she would've looked over at me. But she never looked over at me. So that means that she doesn't want me to buy her a drink.

3.2 Evaluating Deductive Arguments

If validity is so critical in evaluating deductive arguments, how do we go about determining whether an argument is valid or invalid? In deductive reasoning, the key is to look at the pattern of an argument , which is called its **logical form**. As an example, see if you can tell whether the following argument is valid:

All quidnuncs are shunpikers.
All shunpikers are flibbertigibbets.
Therefore, all quidnuncs are flibbertigibbets.

You could likely tell that the argument is valid even though you do not know the meanings of the words. This is an important point. We can often tell whether an argument is valid even if we are not in a position to know whether any of its propositions are true or false. This is because deductive validity typically depends on certain patterns of argument. In fact, even nonsense arguments can be valid. Lewis Carroll (a pen name for C. L. Dodgson) was not only the author of *Alice's Adventures in Wonderland*, but also a clever logician famous for both his use of nonsense words and his tricky logic puzzles.

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In addition to his well-known literary works, Lewis Carroll wrote several mathematical works, including three books on logic: *Symbolic Logic* Parts 1 and 2, and *The Game of Logic*, which was intended to introduce logic to children.

We will look at some of Carroll's puzzles in this chapter's sections on categorical logic, but for now, let us look at an argument using nonsense words from his poem "Jabberwocky." See if you can tell whether the following argument is valid:

All bandersnatches are slithy toves.
All slithy toves are uffish.
Therefore, all bandersnatches are uffish.

If you could tell the argument about quidnuncs was valid, you were probably able to tell that this argument is valid as well. Both arguments have the same pattern, or logical form.

Representing Logical Form

Logical form is generally represented by using variables or other symbols to highlight the pattern. In this case the logical form can be represented by substituting capital letters for certain parts of the propositions. Our argument then has the form:

All S are M.
All M are P.
Therefore, all S are P.

Any argument that follows this pattern, or form, is valid. Try it for yourself. Think of any three plural nouns; they do not have to be related to each other. For example, you could use *submarines*, *candy bars*, and *mountains*. When you have thought of three, substitute them for the letters in the pattern given. You can put them in any order you like, but the same word has to replace the same letter. So you will put one noun in for *S* in the first and third lines, one noun for both instances of *M*, and your last noun for both cases of *P*. If we use the suggested nouns, we would get:

All submarines are candy bars.
All candy bars are mountains.
Therefore, all submarines are mountains.

This argument may be close to nonsense, but it is logically valid. It would not be possible to make up a story in which the premises were true but the conclusion was false. For example, if one wizard turns all submarines into candy bars, and then a second wizard turns all candy bars into mountains, the story would not make any sense (nor would it be logical) if, in the end, all submarines were not mountains. Any story that makes the premises true would have to also make the conclusion true, so that the argument is valid.

As mentioned, the form of an argument is what you get when you remove the specific meaning of each of the nonlogical words in the argument and talk about them in terms of variables. Sometimes, however, one has to change the wording of a claim to make it fit the required form. For example, consider the premise "All men like dogs." In this case the first category would be "men," but the second category is not represented by a plural noun but by a predicate phrase, "like dogs." In such cases we turn the expression "like dogs" into the noun phrase "people who like dogs." In that case the form of the sentence is still "All A are B," in which B is "people who like dogs." As another example, the argument:

All whales are mammals.
Some mammals live in the water.
Therefore, at least some whales live in the water.

can be rewritten with plural nouns as:

All whales are mammals.
Some mammals are things that live in the water.
Therefore, at least some whales are things that live in the water.

and has the form:

All A are B.
Some B are C.
Therefore, at least some A are C.

The variables can represent *anything* (anything that fits grammatically, that is). When we substitute specific expressions (of the appropriate grammatical category) for each of the variables, we get an **instance** of that form. So another instance of this form could be made by replacing *A* with *Apples*, *B* with *Bananas*, and *C* with *Cantaloupes*. This would give us

All Apples are Bananas.
Some Bananas are Cantaloupes.
Therefore, at least some Apples are Cantaloupes.

It does not matter at this stage whether the sentences are true or false or whether the reasoning is valid or invalid. All we are concerned with is the form or pattern of the argument.

We will see many different patterns as we study deductive logic. Different kinds of deductive arguments require different kinds of forms. The form we just used is based on categories; the letters represented groups of things, like dogs, whales, mammals, submarines, or candy bars. That is why in these cases we use plural nouns. Other patterns will require substituting entire sentences for letters. We will study forms of this type in Chapter 4. The patterns you need to know will be introduced as we study each kind of argument, so keep your eyes open for them.

Using the Counterexample Method

By definition, an argument form is valid if and only if all of its instances are valid. Therefore, if we can show that a logical form has even one invalid instance, then we may infer that the argument form is invalid. Such an instance is called a *counterexample* to the argument form's validity; thus, the **counterexample method** for showing that an argument form is invalid involves creating an argument with the exact same form but in which the premises are true and the conclusion is false. (We will examine other methods in this chapter and in later chapters.) In other words, finding a counterexample demonstrates the invalidity of the argument's form.

Consider the invalid argument example from the prior section:

All donuts have added sugar.
All donuts are bad for you.
Therefore, everything with added sugar is bad for you.

By replacing predicate phrases with noun phrases, this argument has the form:

All A are B.
All A are C.
Therefore, all B are C.

This is the same form as that of the following, clearly invalid argument:

All birds are animals.
All birds have feathers.
Therefore, all animals have feathers.

Because we can see that the premises of this argument are true and the conclusion is false, we know that the argument is invalid. Since we have identified an invalid instance of the form, we know that the form is invalid. The invalid instance is a counterexample to the form. Because we have a counterexample, we have good reason to think that the argument about donuts is not valid.

S. Harris/Cartoonstock

Can you think of a counterexample that can prove this dog's argument is invalid?

One of our recent examples has the form:

All A are B.
Some B are C.
Therefore, at least some A are C.

Here is a counterexample that challenges this argument form's validity:

All dogs are mammals.
Some mammals are cats.
Therefore, at least some dogs are cats.

By substituting *dogs* for *A*, *mammals* for *B*, and *cats* for *C*, we have found an example of the argument's form that is clearly invalid because it moves from true premises to a false conclusion. Therefore, the argument form is invalid.

Here is another example of an argument:

All monkeys are primates.
No monkeys are reptiles.
Therefore, no primates are reptiles.

The conclusion is true in this example, so many may mistakenly think that the reasoning is valid. However, to better investigate the validity of the reasoning, it is best to focus on its form. The form of this argument is:

All A are B.
No A are C.
Therefore, no B are C.

To demonstrate that this form is invalid, it will suffice to demonstrate that there is an argument of this exact form that has all true premises and a false conclusion. Here is such a counterexample:

All men are human.
No men are women.
Therefore, no humans are women.

Clearly, there is something wrong with this argument. Though this is a different argument, the fact that it is clearly invalid, even though it has the exact same form as our original argument, means that the original argument's form is also invalid.

3.3 Types of Deductive Arguments

Once you learn to look for arguments, you will see them everywhere. Deductive arguments play very important roles in daily reasoning. This section will discuss some of the most important types of deductive arguments.

Mathematical Arguments

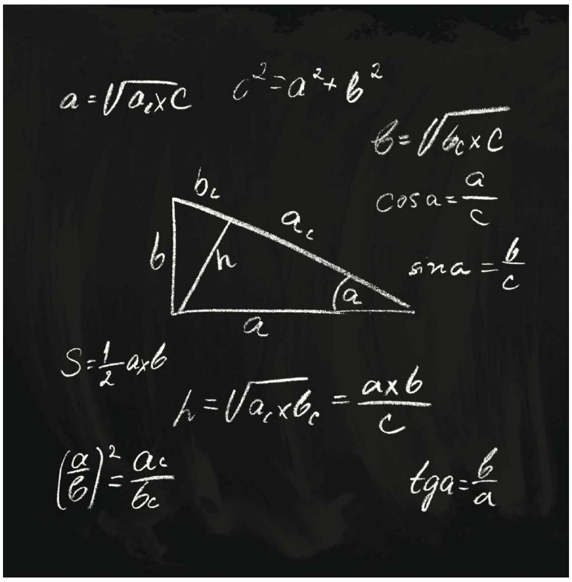
Arguments about or involving mathematics generally use deductive reasoning. In fact, one way to think about deductive reasoning is that it is reasoning that tries to establish its conclusion with mathematical certainty. Let us consider some examples.

Suppose you are splitting the check for lunch with a friend. In calculating your portion, you reason as follows:

I had the chicken sandwich plate for $8.49.
I had a root beer for $1.29.
I had nothing else.
$8.49 + $1.29 = $9.78.
Therefore, my portion of the bill, excluding tip and tax, is $9.78.

Notice that if the premises are all true, then the conclusion must be true also. Of course, you might be mistaken about the prices, or you might have forgotten that you had a piece of pie for dessert. You might even have made a mistake in how you added up the prices. But these are all premises. So long as your premises are correct and the argument is valid, then the conclusion is certain to be true.

But wait, you might say—aren't we often mistaken about things like this? After all, it is common for people to make mistakes when figuring out a bill. Your friend might even disagree with one of your premises: For example, he might think the chicken sandwich plate was really $8.99. How can we say that the conclusion is established with mathematical certainty if we are willing to admit that we might be mistaken?



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A mathematical proof is a valid deductive argument that attempts to prove the conclusion. Because mathematical proofs are deductively valid, mathematicians establish mathematical truth with complete certainty (as long as they agree on the premises).

These are excellent questions, but they pertain to our certainty of the truth of the premises. The important feature of valid arguments is that the reasoning is so strong that the conclusion is just as certain to be true as the premises. It would be a very strange friend indeed who agreed with all of your premises and yet insisted that your portion of the bill was something other than $9.78. Still, no matter how good our reasoning, there is almost always some possibility that we are mistaken about our premises.

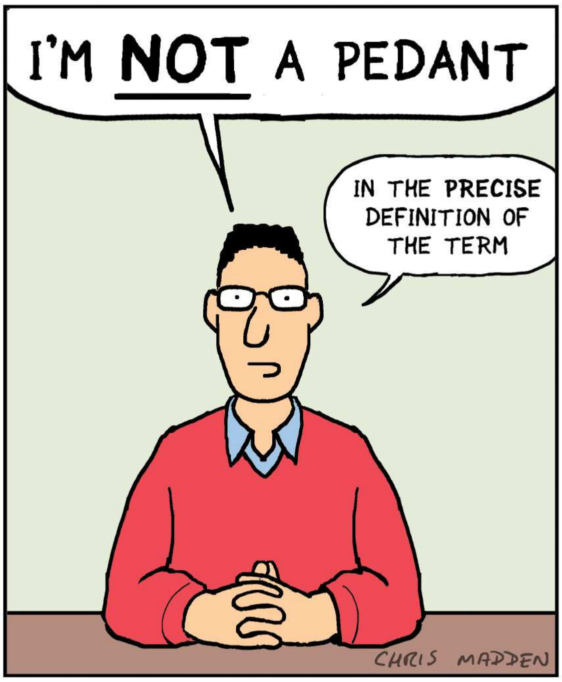
Arguments From Definitions

Another common type of deductive argument is **argument from definition**. This type of argument typically has two premises. One premise gives the definition of a word; the second premise says that something meets the definition. Here is an example:

Bachelor means "unmarried male."
John is an unmarried male.
Therefore, John is a bachelor.

Notice that as with arguments involving math, we may disagree with the premises, but it is very hard to agree with the premises and disagree with the conclusion. When the argument is set out in standard form, it is typically relatively easy to see that the argument is valid.

On the other hand, it can be a little tricky to tell whether the argument is sound. Have we really gotten the definition right? We have to be very careful, as definitions often sound right even though they are a little bit off. For example, the stated definition of *bachelor* is not quite right. At the very least, the definition should apply only to human males, and probably only adult ones. We do not normally call children or animals "bachelors."



Chris Madden/Cartoonstock

When crafting or evaluating a deductive argument via definition, special attention should be paid to the clarity of the definition.

An interesting feature of definitions is that they can be understood as going both ways. In other words, if *bachelor* means "unmarried male," then we can reason either from the man being an unmarried male to his being a bachelor, as in the previous example, or from the man being a bachelor to his being an unmarried male, as in the following example.

> *Bachelor* means "unmarried male."
> John is a bachelor.
> Therefore, John is an unmarried male.

Arguments from definition can be very powerful, but they can also be misused. This typically happens when a word has two meanings or when the definition is not fully accurate. We will learn more about this when we study fallacies in Chapter 7, but here is an example to consider:

> Murder is the taking of an innocent life.
> Abortion takes an innocent life.
> Therefore, abortion is murder.

This is an argument from definition, and it is valid—the premises guarantee the truth of the conclusion. However, are the premises true? Both premises could be disputed, but the first premise is probably not right as a definition. If the word *murder* really just meant "taking an innocent life," then it would be impossible to commit murder by killing someone who was not innocent. Furthermore, there is nothing in this definition about the victim being a human or the act being intentional. It is very tricky to get definitions right, and we should be very careful about reaching conclusions based on oversimplified definitions. We will come back to this example from a different angle in the next section when we study syllogisms.

Categorical Arguments

Historically, some of the first arguments to receive a detailed treatment were *categorical arguments*, having been thoroughly explained by Aristotle himself (Smith, 2014). **Categorical arguments** are arguments whose premises and conclusions are statements about categories of things. Let us revisit an example from earlier in this chapter:

> All whales are mammals.
> All mammals breathe air.
> Therefore, all whales breathe air.

In each of the statements of this argument, the membership of two categories is compared. The categories here are whales, mammals, and air breathers. As discussed in the previous section on evaluating deductive arguments, the validity of these arguments depends on the repetition of the category terms in certain patterns; it has nothing to do with the specific categories being compared. You can test this by changing the category terms *whales*, *mammals*, and *air breathers* with any other category terms you like. Because this argument's form is valid, any other argument with the same form will be valid. The branch of deductive reasoning that deals with categorical arguments is known as *categorical logic*. We will discuss it in the next two sections.

Propositional Arguments

Propositional arguments are a type of reasoning that relates sentences to each other rather than relating categories to each other. Consider this example:

> Either Jill is in her room, or she's gone out to eat.
> Jill is not in her room.
> Therefore, she's gone out to eat.

Notice that in this example the pattern is made by the sentences "Jill is in her room" and "she's gone out to eat." As with categorical arguments, the validity of propositional arguments can be determined by examining the form, independent of the specific sentences used. The branch of deductive reasoning that deals with propositional arguments is known as *propositional logic*, which we will discuss in Chapter 4.

3.4 Categorical Logic: Introducing Categorical Statements

The field of deductive logic is a rich and productive one; one could spend an entire lifetime studying it. (See *A Closer Look: More Complicated Types of Deductive Reasoning*.) Because the focus of this book is critical thinking and informal logic (rather than formal logic), we will only look closely at categorical and propositional logic, which focus on the basics of argument. If you enjoy this introductory exposure, you might consider looking for more books and courses in logic.

A Closer Look: More Complicated Types of Deductive Reasoning

As noted, deductive logic deals with a precise kind of reasoning in which logical validity is based on logical form. Within logical forms, we can use letters as variables to replace English words. Logicians also frequently replace other words that occur within arguments—such as *all*, *some*, *or*, and *not*—to create a kind of symbolic language. Formal logic represented in this type of symbolic language is called *symbolic logic*.

Because of this use of symbols, courses in symbolic logic end up looking like math classes. An introductory course in symbolic logic will typically begin with propositional logic and then move to something called predicate logic. Predicate logic combines everything from categorical and propositional logic but allows much more flexibility in the use of *some* and *all*. This flexibility allows it to represent much more complex and powerful statements.

Predicate logic forms the basis for even more advanced types of logic. Modal logic, for example, can be used to represent many deductive arguments about possibility and necessity that cannot be symbolized using predicate logic alone. Predicate logic can even help provide a foundation for mathematics. In particular, when predicate logic is combined with a mathematical field called set theory, it is possible to prove the fundamental truths of arithmetic. From there it is possible to demonstrate truths from many important fields of mathematics, including calculus, without which we could not do physics, engineering, or many other fascinating and useful fields. Even the computers that now form such an essential part of our lives are founded, ultimately, on deductive logic.

Categorical arguments have been studied extensively for more than 2,000 years, going back to Aristotle. **Categorical logic** is the logic of argument made up of categorical statements. It is a logic that is concerned with reasoning about certain relationships between categories of things. To learn more about how categorical logic works, it will be useful to begin by analyzing the nature of categorical statements, which make up the premises and conclusions of categorical arguments. A **categorical statement** talks about two categories or groups. Just to keep things simple, let us start by talking about dogs, cats, and animals.

One thing we can say about these groups is that all dogs are animals. Of course, all cats are animals, too. So we have the following true categorical statements:

All dogs are animals.

All cats are animals.

In categorical statements, the first group name is called the **subject term**; it is what the sentence is about. The second group name is called the **predicate term**. In the categorical sentences just mentioned, *dogs* and *cats* are both in the subject position, and *animals* is in the predicate position. Group terms can go in either position, but of course, the sentence might be false. For example, in the sentence "All animals are dogs" the term *dogs* is in the predicate position.

You may recall that we can represent the logical form of these types of sentences by replacing the category terms with single letters. Using this method, we can represent the form of these categorical statements in the following way:

All D are A.

All C are A.

Another true statement we can make about these groups is "No dogs are cats." Which term is in subject position, and which is in predicate position? If you said that *dogs* is the subject and *cats* is the predicate, you're right! The logical form of "No dogs are cats" can be given as "No D are C."

We now have two sentences in which the category *dogs* is the subject: "All dogs are animals" and "No dogs are cats." Both of these statements tell us something about every dog. The first, which starts with *all*, tells us that each dog is an animal. The second, which begins with *no*, tells us that each dog is not a cat. We say that both of these types of sentences are *universal* because they tell us something about every member of the subject class.

Not all categorical statements are universal. Here are two statements about dogs that are not universal:

Some dogs are brown.

Some dogs are not tall.

Statements that talk about some of the things in a category are called *particular* statements. The distinction between a statement being universal or particular is a distinction of **quantity**.

Another distinction is that we can say that the things mentioned are in or not in the predicate category. If we say the things *are in* that category, our statement is *affirmative*. If we say the things *are not in* that category, our statement is *negative*. The distinction between a statement being affirmative or negative is a distinction of **quality**. For example, when we say "Some dogs are brown," the thing mentioned (dogs) *is in* the predicate category (brown things), making this an affirmative statement. When we say "Some dogs are not tall," the thing mentioned (dogs) *is not in* the predicate category (tall things), and so this is a negative statement.

Taking both of these distinctions into account, there are four types of categorical statements: *universal affirmative*, *universal negative*, *particular affirmative*, and *particular negative*. Table 3.1 shows the form of each statement along with its quantity and quality.

Table 3.1: Types of categorical statements

	Quantity	**Quality**
All S is P	Universal	Affirmative
No S is P	Universal	Negative
Some S is P	Particular	Affirmative
Some S is not P	Particular	Negative

To abbreviate these categories of statement even further, logicians over the millennia have used letters to represent each type of statement. The abbreviations are as follows:

A: Universal affirmative (All S is P)

E: Universal negative (No S is P)

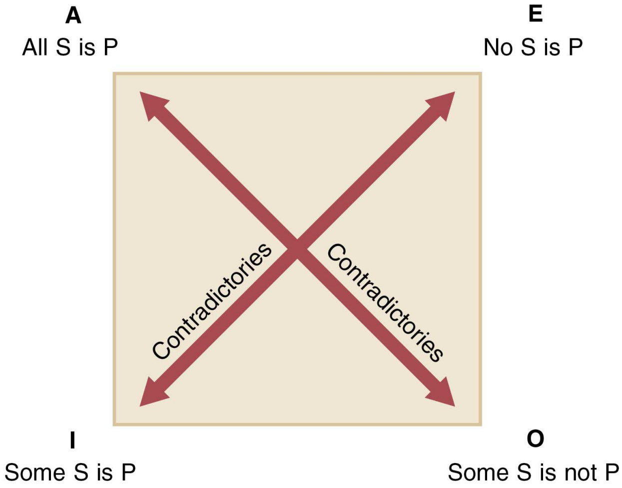
I: Particular positive (Some S is P)

O: Particular negative (Some S is not P)

Accordingly, the statements are known as *A propositions*, *E propositions*, *I propositions*, and *O propositions.* Remember that the single capital letters in the statements themselves are just placeholders for category terms; we can fill them in with any category terms we like. Figure 3.1 shows a traditional way to arrange the four types of statements by quantity and quality.

Figure 3.1: The square of opposition

The square of opposition serves as a quick reference point when evaluating categorical statements. Note that A statements and O statements always contradict one another; when one is true, the other is false. The same is true of E statements and I statements.



Now we need to get just a bit clearer on what the four statements mean. Granted, the meaning of categorical statements seems clear: To say, for example, that "no dogs are reptiles" simply means that there are no things that are both dogs and reptiles. However, there are certain cases in which the way that logicians understand categorical statements may differ somewhat from how they are commonly understood in everyday language. In particular, there are two specific issues that can cause confusion.

Clarifying Particular Statements

The first issue is with particular statements (I and O propositions). When we use the word *some* in everyday life, we typically mean more than one. For example, if someone says that she has *some* apples, we generally think that this means that she has more than one. However, in logic, we take the word *some* simply to mean *at least one*. Therefore, when we say that some S is P, we mean only that at least one S is P. For example, we can say "Some dogs live in the White House" even if only one does.

Clarifying Universal Statements

The second issue involves universal statements (A and E propositions). It is often called the "issue of existential presupposition"—the issue concerns whether a universal statement implies a particular statement. For example, does the fact that all dogs are animals imply that some dogs are animals? The question really becomes an issue only when we talk about things that do not really exist. For example, consider the claim that all the survivors of the Civil War live in New York. Given that there are no survivors of the Civil War anymore, is the statement true or not?

The Greek philosopher Aristotle, the inventor of categorical logic, would have said the statement is false. He thought that "All S is P" could only be true if there was at least one S (Parsons, 2014). Modern logicians, however, hold that that "All S is P" is true even when no S exists. The reasons for the modern view are somewhat beyond the scope of this text—see *A Closer Look: Existential Import* for a bit more of an explanation—but an example will help support the claim that universal statements are true when no member of the subject class exists.

Suppose we are driving somewhere and stop for snacks. We decide to split a bag of M&M's. For some reason, one person in our group really wants the brown M&M's, so you promise that he can have all of them. However, when we open the bag, it turns out that there are no brown candies in it. Since this friend did not get any brown M&M's, did you break your promise? It seems clear that you did not. He did get all of the brown M&M's that were in the bag; there just weren't any. In order for you to have broken your promise, there would have to be a brown M&M that you did not let your friend have. Therefore, it is true that your friend got all the brown M&M's, even though he did not get any.

This is the way that modern logicians think about universal propositions when there are no members of the subject class. Any universal statement with an empty subject class is true, regardless of whether the statement is positive or negative. It is true that all the brown M&M's were given to your friend and also true that no brown M&M's were given to your friend.

A Closer Look: Existential Import

It is important to remember that particular statements in logic (I and O propositions) refer to things that actually exist. The statement “Some dogs are mammals” is essentially saying, “There is at least one dog that exists in the universe, and that dog is a mammal.” The way that logicians refer to this attribute of I and O statements is that they have “existential import.” This means that for them to be true, there must be something that actually exists that has the property mentioned in the statement.

The 19th-century mathematician George Boole, however, presented a problem. Boole agreed with Aristotle that the existential statements I and O had to refer to existing things to be true. Also, for Aristotle, all A statements that are true necessarily imply the truth of their corresponding I statements. The same goes with E and O statements.

Boole pointed out that some true A and E statements refer to things that do not actually exist. Consider the statement “All vampires are creatures that drink blood.” This is a true statement. That means that the corresponding I statement, “Some vampires are creatures that drink blood,” would also be true, according to Aristotle. However, Boole noted that there are no existing things that are vampires. If vampires do not exist, then the I statement, “Some vampires are creatures that drink blood,” is not true: The truth of this statement rests on the idea that there is an actually existing thing called a vampire, which, at this point, there is no evidence of.

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George Boole, for whom Boolean logic is named, challenged Aristotle’s assertion that the truth of A statements implies the truth of corresponding I statements. Boole suggested that some valid forms of syllogisms had to be excluded.

Boole reasoned that Aristotle’s ideas did not work in cases where A and E statements refer to nonexisting classes of objects. For example, the E statement “No vampires are time machines” is a true statement. However, both classes in this statement refer to things that do not actually exist. Therefore, the statement “Some vampires are not time machines” is not true, because this statement could only be true if vampires and time machines actually existed.

Boole reasoned that Aristotle’s claim that true A and E statements led necessarily to true I and O statements was not universally true. Hence, Boole claimed that there needed to be a revision of the forms of categorical syllogisms that are considered valid. Because one cannot generally claim that an existential statement (I or O) is true based on the truth of the corresponding universal (A or E), there were some valid forms of syllogisms that had to be excluded under the Boolean (modern) perspective. These syllogisms were precisely those that reasoned from universal premises to a particular conclusion.

Of course, we all recognize that in everyday life we can logically infer that if all dogs are mammals, then it must be true that some dogs are mammals. That is, we know that there is at least one existing dog that is a mammal. However, because our logical rules of evaluation need to apply to all instances of syllogisms, and because there are other instances where universals do not lead of necessity to the truth of particulars, the rules of evaluation had to be reformed after Boole presented his analysis. It is important to avoid committing the existential fallacy, or assuming that a class has members and then drawing an inference about an actually existing member of the class.

Accounting for Conversational Implication

These technical issues likely sound odd: We usually assume that *some* implies that there is more than one and that *all* implies that something exists. This is known as *conversational implication* (as opposed to logical implication). It is quite common in everyday life to make a conversational implication and take a statement to suggest that another statement is true as well, even though it does not *logically* imply that the other must be true. In logic, we focus on the literal meaning.

One of the common reasons that a statement is taken to conversationally imply another is that we are generally expected to make the most fully informative statement that we can in response to a question. For example, if someone asks what time it is and you say, “Sometime after 3,” your statement seems to imply that you do not know the exact time. If you knew it was 3:15 exactly, then you probably should have given this more specific information in response to the question.

For example, we all know that all dogs are animals. Suppose, however, someone says, “Some dogs are animals.” That is an odd thing to say: We generally would not say that some dogs are animals unless we thought that some of them are *not* animals. However, that would be making a conversational implication, and we want to make logical implications. For the purposes of logic, we want to know whether the statement “some dogs are animals” is true or false. If we say it is false, then we seem to have stated it is *not* true that some dogs are animals; this, however, would seem to mean that there are *no* dogs that are animals. That cannot be right. Therefore, logicians take the statement "Some dogs are animals" simply to mean that there is at least one dog that is an animal, which is true. The statement "Some dogs are not animals" is not part of the meaning of the statement "Some dogs are animals." In the language of logic, the statement that some S are not P is not part of the meaning of the statement that some S are P.

Of course, it would be odd to make the less informative statement that some dogs are animals, since we know that all dogs are animals. Because we tend to assume someone is making the most informative statement possible, the statement "Some dogs are animals" may conversationally imply that they are not all animals, even though that is not part of the literal meaning of the statement.

In short, a particular statement is true when there is at least one thing that makes it true, even if the universal statement would also be true. In fact, sometimes we emphasize that we are not talking about the whole category by using the words *at least*, as in, "At least some planets orbit stars." Therefore, it appears to be nothing more than conversational implication, not literal meaning, that leads our statement "Some dogs are animals" to suggest that some also are not. When looking at categorical statements, be sure that you are thinking about the actual meaning of the sentence rather than what might be conversationally implied.

Practice Problems 3.2

Complete the following problems. Click here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems3.2.pdf) to check your answers.

1. "All dinosaurs are things that are extinct." Which of the following is the subject term in this statement?
   a. dinosaurs
   b. things that are extinct
2. "No Honda Civics are Lamborghinis." Which of the following is the predicate term in this statement?
   a. Lamborghinis
   b. Honda Civics
3. "Some authors are people who write horror." Which of the following is the predicate term in this statement?
   a. authors
   b. people who write horror
4. "Some politicians are not people who can be trusted." Which of the following is the subject term in this statement?
   a. politicians
   b. people who can be trusted
5. "All mammals are pieces of cheese." Which of the following is the predicate term in this statement?
   a. pieces of cheese
   b. mammals
6. What is the quantity of the following statement? "All dinosaurs are things that are extinct."
   a. universal
   b. particular
   c. affirmative
   d. negative
7. What is the quality of the following statement? "No Honda Civics are Lamborghinis."
   a. universal
   b. particular
   c. affirmative
   d. negative
8. What is the quality of the following statement? "Some authors are people who write horror."
   a. universal
   b. particular
   c. affirmative

   d. negative

9. What is the quantity of the following statement? “Some politicians are not people who can be trusted.”
   a. universal
   b. particular
   c. affirmative
   d. negative

10. What is the quality of the following statement? “All mammals are pieces of cheese.”
   a. universal
   b. particular
   c. affirmative
   d. negative

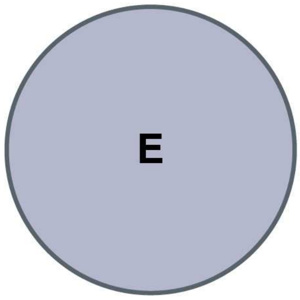
3.5 Categorical Logic: Venn Diagrams as Pictures of Meaning

Given that it is sometimes tricky to parse out the meaning and implications of categorical statements, a logician named John Venn devised a method that uses diagrams to clarify the literal meanings and logical implications of categorical claims. These diagrams are appropriately called **Venn diagrams** (Stapel, n.d.). Venn diagrams not only give a visual picture of the meanings of categorical statements, they also provide a method by which we can test the validity of many categorical arguments.

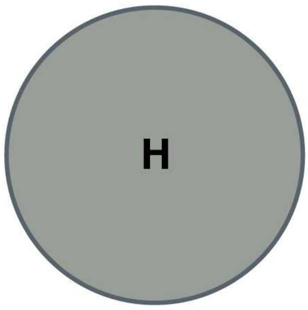
Drawing Venn Diagrams

Here is how the diagramming works: Imagine we get a bunch of people together and all go to a big field. We mark out a big circle with rope on the field and ask everyone with brown eyes to stand in the circle. Would you stand inside the circle or outside it? Where would you stand if we made another circle and asked everyone with brown hair to stand inside? If your eyes or hair are sort of brownish, just pick whether you think you should be inside or outside the circles. No standing on the rope allowed! Remember your answers to those two questions.

Here is an image of the brown-eye circle, labeled "E" for "eyes"; touch inside or outside the circle indicating where you would stand.

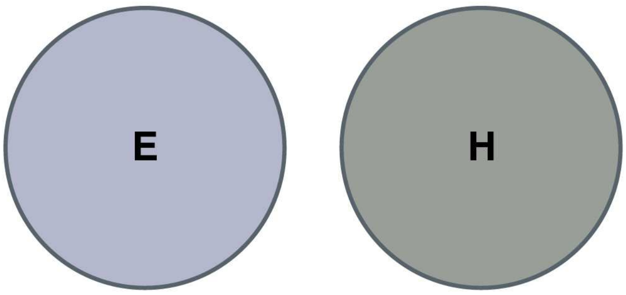


Here is a picture of the brown-hair circle, labeled "H" for "hair"; touch inside or outside the circle indicating where you would stand.

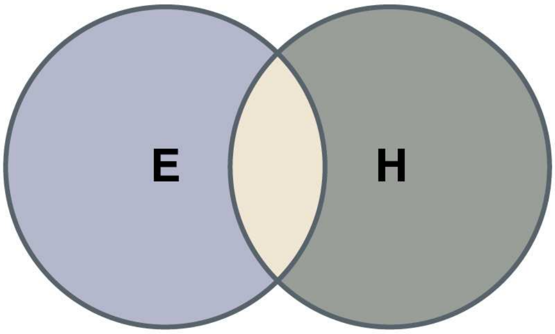


Notice that each circle divides the people into two groups: Those inside the circle have the feature we are interested in, and those outside the circle do not.

Where would you stand if we put both circles on the ground at the same time?

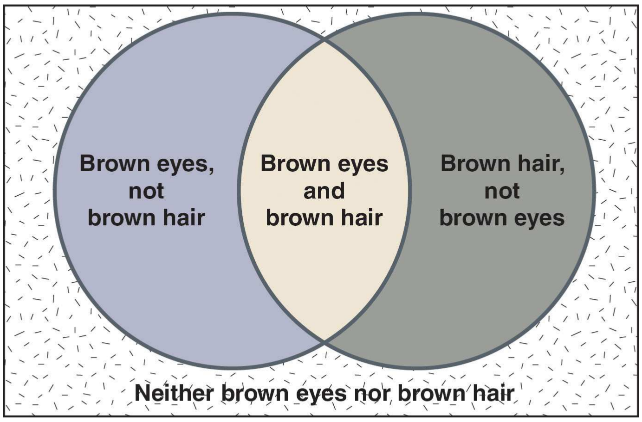


As long as you do not have both brown eyes and brown hair, you should be able to figure out where to stand. But where would you stand if you have brown eyes and brown hair? There is not any spot that is in both circles, so you would have to choose. In order to give brown-eyed, brown-haired people a place to stand, we have to overlap the circles.



Now there is a spot where people who have both brown hair and brown eyes can stand: where the two circles overlap. We noted earlier that each circle divides our bunch of people into two groups, those inside and those outside. With two circles, we now have four groups. Figure 3.2 shows what each of those groups are and where people from each group would stand.

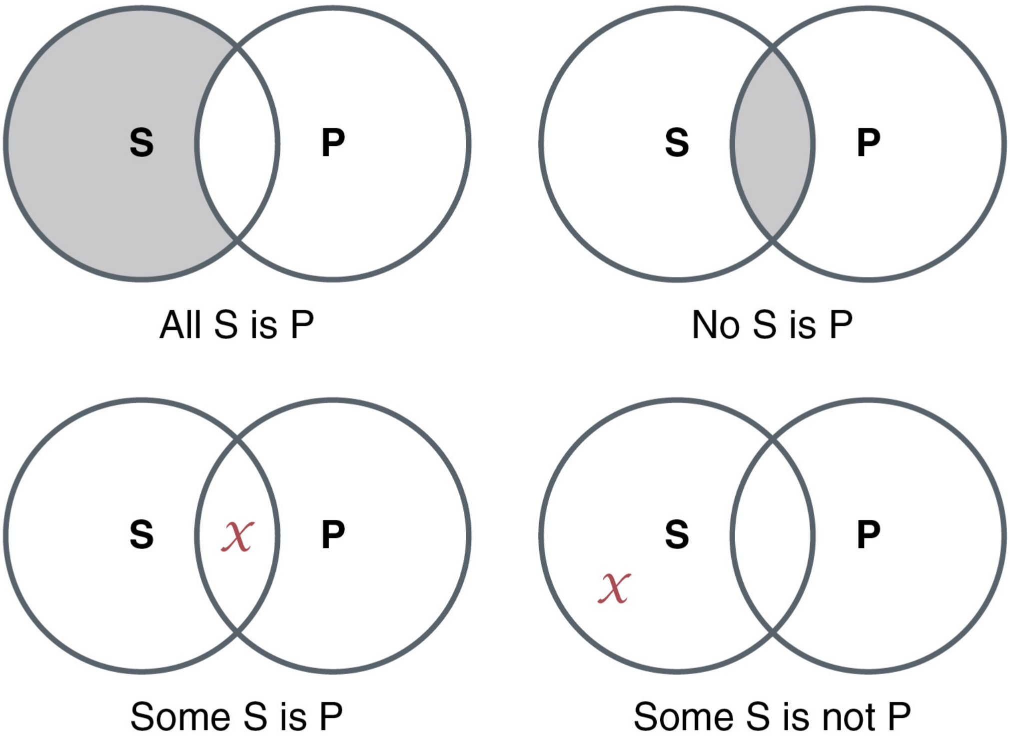
Figure 3.2: Sample Venn diagram



With this background, we can now draw a picture for each categorical statement. When we know a region is empty, we will darken it to show there is nobody there. If we know for sure that someone is in a region, we will put an *x* in it to represent a person standing there. Figure 3.3 shows the pictures for each of the four kinds of statements.

Figure 3.3: Venn diagrams of categorical statements

Each of the four categorical statements can be represented visually with a Venn diagram.



In drawing these pictures, we adopt the convention that the subject term is on the left and the predicate term is on the right. There is nothing special about this way of doing it, but diagrams are easier to understand if we draw them the same way as much as possible. The important thing to remember is that a Venn diagram is just a picture of the meaning of a statement. We will use this fact in our discussion of inferences and arguments.

Drawing Immediate Inferences

As mentioned, Venn diagrams help us determine what inferences are valid. The most basic of such inferences, and a good place to begin, is something called *immediate inference*. **Immediate inferences** are arguments from one categorical statement as premise to another as conclusion. In other words, we immediately infer one statement from another. Despite the fact that these inferences have only one premise, many of them are logically valid. This section will use Venn diagrams to help discern which immediate inferences are valid.

The basic method is to draw a diagram of the premises of the argument and determine if the diagram thereby shows the conclusion is true. If it does, then the argument is valid. In other words, if drawing a diagram of just the premises automatically creates a diagram of the conclusion, then the argument is valid. The diagram shows that any way of making the premises true would also make the conclusion true; it is impossible for the conclusion to be false when the premises are true. We will see how to use this method with each of the immediate inferences and later extend the method to more complicated arguments.

Conversion

Conversion is just a matter of switching the positions of the subject and predicate terms. The resulting statement is called the converse of the original statement. Table 3.2 shows the converse of each type of statement.

Table 3.2: Conversion

Statement	**Converse**
All S is P.	All P is S.
No S is P.	No P is S.
Some S is P.	Some P is S.
Some S is not P.	Some P is not S.

Forming the converse of a statement is easy; just switch the subject and predicate terms with each other. The question now is whether the immediate inference from a categorical statement to its converse is *valid* or not. It turns out that the argument from a statement to its converse is valid for some statement types, but not for others. In order to see which, we have to check that the converse is true whenever the original statement is true.

An easy way to do this is to draw a picture of the two statements and compare them. Let us start by looking at the universal negative statement, or E proposition, and its converse. If we form an argument from this statement to its converse, we get the following:

No S is P.

Therefore, no P is S.

Figure 3.4 shows the Venn diagrams for these statements.

As you can see, the same region is shaded in both pictures—the region that is inside both circles. It does not matter which order the circles are in, the picture is the same. This means that the two statements have the same meaning; we call such statements *equivalent*.

The Venn diagrams for these statements demonstrate that all of the information in the conclusion is present in the premise. We can therefore infer that the inference is *valid.* A shorter way to say it is that conversion is valid for universal negatives.

We see the same thing when we look at the particular affirmative statement, or I proposition.

In the case of particular affirmatives as well, we can see that all of the information in the conclusion is contained within the premises. Therefore, the immediate inference is *valid*. In fact, because the diagram for "Some S is P" is the same as the diagram for its converse, "Some P is S" (see Figure 3.5), it follows that these two statements are equivalent as well.

Figure 3.4: Universal negative statement and its converse

In this representation of "No S is P. Therefore, no P is S," the areas shaded are the same, meaning the statements are equivalent.

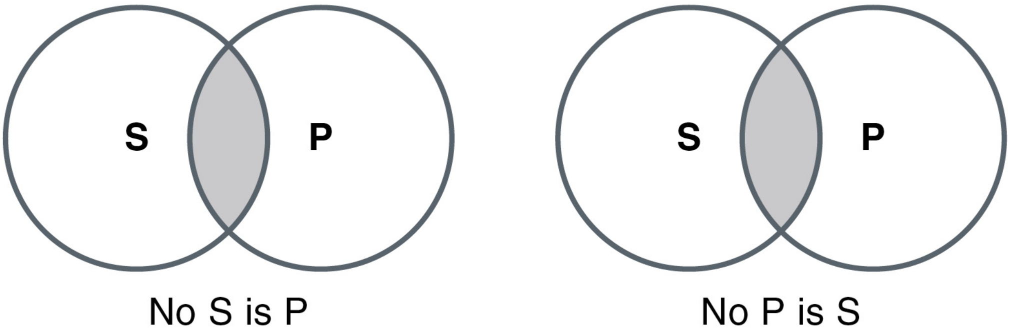
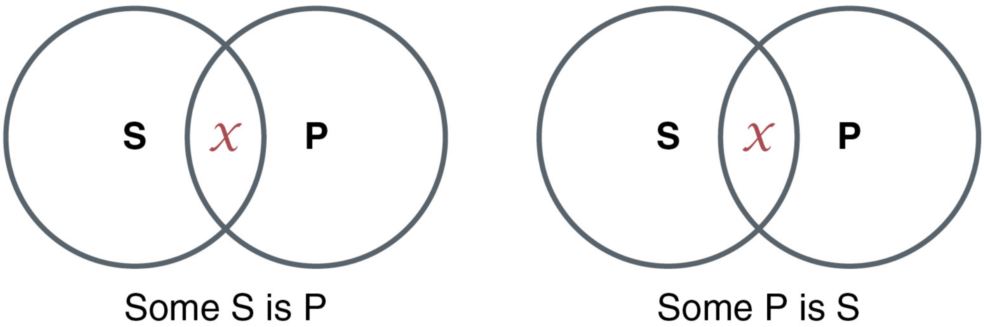


Figure 3.5: Particular affirmative statement and its converse

As with the E proposition, all of the information contained in the conclusion of the I proposition is also contained within the premises, making the inference valid.



However, there will be a big difference when we draw pictures of the universal affirmative (A proposition), the particular negative (O proposition), and their converses (see Figure 3.6 and Figure 3.7).

In these two cases we get different pictures, so the statements do not mean the same thing. In the original statements, the marked region is inside the S circle but not in the P circle. In the converse statements, the marked region is inside the P circle but not in the S circle. Because there is information in the conclusions of these arguments that is not present in the premises, we may infer that conversion is invalid in these two cases.

Figure 3.6: Universal affirmative statement and its converse

Unlike Figures 3.4 and 3.5 where the diagrams were identical, we get two different diagrams for A propositions. This tells us that there is information contained in the conclusion that was not included in the premises, making the inference invalid.

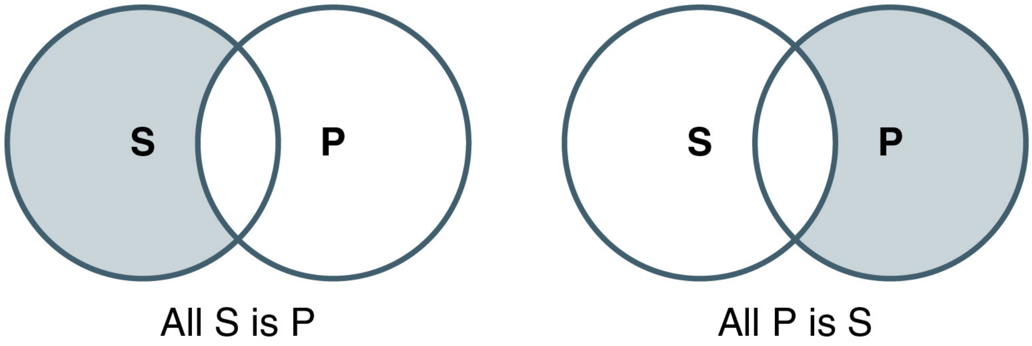
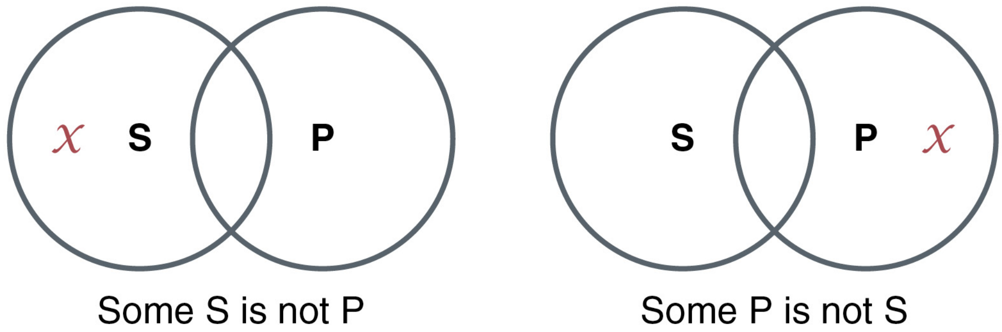


Figure 3.7: Particular negative statement and its converse

As with A propositions, O propositions present information in the conclusion that was not present in the premises, rendering the inference invalid.



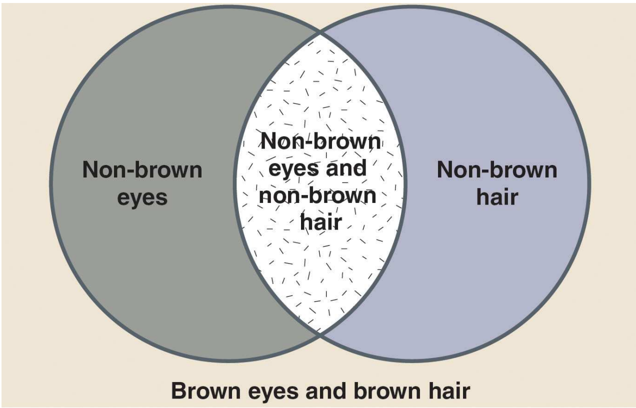
Let us consider another type of immediate inference.

Contraposition

Before we can address *contraposition*, it is necessary to introduce the idea of a **complement class**. Remember that for any category, we can divide things into those that are in the category and those that are out of the category. When we imagined rope circles on a field, we asked all the brown-haired people to step inside one of the circles. That gave us two groups: the brown-haired people inside the circle, and the non-brown-haired people outside the circle. These two groups are complements of each other. The complement of a group is everything that is not in the group. When we have a term that gives us a category, we can just add *non-* before the term to get a term for the complement group. The complement of term *S* is *non-S*, the complement of term *animal* is *nonanimal*, and so on. Let us see what complementing a term does to our Venn diagrams.

Recall the diagram for brown-eyed people. You were inside the circle if you have brown eyes, and outside the circle if you do not. (Remember, we did not let people stand on the rope; you had to be either in or out.) So now consider the diagram for non-brown-eyed people.

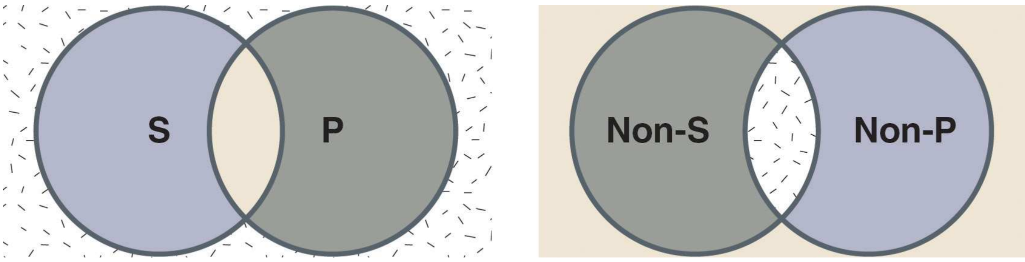
If you were inside the brown-eyed circle, you would be outside the non-brown-eyed circle. Similarly, if you were outside the brown-eyed circle, you would be inside the non-brown-eyed circle. The same would be true for complementing the brown-haired circle. Complementing just switches the inside and outside of the circle.



Do you remember the four regions from Figure 3.2? See if you can find the regions that would have the same people in the complemented picture. Where would someone with blue eyes and brown hair stand in each picture? Where would someone stand if he had red hair and green eyes? How about someone with brown hair and brown eyes?

In Figure 3.8, the regions are colored to indicate which ones would have the same people in them. Use the diagram to help check your answers from the previous paragraph. Notice that the regions in both circles and outside both circles trade places and that the region in the left circle only trades places with the region in the right circle.

Figure 3.8: Complement class



Now that we know what a complement is, we are ready to look at the immediate inference of contraposition. **Contraposition** combines conversion and complementing; to get the contrapositive of a statement, we first get the converse and then find the complement of both terms.

Let us start by considering the universal affirmative statement, "All S is P." First we form its converse, "All P is S," and then we complement both class terms to get the contrapositive, "All non-P is non-S." That may sound like a mouthful, but you should see that there is a simple, straightforward process for getting the contrapositive of any statement. Table 3.3 shows the process for each of the four types of categorical statements.

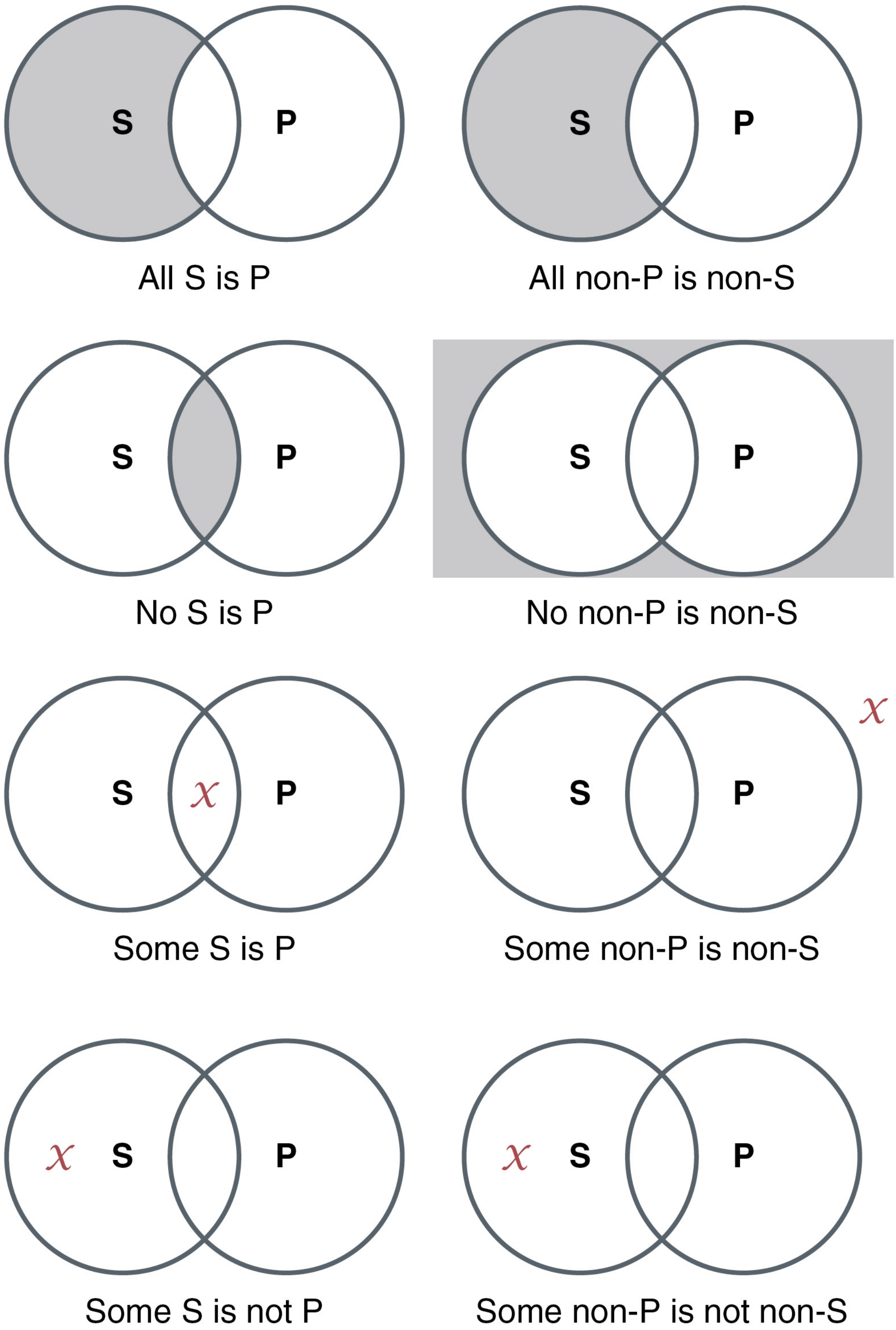
Table 3.3: Contraposition

Original	Converse	Contrapositive
All S is P.	All P is S.	All non-P is non-S.
No S is P.	No P is S.	No non-P is non-S.
Some S is P.	Some P is S.	Some non-P is non-S.
Some S is not P.	Some P is not S.	Some non-P is not non-S.

Figure 3.9 shows the diagrams for the four statement types and their contrapositives, colored so that you can see which regions represent the same groups.

Figure 3.9: Contrapositive Venn diagrams

Using the converse and contrapositive diagrams, you can infer the original statement.



As you can see, contraposition preserves meaning in universal affirmative and particular negative statements. So from either of these types of statements, we can immediately infer their contrapositive, and from the contrapositive, we can infer the original statement. In other words, these statements are equivalent; therefore, in those two cases, the contrapositive is valid.

In the other cases, particular affirmative and universal negative, we can see that there is information in the conclusion that is not present in diagram of the premise; these immediate inferences are invalid.

There are more immediate inferences that can be made, but our main focus in this chapter is on arguments with multiple premises, which tend to be more interesting, so we are going to move on to *syllogisms*.

Practice Problems 3.3

Answer the following questions about conversion and contraposition. Click [here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems3.3.pdf)](https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems3.3.pdf) to check your answers.

1. What is the converse of the statement “No humperdinks are picklebacks”?
    a. No humperdinks are picklebacks.
    b. All picklebacks are humperdinks.
    c. Some humperdinks are picklebacks.
    d. No picklebacks are humperdinks.

2. What is the converse of the statement “Some mammals are not dolphins”?
    a. Some dolphins are mammals.
    b. Some dolphins are not mammals.
    c. All dolphins are mammals.
    d. No dolphins are mammals.

3. What is the contrapositive of the statement “All couches are pieces of furniture”?
    a. All non-couches are non-pieces of furniture.
    b. All pieces of furniture are non-couches.
    c. All non-pieces of furniture are couches.
    d. All non-pieces of furniture are non-couches.

4. What is the contrapositive of the statement “Some apples are not vegetables”?
    a. Some non-apples are not non-vegetables.
    b. Some non-vegetables are not non-apples.
    c. Some non-vegetables are non-apples.
    d. Some non-vegetables are apples.

5. What is the converse of the statement “Some men are bachelors”?
    a. Some bachelors are men.
    b. Some bachelors are non-men.
    c. All bachelors are men.
    d. No women are bachelors.

3.6 Categorical Logic: Categorical Syllogisms

Whereas contraposition and conversion can be seen as arguments with only one premise, a **syllogism** is a deductive argument with two premises. The categorical syllogism, in which a conclusion is derived from two categorical premises, is perhaps the most famous—and certainly one of the oldest—forms of deductive argument. The categorical syllogism—which we will refer to here as just "syllogism"—presented by Aristotle in his *Prior Analytics* (350 BCE/1994), is a very specific kind of deductive argument and was subsequently studied and developed extensively by logicians, mathematicians, and philosophers.

Terms

We will first discuss the syllogism's basic outline, following Aristotle's insistence that syllogisms are arguments that have two premises and a conclusion. Let us look again at our standard example:

All S are M.
All M are P.
Therefore, all S are P.

There are three total terms here: S, M, and P. The term that occurs in the predicate position in the conclusion (in this case, P) is the *major term*. The term that occurs in the subject position in the conclusion (in this case, S) is the *minor term*. The other term, the one that occurs in both premises but not the conclusion, is the *middle term* (in this case, M).

The premise that includes the major term is called the *major premise*. In this case it is the first premise. The premise that includes the minor term, the second one here, is called the *minor premise*. The conclusion will present the relationship between the predicate term of the major premise (P) and the subject term of the minor premise (S) (Smith, 2014).

There are 256 possible different forms of syllogisms, but only a small fraction of those are valid, which can be shown by testing syllogisms through the traditional rules of the syllogism or by using Venn diagrams, both of which we will look at later in this section.

"Syllogisms won't do you any good here, Mr. Aristotle."

Ron Morgan/Cartoonstock

Aristotle's categorical syllogism uses two categorical premises to form a deductive argument.

Distribution

As Aristotle understood logical propositions, they referred to classes, or groups: sets of things. So a universal affirmative (type A) proposition that states "All Clydesdales are horses" refers to the group of Clydesdales and says something about the relationship between all of the members of that group and the members of the group "horses." However, nothing at all is said about those horses that might not be Clydesdales, so not all members of the group of horses are referred to. The idea of referring to members of such groups is the basic idea behind **distribution**: If all of the members of a group are referred to, the term that refers to that group is said to be distributed.

Using our example, then, we can see that the proposition "All Clydesdales are horses" refers to all the members of that group, so the term *Clydesdales* is said to be distributed. Universal affirmatives like this one distribute the term that is in the first, or subject, position.

However, what if the proposition were a universal negative (type E) proposition, such as "No koala bears are carnivores"? Here all the members of the group "koala bears" (the subject term) are referred to, but all the members of the group "carnivores" (the predicate term) are also referred to. When we say that no koala bears are carnivores, we have said something about all koala bears (that they are not carnivores) and also something about all carnivores (that they are not koala bears). So in this universal negative proposition, both of its terms are distributed.

To sum up distribution for the universal propositions, then: Universal affirmative (A) propositions distribute only the first (subject) term, and universal negative (E) propositions distribute both the first (subject) term and the second (predicate) term.

The distribution pattern follows the same basic idea for particular propositions. A particular affirmative (type I) proposition, such as "Some students are football players," refers only to at least one member of the subject class ("students") and only to at least one member of the predicate class ("football players"). Thus, remembering that *some* is interpreted as meaning "at least one," the particular affirmative proposition distributes neither term, for this proposition does not refer to all the members of either group.

Finally, a particular negative (type O) proposition, such as "Some Floridians are not surfers," only refers to at least one Floridian—but says that at least one Floridian does not belong to the entire class of surfers or is excluded from the entire class of surfers. In this way, the particular negative proposition distributes only the term that refers to surfers, or the predicate term.

To sum up distribution for the particular propositions, then: particular affirmative (I) propositions distribute neither the first (subject) nor the second (predicate) term, and particular negative (O) propositions distribute only the second (predicate) term. This is a lot of detail, to be sure, but it is summarized in Table 3.4.

Table 3.4: Distribution

Proposition	Subject	Predicate
A	Distributed	Not
E	Distributed	Distributed
I	Not	Not
O	Not	Distributed

Once you understand how distribution works, the rules for determining the validity of syllogisms are fairly straightforward. You just need to see that in any given syllogism, there are three terms: a subject term, a predicate term, and a middle term. But there are only two *positions*, or "slots," a term can appear in, and distribution relates to those positions.

Rules for Validity

Once we know how to determine whether a term is distributed, it is relatively easy to learn the rules for determining whether a categorical syllogism is valid. The traditional rules of the syllogism are given in various ways, but here is one standard way:

Rule 1: The middle term must be distributed at least once.

Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.

Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.

Rule 4: The syllogism cannot have two negative premises.

Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.

A syllogism that satisfies all five of these rules will be valid; a syllogism that does not will be invalid. Perhaps the easiest way of seeing how the rules work is to go through a few examples. We can start with our standard syllogism with all universal affirmatives:

All M are P.
All S are M.
Therefore, all S are P.

Rule 1 is satisfied: The middle term is distributed by the first premise; a universal affirmative (A) proposition distributes the term in the first (subject) position, which here is M. Rule 2 is satisfied because the subject term that is distributed by the conclusion is also distributed by the second premise. In both the conclusion and the second premise, the universal affirmative proposition

The Origins of Logic

The text describes five rules for determining a syllogism's validity, but Aristotle's fundamental rules were far more basic.

Origins of Logic
From Title: *Logic: The Structure of Reason*
(https://fod.infobase.com/PortalPlaylists.aspx?wID=100753&xtid=32714)

Critical Thinking Questions

1. The law of noncontradiction and the excluded middle establish that a proposition cannot be both true and false and must be either true or false. Can you think of a proposition that violates either of these rules?
2. Aristotle's syllogism form, or the standard argument form, allows us to condense arguments into their fundamental pieces for easier evaluation. Try putting an argument you have heard into the standard form.

distributes the term in the first position. Rule 3 is also satisfied because there is not a negative premise without a negative conclusion, or a negative conclusion without a negative premise (all the propositions in this syllogism are affirmative). Rule 4 is passed because both premises are affirmative. Finally, Rule 5 is passed as well because there is a universal conclusion. Since this syllogism passes all five rules, it is valid.

These get easier with practice, so we can try another example:

> Some M are not P.
> All M are S.
> Therefore, some S are not P.

Rule 1 is passed because the second premise distributes the middle term, M, since it is the subject in the universal affirmative (A) proposition. Rule 2 is passed because the major term, P, that is distributed in the O conclusion is also distributed in the corresponding O premise (the first premise) that includes that term. Rule 3 is passed because there is a negative conclusion to go with the negative premise. Rule 4 is passed because there is only one negative premise. Rule 5 is passed because the first premise is a particular premise (O). Since this syllogism passes all five rules, it is valid; there is no way that all of its premises could be true and its conclusion false.

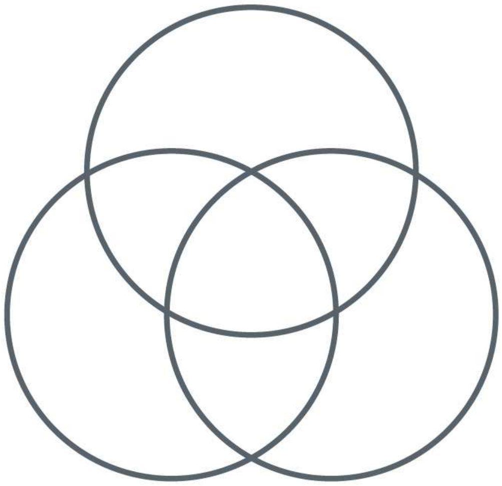
Both of these have been valid; however, out of the 256 possible syllogisms, most are invalid. Let us take a look at one that violates one or more of the rules:

> No P are M.
> Some S are not M.
> Therefore, all S are P.

Rule 1 is passed. The middle term is distributed in the first (major) premise. However, Rule 2 is violated. The subject term is distributed in the conclusion, but not in the corresponding second (minor) premise. It is not necessary to check the other rules; once we know that one of the rules is violated, we know that the argument is invalid. (However, for the curious, Rule 3 is violated as well, but Rules 4 and 5 are passed).

Venn Diagram Tests for Validity

Another value of Venn diagrams is that they provide a nice method for evaluating the validity of a syllogism. Because every valid syllogism has three categorical terms, the diagrams we use must have three circles:



The idea in diagramming a syllogism is that we diagram each premise and then check to see if the conclusion has been automatically diagrammed. In other words, we determine whether the conclusion must be true, according to the diagram of the premises.

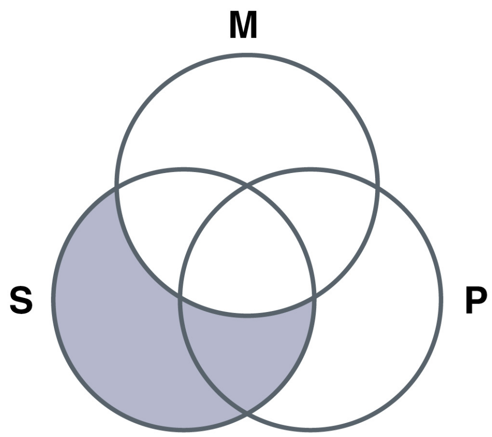
It is important to remember that we never draw a diagram of the conclusion. If the argument is valid, diagramming the premises will automatically provide a diagram of the conclusion. If the argument is invalid, diagramming the premises will not provide a diagram of the conclusions.

Diagramming Syllogisms With Universal Statements

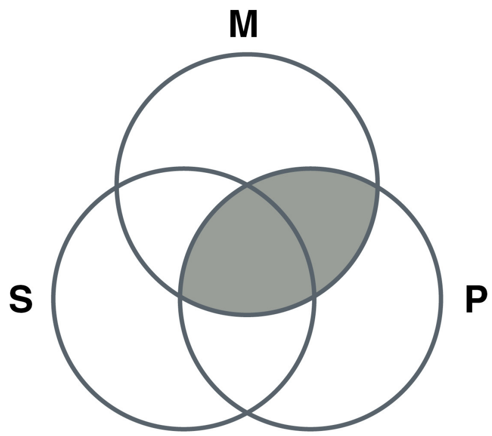
Particular statements are slightly more difficult in these diagrams, so we will start by looking at a syllogism with only universal statements. Consider the following syllogism:

All S is M.
No M is P.
Therefore, no S is P.

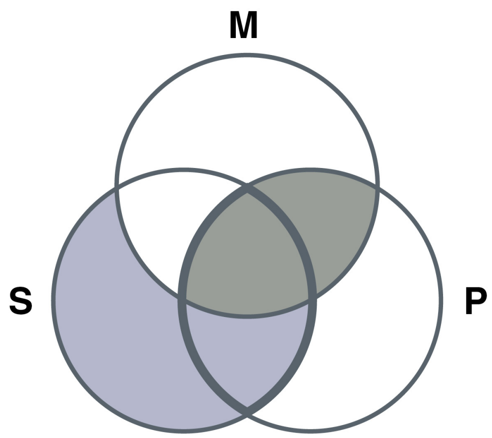
Remember, we are only going to diagram the two premises; we will not diagram the conclusion. The easiest way to diagram each premise is to temporarily ignore the circle that is not relevant to the premise. Looking just at the S and M circles, we diagram the first premise like this:



Here is what the diagram for the second premise looks like:



Now we can take those two diagrams and superimpose them, so that we have one diagram of both premises:

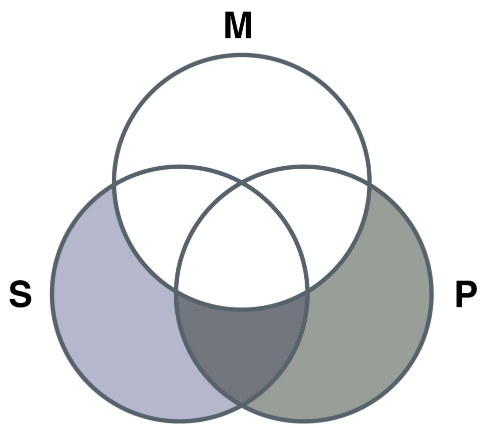


Now we can check whether the argument is valid. To do this, we see if the conclusion is true according to our diagram. In this case our conclusion states that no S is P; is this statement true, according to our diagram? Look at just the S and P circles; you can see that the area between the S and P circles (outlined) is fully shaded. So we have a diagram of the conclusion. It does not matter if the S and P circles have some extra shading in them, so long as the diagram has all the shading needed for the truth of the conclusion.

Let us look at an invalid argument next.

All S is M.
All P is M.
Therefore, all S is P.

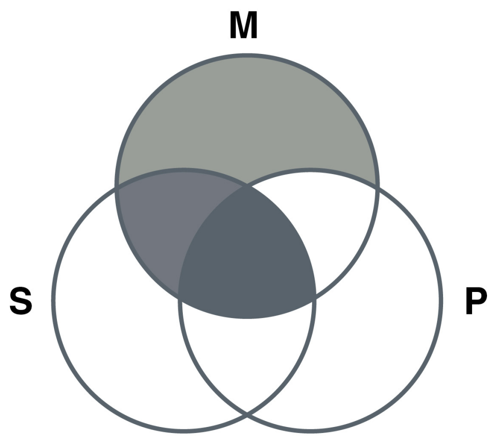
Again, we diagram each premise and look to see if we have a diagram of the conclusion. Here is what the diagram of the premises looks like:



Now we check to see whether the conclusion must be true, according to the diagram. Our conclusion states that all S is P, meaning that no unshaded part of the S circle can be outside of the P circle. In this case you can see that we do not have a diagram of the conclusion. Since we have an unshaded part of S outside of P (outlined), the argument is invalid.

Let us do one more example with all universals.

All M are P.
No M is S.
Therefore, no S is P.
Here is how to diagram the premises:



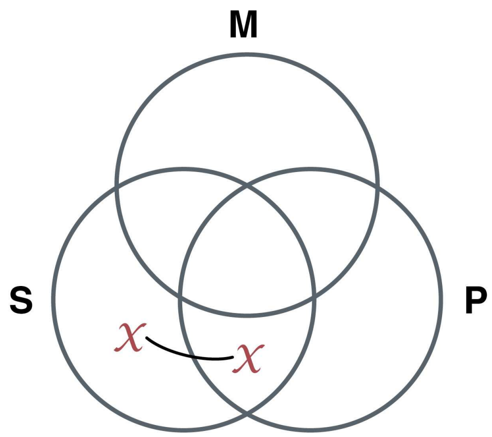
Is the conclusion true in this diagram? In order to know that the conclusion is true, we would need to know that there are no S that are P. However, we see in this diagram that there is room for some S to be P. Therefore, these premises do not guarantee the truth of this conclusion, so the argument is invalid.

Diagramming Syllogisms With Particular Statements

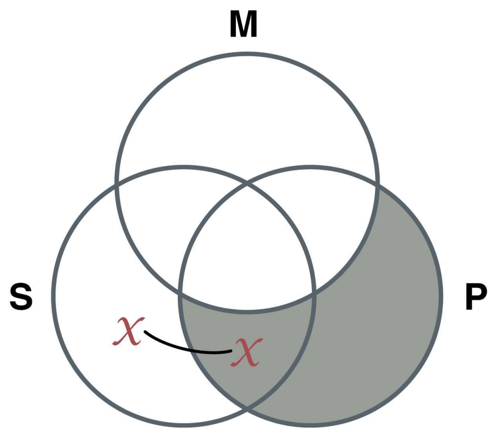
Particular statements (I and O) are a bit trickier, but only a bit. The problem is that when you diagram a particular statement, you put an *x* in a region. If that region is further divided by a third circle, then the single *x* will end up in one of those subregions even though we do not know which one it should go in. As a result, we have to adopt a convention to indicate that the *x* may be in either of them. To do this, we will draw an *x* in each subregion and connect them with a line to show that we mean the individual might be in either subregion. To see how this works, let us consider the following syllogism.

Some S is not M.
All P are M.
Therefore, some S is not P.

We start by diagramming the first premise:



Then we add the diagram for the second premise:



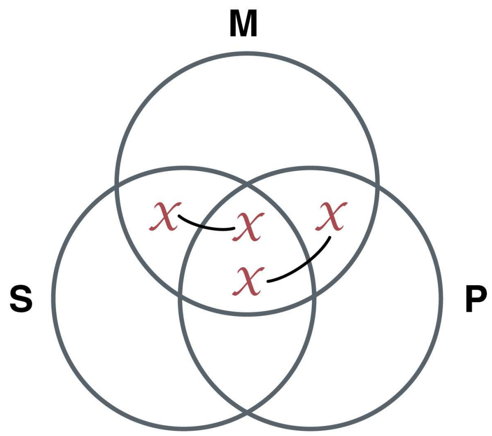
Notice that in diagramming the second premise, we shaded over one of the linked *x*'s. This leaves us with just one *x*. When we look at just the S and P circles, we can see that the remaining is inside the S circle but outside the P circle.

To see if the argument is valid, we have to determine whether the conclusion must be true according to this diagram. The truth of our conclusion depends on there being at least one S that is not P. Here we have just such an entity: The remaining *x* is in the S circle but not in the P circle, so the conclusion must be true. This shows that the conclusion validly follows from the premises.

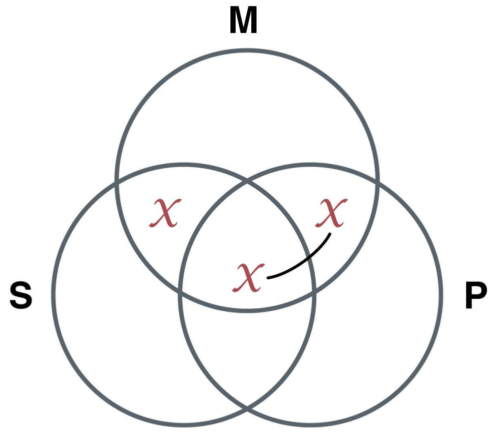
Here is an example of an invalid syllogism.

> Some S is M.
> Some M is P.
> Therefore, some S is P.

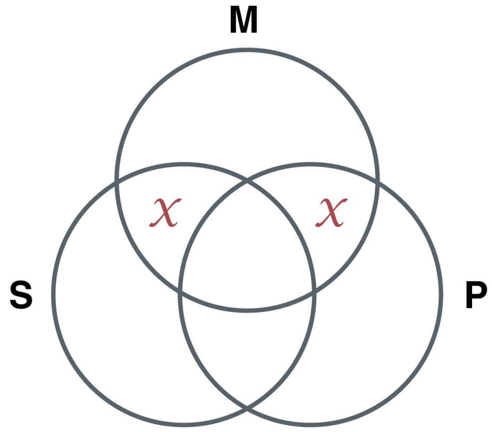
Here is the diagram with both premises represented:



Now it seems we have *x*'s all over the place. Remember, our job now is just to see if the conclusion is already diagrammed when we diagram the premises. The diagram of the conclusion would have to have an *x* that was in the region between where the S and P circles overlap. We can see that there are two in that region, each linked to an *x* outside the region. The fact that they are linked to other *x*'s means that neither *x has to be* in the middle region; they might both be at the other end of the link. We can show this by carefully erasing one of each pair of linked *x*'s. In fact, we will erase one *x* from each linked pair, trying to do so in a way that makes the conclusion false. First we erase the right-hand *x* from the pair in the S circle. Here is what the diagram looks like now:



Now we erase the left-hand *x* from the remaining pair. Here is the final diagram:



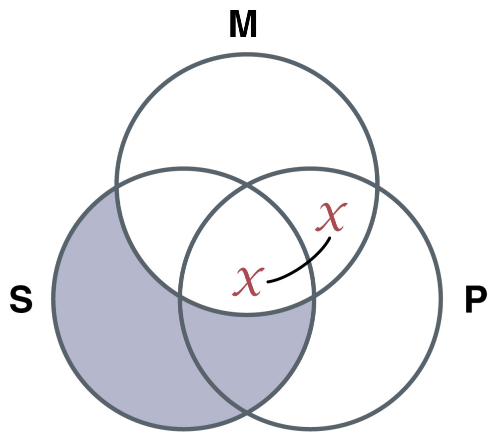
Notice that there are no *x*'s remaining in the overlapped region of S and P. This modification of the diagram still makes both premises true, but it also makes the conclusion false. Because this combination is possible, that means that the argument must be invalid.

Here is a more common example of an invalid categorical syllogism:

All S are M.
Some M are P.
Therefore, some S are P.

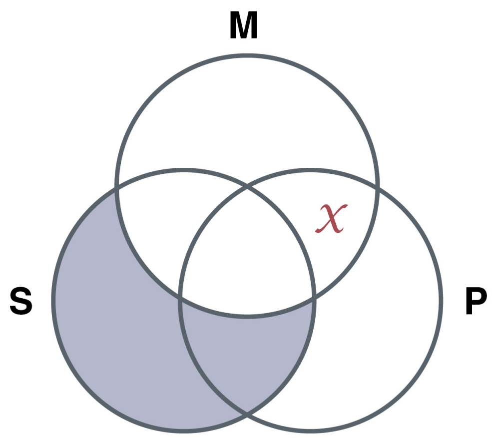
This argument form looks valid, but it is not. One way to see that is to notice that Rule 1 is violated: The middle term does not distribute in either premise. That is why this argument form represents an example of the common deductive error in reasoning known as the "undistributed middle."

A perhaps more intuitive way to see why it is invalid is to look at its Venn diagram. Here is how we diagram the premises:



The two *x*'s represent the fact that our particular premise states that some M are P and does not state whether or not they are in the S circle, so we represent both possibilities here. Now we simply need to check if the conclusion is necessarily true.

We can see that it is not, because although one *x* is in the right place, it is linked with another *x* in the wrong place. In other words, we do not know whether the *x* in "some M are P" is inside or outside the S boundary. Our conclusion requires that the *x* be inside the S boundary, but we do not know that for certain whether it is. Therefore, the argument is invalid. We could, for example, erase the linked *x* that is inside of the S circle, and we would have a diagram that makes both premises be true and the conclusion false.



Because this diagram shows that it is possible to make the premises true and the conclusion false, it follows that the argument is invalid.

A final way to understand why this form is invalid is to use the counterexample method and consider that it has the same form as the following argument:

> All dogs are mammals.
> Some mammals are cats.
> Therefore, some dogs are cats.

This argument has the same form and has all true premises and a false conclusion. This counterexample just verifies that our Venn diagram test got the right answer. If applied correctly, the Venn diagram test works every time. With this example, all three methods agree that our argument is invalid.

Moral of the Story: The Venn Diagram Test for Validity

Here, in summary, are the steps for doing the Venn diagram test for validity:

1. Draw the three circles, all overlapping.
2. Diagram the premises.

   a. Shade in areas where nothing exists.
   b. Put an *x* for areas where something exists.
   c. If you are not sure what side of a line the *x* should be in, then put two linked *x*'s, one on each side.

3. Check to see if the conclusion is (must be) true in this diagram.
   a. If there are two linked *x*'s, and one of them makes the conclusion true and the other does not, then the argument is invalid because the premises do not guarantee the truth of the conclusion.
   b. If the conclusion must be true in the diagram, then the argument is valid; otherwise it is not.

Practice Problems 3.4

Answer the following questions. Note that some questions may have more than one answer. Click here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems3.4_corrected.pdf) to check your answers.

1. Which rules does the following syllogism pass?

   All M are P.
   Some M are S.
   Therefore, some S are P.

   a. Rule 1: The middle term must be distributed at least once.
   b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
   c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
   d. Rule 4: The syllogism cannot have two negative premises.
   e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
   f. All the rules

2. Which rules does the following syllogism fail?

   No P are M.
   All S are M.
   Therefore, all S are P.

   a. Rule 1: The middle term must be distributed at least once.
   b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
   c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
   d. Rule 4: The syllogism cannot have two negative premises.
   e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
   f. All the rules

3. Which rules does the following syllogism fail?

   Some M are P.
   Some S are not M.
   Therefore, some S are not P.

   a. Rule 1: The middle term must be distributed at least once.
   b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
   c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
   d. Rule 4: The syllogism cannot have two negative premises.
   e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.

f. All the rules

4. Which rules does the following syllogism fail?

   No P are M.
   No M are S.
   Therefore, no S are P.

   a. Rule 1: The middle term must be distributed at least once.
   b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
   c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
   d. Rule 4: The syllogism cannot have two negative premises.
   e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
   f. All the rules

5. Which rules does the following syllogism fail?

   All M are P.
   Some M are not S.
   Therefore, no S are P.

   a. Rule 1: The middle term must be distributed at least once.
   b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
   c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
   d. Rule 4: The syllogism cannot have two negative premises.
   e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
   f. All the rules

6. Which rules does the following syllogism fail?

   All humans are dogs.
   Some dogs are mammals.
   Therefore, no humans are mammals.

   a. Rule 1: The middle term must be distributed at least once.
   b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
   c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
   d. Rule 4: The syllogism cannot have two negative premises.
   e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
   f. None of the rules

7. Which rules does the following syllogism fail?

   Some books are hardbacks.
   All hardbacks are published materials.
   Therefore, some books are published materials.

   a. Rule 1: The middle term must be distributed at least once.
   b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
   c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
   d. Rule 4: The syllogism cannot have two negative premises.
   e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
   f. None of the rules

8. Which rules does the following syllogism fail?

No politicians are liars.
Some politicians are men.
Therefore, some men are not liars.

a. Rule 1: The middle term must be distributed at least once.
b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
d. Rule 4: The syllogism cannot have two negative premises.
e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
f. None of the rules

9. Which rules does the following syllogism fail?

Some Macs are computers.
No PCs are Macs.
Therefore, all PCs are computers.

a. Rule 1: The middle term must be distributed at least once.
b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
d. Rule 4: The syllogism cannot have two negative premises.
e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
f. None of the rules

10. Which rules does the following syllogism fail?

All media personalities are people who manipulate the masses.
No professors are media personalities.
Therefore, no professors are people who manipulate the masses.

a. Rule 1: The middle term must be distributed at least once.
b. Rule 2: Any term distributed in the conclusion must be distributed in its corresponding premise.
c. Rule 3: If the syllogism has a negative premise, it must have a negative conclusion, and if the syllogism has a negative conclusion, it must have a negative premise.
d. Rule 4: The syllogism cannot have two negative premises.
e. Rule 5: If the syllogism has a particular premise, it must have a particular conclusion, and if the syllogism has a particular conclusion, it must have a particular premise.
f. None of the rules

11. Examine the following syllogisms. In the first pair, the terms that are distributed are marked in bold. Can you explain why? The second pair is left for you to determine which terms, if any, are distributed.

Some P are M.
Some M are not **S**.
Therefore, some S are not **P**.

No **P** are **M**.
All **M** are S.
Therefore, no **S** are **P**.

All M are P.
All M are S.
Therefore, all S are P.

Some P are not M.
No S are M.
Therefore, no S are P.

3.7 Categorical Logic: Types of Categorical Arguments

Many examples of deductively valid arguments that we have considered can seem quite simple, even if the theory and rules behind them can be a bit daunting. You might even wonder how important it is to study deduction if even silly arguments about the moon being tasty are considered valid. Remember that this is just a brief introduction to deductive logic. Deductive arguments can get quite complex and difficult, even though they are built from smaller pieces such as those we have covered in this chapter. In the same way, a brick is a very simple thing, interesting in its form, but not much use all by itself. Yet someone who knows how to work with bricks can make a very complex and sturdy building from them.

Thus, it will be valuable to consider some of the more complex types of categorical arguments, sorites and enthymemes. Both of these types of arguments are often encountered in everyday life.

Sorites

A *sorites* is a specific kind of argument that strings together several subarguments. The word *sorites* comes from the Greek word meaning a "pile" or a "heap"; thus, a sorites-style argument is a collection of arguments piled together. More specifically, a **sorites** is any categorical argument with more than two premises; the argument can then be turned into a string of categorical syllogisms. Here is one example, taken from Lewis Carroll's book *Symbolic Logic* (1897/2009):

> The only animals in this house are cats;
> Every animal is suitable for a pet, that loves to gaze at the moon;
> When I detest an animal, I avoid it;
> No animals are carnivorous, unless they prowl at night;
> No cat fails to kill mice;
> No animals ever take to me, except what are in this house;
> Kangaroos are not suitable for pets;
> None but carnivora kill mice;
> I detest animals that do not take to me;
> Animals, that prowl at night, always love to gaze at the moon.
> Therefore, I always avoid kangaroos. (p. 124)

Figuring out the logic in such complex sorites can be challenging and fun. However, it is easy to get lost in sorites arguments. It can be difficult to keep all the premises straight and to make sure the appropriate relationships are established between each premise in such a way that, ultimately, the conclusion follows.

Carroll's sorites sounds ridiculous, but as discussed earlier in the chapter, many of us develop complex arguments in daily life that use the conclusion of an earlier argument as the premise of the next argument. Here is an example of a relatively short one:

> All of my friends are going to the party.
> No one who goes to the party is boring.
> People that are not boring interest me.
> Therefore, all of my friends interest me.

Here is another example that we might reason through when thinking about biology:

> All lizards are reptiles.
> No reptiles are mammals.
> Only mammals nurse their young.
> Therefore, no lizards nurse their young.

There are many examples like these. It is possible to break them into smaller syllogistic subarguments as follows:

> All lizards are reptiles.
> No reptiles are mammals.
> Therefore, no lizards are mammals.
> No lizards are mammals.
> Only mammals nurse their young.
> Therefore, no lizards nurse their young.

Breaking arguments into components like this can help improve the clarity of the overall reasoning. If a sorites gets too long, we tend to lose track of what is going on. This is part of what can make some arguments hard to understand. When constructing your own arguments, therefore, you should beware of bunching premises together unnecessarily. Try to break a long argument into a series of smaller arguments instead, including subarguments, to improve clarity.

Enthymemes

While sorites are sets of arguments strung together into one larger argument, a related argument form is known as an **enthymeme**, a syllogistic argument that omits either a premise or a conclusion. There are also many nonsyllogistic arguments that leave out premises or conclusions; these are sometimes also called enthymemes as well, but here we will only consider enthymemes based on syllogisms.

A good question is why the arguments are missing premises. One reason that people may leave a premise out is that it is considered to be too obvious to mention. Here is an example:

> All dolphins are mammals.
> Therefore, all dolphins are animals.

Here the suppressed premise is "All mammals are animals." Such a statement probably does not need to be stated because it is common knowledge, and the reader knows how to fill it in to get to the conclusion. Technically speaking, we are said to "suppress" the premise that does not need to be stated.

Sometimes people even leave out conclusions if they think that the inference involved is so clear that no one needs the conclusion stated explicitly. Arguments with unstated conclusions are considered enthymematic as well. Let us suppose a baseball fan complains, "You have to be rich to get tickets to game 7, and none of my friends is rich." What is the implied conclusion? Here is the argument in standard form:

> Everyone who can get tickets to game 7 is rich.
> None of my friends is rich.
> Therefore, ???

In this case we may validly infer that none of the fan's friends can get tickets to game 7.

To be sure, you cannot always assume your audience has the required background knowledge, and you must attempt to evaluate whether a premise or conclusion does need to be stated explicitly. Thus, if you are talking about math to professional physicists, you do not need to spell out precisely what the hypotenuse of an angle is. However, if you are talking to third graders, that is certainly not a safe assumption. Determining the background knowledge of those with whom one is talking—and arguing—is more of an art than a science.

Validity in Complex Arguments

Recall that a valid argument is one whose premises guarantee the truth of the conclusion. Sorites are illustrations of how we can "stack" smaller valid arguments together to make larger valid arguments. Doing so can be as complicated as building a cathedral from bricks, but so long as each piece is valid, the structure as a whole will be valid.

How do we begin to examine a complex argument's validity? Let us start by looking at another example of sorites from Lewis Carroll's book *Symbolic Logic* (1897/2009):

> Babies are illogical.
> Nobody is despised who can manage a crocodile.
> Illogical persons are despised.
> Therefore, no babies can manage a crocodile. (p. 112)

Is this argument valid? We can see that it is by breaking it into a pair of syllogisms. Start by considering the first and third premises. We will rewrite them slightly to show the *All* that Carroll has assumed. With those two premises, we can build the following valid syllogism:

> All babies are illogical.
> All illogical persons are despised.
> Therefore, all babies are despised.

Using the tools from this chapter (the rules, Venn diagrams, or just by thinking it through carefully), we can check that the syllogism is valid. Now we can use the conclusion of our syllogism along with the remaining premise and conclusion from the original argument to construct another syllogism.

All babies are despised.
No despised persons can manage a crocodile.
Therefore, no babies can manage a crocodile.

Again, we can check that this syllogism is valid using the tools from this chapter. Since both of these arguments are valid, the string that combines them is valid as well. Therefore, the original argument (the one with three premises) is valid.

This process is somewhat like how we might approach adding a very long list of numbers. If you need to add a list of 100 numbers (suppose you are checking a grocery bill), you can do it by adding them together in groups of 10, and then adding the subtotals together. As long as you have done the addition correctly at each stage, your final answer will be the correct total. This is one reason validity is important. It allows us to have confidence in complex arguments by examining the smaller arguments from which they are, or can be, built. If one of the smaller arguments was not valid, then we could not have complete confidence in the larger argument.

But what about soundness? What use is the argument about babies managing crocodiles when we know that babies are not generally despised? Again, let us make a comparison to adding up your grocery bill. Arithmetic can tell you if your bill is added correctly, but it cannot tell you if the prices are correct or if the groceries are really worth the advertised price. Similarly, logic can tell you whether a conclusion validly follows from a set of premises, but it cannot generally tell you whether the premises are true, false, or even interesting. By themselves, random deductive arguments are as useful as sums of random numbers. They may be good practice for learning a skill, but they do not tell us much about the world unless we can somehow verify that their premises are, in fact, true. To learn about the world, we need to apply our reasoning skills to accurate facts (usually outside of arithmetic and logic) known to be true about the world.

This is why logicians are not as concerned with soundness as they are with validity, and why a mathematician is only concerned with whether you added correctly, and not with whether the prices were correctly recorded. Logic and mathematics give us skills to apply valid reasoning to the information around us. It is up to us, and to other fields, to make sure the information that we use in the premises is correct.

Practice Problems 3.5

Answer the following questions. Click here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems3.5.pdf) to check your answers.

1. This is the name that is given to an argument that has two premises and one conclusion.
   a. syllogism
   b. creative syllogism
   c. enthymeme
   d. sorites
   e. none of the above

2. The discovery of categorical logic is often attributed to this philosopher.
   a. Plato
   b. Boole
   c. Aristotle
   d. Kant
   e. Hume

3. Which of the following is a type of deductive argument?
   a. generalization
   b. categorical syllogism
   c. argument by analogy
   d. *modus spartans*
   e. none of the above

4. All categorical statements have which of the following?
   a. mood and placement
   b. figure and form
   c. number and validity
   d. quantity and quality

e. all of the above

5. The premise that contains the predicate term of the conclusion in a categorical syllogism is _________.
    a. the minor premise
    b. the major premise
    c. the necessary premise
    d. the conclusion
    e. none of the above

Summary and Resources

Chapter Summary

Validity is the central concept of deductive reasoning. An argument is valid when the truth of the premises absolutely guarantees the truth of the conclusion. For valid arguments, if the premises are true, then the conclusion must be true also. Valid arguments need not have true premises, but if they do, then they are sound arguments. Because they use valid reasoning and have true premises, sound arguments are guaranteed to have true conclusions.

Deductive arguments can include mathematical arguments, arguments from definitions, categorical arguments, and propositional arguments. Categorical arguments allow us to reason about things based on their properties. Categorical arguments with two premises are called syllogisms. The validity of syllogisms can be evaluated either with a system of rules or by using Venn diagrams.

Syllogisms often leave one premise or the conclusion unstated. These are called enthymemes. Sometimes strings of syllogisms are combined into a larger argument called a sorites. If we have a string of valid arguments that are combined to make a larger argument, then we may infer that the long argument composed of these parts is valid as well.

The process of using subarguments to create longer ones allows us to make rather complex valid arguments out of simple parts. This is an important motivation for studying deductive logic. As with arithmetic, computer programming, and structural engineering, combining smaller steps in a careful way allows us to create complex structures that are fully reliable because they are built out of reliable parts.

Critical Thinking Questions

1. How does the logical definition of *validity* differ from the way that the term *valid* is used in everyday speech? How do you plan on differentiating the two as you continue studying logic?
2. In the chapter, you read a section about the importance of having evidence that supports your arguments. Is it important to claim to believe things only when one has evidence, or are there some things that people can justifiably believe without evidence? Why?
3. How would you describe what a deductive argument is to someone who does not know the technical terms that apply to arguments? What examples would you use to demonstrate deduction?
4. What is the point of being able to understand if a deductive argument is valid or sound? Why is it important to be able to determine these things? If you do not think it is important, how would you justify your claims that it is not important to be able to determine validity?
5. Has there ever been a time that you presented an argument in which you had little or no evidence to support your claims? What types of claims did you use in the place of premises? What types of techniques did you use to try to present an argument with no information to back up your conclusion(s)? What is a better method to use in the future?

Web Resources

http://www.philosophyexperiments.com/validorinvalid/Default.aspx (http://www.philosophyexperiments.com/validorinvalid/Default.aspx)
This game at the Philosophy Experiments website tests your ability to determine whether an argument is valid.

http://www.thefirstscience.org/syllogistic-machine (http://www.thefirstscience.org/syllogistic-machine)
This professor's blog includes an online syllogism solver that allows you to explore fallacies, figures, terms, and modes of syllogisms. Click on "Notes on Syllogistic Logic" for more coverage of topics discussed in this chapter.

Key Terms

argument from definition
An argument in which one premise is a definition.

categorical argument
An argument entirely composed of categorical statements.

categorical logic
The branch of deductive logic that is concerned with categorical arguments.

categorical statement
A statement that relates one category or class to another. Specifically, if S and P are categories, the categorical statements relating them are: All S is P, No S is P, Some S is P, and Some S is not P.

complement class
For a given class, the complement class consists of all things that are not in the given class. For example, if S is a class, its complement class is non-S.

contraposition
The immediate inference obtained by switching the subject and predicate terms with each other and complementing them both.

conversion
The immediate inference obtained by switching the subject and predicate terms with each other.

counterexample method
The method of proving an argument form to be not valid by constructing an instance of it with true premises and a false conclusion.

deductive argument
An argument that is presented as being valid—if the primary evaluative question about the argument is whether it is valid.

distribution
Referring to members of groups. If all the members of a group are referred to, the term that refers to that group is said to be distributed.

enthymeme
An argument in which one or more claims are left unstated.

immediate inferences
Arguments from one categorical statement as premise to another as conclusion. In other words, we immediately infer one statement from another.

instance
A term in logic that describes the sentence that results from replacing each variable within the form with specific sentences.

logical form
The pattern of an argument or claim.

predicate term
The second term in a categorical proposition.

quality
In logic, the distinction between a statement being affirmative or negative.

quantity
In logic, the distinction between a statement being universal or particular.

sorites
A categorical argument with more than two premises.

sound
Describes an argument that is valid and in which all of the premises are true.

subject term
The first term in a categorical proposition.

syllogism
A deductive argument with exactly two premises.

valid
An argument in which the premises absolutely guarantee the conclusion, such that is *impossible* for the premises to be true while the conclusion is false.

Venn diagram
A diagram constructed of overlapping circles, with shaded areas or *x*'s, which shows the relationships between the represented groups.

An argument in which one premise is a definition.

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