2.2 Putting Arguments in the Standard Form

Presenting arguments in the standard argument form is crucial because it provides us with a dispassionate method that will allow us to find out whether the argument is good, regardless of how we feel about the subject matter. The first step is to identify the fundamental argument being presented.

At first it might seem a bit daunting to identify an argument, because arguments typically do not come neatly presented in the standard argument form. Instead, they may come in confusing and unclear language, much like this statement by Special Prosecutor Francis Schmitz of Wisconsin regarding Governor Scott Walker:

> Governor Walker was not a target of the investigation. At no time has he been served with a subpoena. . . . While these documents outlined the prosecutor's legal theory, they did not establish the existence of a crime; rather, they were arguments in support of further investigation to determine if criminal charges against any person or entity are warranted. (Crocker, 2014, para. 7 & 10)

This was a position presented in regard to the investigation of an alleged illegal campaign finance coordination during the 2011–2012 recall elections (Stein, 2014). Does it claim a vindication of Walker? Or does it suggest that there may be sufficient evidence to make Walker a central figure in the investigation? How would you even begin to make heads or tails of such a confusing argument? Do not despair. The remainder of this section will show you exactly what to look for in order to make sense of the most complicated argument. With a little practice, you will be able to do this without much effort.

Find the Conclusion First

Although the conclusion is last in the standard form, the conclusion is the first thing to find because the conclusion is the main claim in an argument. The other claims—the premises—are present for the sole purpose of supporting the conclusion. Accordingly, if you are able to find the conclusion, then you should be able to find the premises.

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Punctuation, parentheses, and conclusion indicators all serve as signposts to assist us when deconstructing an argument. They provide important clues about where to find the conclusion as well as supporting claims.

The good news is that language is not only a means for expressing ideas; it also offers a road map for the ideas presented. Chapter 1 underscored the fundamental importance of clear, precise, and correct language in logical reasoning. When used properly, language also offers structures and directions for communicating meaning, thus facilitating our understanding of what others are saying. One punctuation mark—the question mark—tells us that we are confronting a question. A different punctuation mark—the parentheses—tells us that we are being given relevant information but only as an aside or afterthought to the main point; if removed, the parenthetical information would not alter the main point. In the case of arguments, some words serve as signposts identifying conclusions. Take the following example of an argument in the standard argument form:

> All men are mortal.
> Socrates is a man.
> Therefore, Socrates is mortal.

The word *therefore* indicates that the sentence is a conclusion. In fact, the word *therefore* is the standard **conclusion indicator** we will use when constructing arguments in the standard argument form. However, there are other conclusion indicators that are used in ordinary arguments, including:

- Consequently . . .
- So . . .
- Hence . . .
- Thus . . .
- Wherefore . . .
- As a result . . .
- It follows that . . .
- For these reasons . . .
- We may conclude that . . .

When a conclusion indicator is present, it can help identify the conclusion in an argument. Unfortunately, many arguments do not come with conclusion indicators. In such cases start by trying to identify the main point. If you can clearly identify a single main point, then that is likely to be the conclusion. But sometimes you will have to look at a passage closely to find the conclusion. Suppose you encounter the following argument:

> Don't you know that driving without a seat belt is dangerous? Statistics show that you are 10 times more likely to be injured in an accident if you are not wearing one. Besides, in our state you can get fined $100 if you are caught not wearing one. You ought to wear one even if you are driving a short distance.

Arguments are often longer and more complicated than this one, but let us work with this simple case before trying more complicated examples. You know that the first thing you need to do is to look for the conclusion. The problem is that the author of the argument does not use a conclusion indicator. Now what? Nothing to worry about. Just remember that the conclusion is the main claim, so the thing to look for is what the author may be trying to defend. Although the first sentence is stated as a question—remember, punctuation marks give us important clues—the author seems to intend to assert that driving without a seat belt is dangerous. In fact, the second sentence offers evidence in support of this claim. On the other hand, the third sentence seems to be important, yet it does not speak to driving without a seat belt being dangerous, only expensive. In the final sentence, we find a claim that is supported by all the others. Because of this, the final sentence presents the conclusion.

Now, it so happens that in this case, the conclusion is at the end of this short argument, but keep in mind that conclusions can be found in various places in essays, such as the beginning or sometimes in the middle. Now that you have identified your first piece of the puzzle, we have this:

> Premise 1: ?
> Premise 2: ?
> Premise 3: ?
> Therefore, you ought to wear a seat belt whenever you drive.

You might have noticed that the conclusion does not appear as it did in the essay. The original sentence is "You ought to wear one even if you are driving a short distance." Why did we modify it? Once again, clarity is of the essence in logical reasoning. Conclusions should make the subject clear, so the pronoun *one* was replaced with the actual subject to which the author is referring: seat belt. In addition, the predicate "even if you are driving a short distance" was rewritten to reflect the more inclusive point that the author seems to be making: that you should wear a seat belt *whenever* you drive.

This modification of language, known as paraphrasing, is part of the construction of arguments in the standard argument form. The act of extracting an argument from a longer piece to its fundamental claims in the standard argument form necessarily involves paraphrasing the original language to the clearest and most precise form possible. This concept will be addressed in greater detail later in this section.

Find the Premises Next

After identifying the conclusion, the next thing to do is look for the reasons the author offers in defense of his or her position. These are the premises. As with conclusions, there are **premise indicators** that serve as signposts that reasons are being offered for the main claim or conclusion. Some examples of premise indicators are:

- Since . . .
- For . . .
- Given that . . .
- Because . . .
- As . . .
- Owing to . . .
- Seeing that . . .
- May be inferred from . . .

To practice identifying premises, let us return to our seat belt example:

> Don't you know that driving without a seat belt is dangerous? Statistics show that you are 10 times more likely to be injured in an accident if you are not wearing one. Besides, in our state you can get fined $100 if you are caught not wearing one. You ought to wear one even if you are driving a short distance.

Notice again that this argument starts with a question: "Don't you know that driving without a seat belt is dangerous?" The author is not really asking whether *you know* that driving without a seat belt is

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Much like a map will get you from point A to point B, putting an argument into the standard argument form will help you navigate from the conclusion to the premises and vice versa.

dangerous. Rather, the author seems to be asking a rhetorical question—a question that does not actually demand an answer—to assert that driving without a seat belt is dangerous. You should avoid asking rhetorical questions in the essays that you write, because the outcome can be highly uncertain. The success of a rhetorical question depends on the reader or listener first understanding the hidden meaning behind the rhetorical question and then correctly articulating the answer you have in mind. This does not always work.

For the sake of this example, however, let us do our best to try to get at the author's intention. We could paraphrase the first premise to the following claim: Driving without a seat belt is dangerous. Does this paraphrased claim serve as a premise in support of the conclusion? In order to answer this, we need to put the conclusion in the form of a question. Again, premises are reasons offered in support of the conclusion, so if we have a well-constructed argument, then the premises should answer why the conclusion is the case. This is what we would have:

Question: Why must you wear a seat belt whenever you drive?

Answer: Because driving without a seat belt is dangerous.

This works, so the paraphrased claim that we drew from the author's rhetorical question is indeed a reason in defense of the conclusion. So now we have one more piece of the puzzle:

Premise 1: Driving without a seat belt is dangerous.
Premise 2: ?
Premise 3: ?
Therefore, you ought to wear a seat belt whenever you drive.

Let us now move to the next sentence: "Statistics show that you are 10 times more likely to be injured in an accident if you are not wearing one." Is this a claim that can be a support for the conclusion? In other words, if we put the conclusion in the form of a question again as we did before, would this sentence be an adequate reason in response? Let us see.

Question: Why must you wear a seat belt whenever you drive?

Answer: Because statistics show that you are 10 times more likely to be injured in an accident if you are not wearing one.

The answer provides a reason in support of the conclusion, and thus, we have another premise. Now we have most of the puzzle completed, as follows:

Premise 1: Driving without a seat belt is dangerous.
Premise 2: Statistics show that you are 10 times more likely to be injured in an accident if you are not wearing one.
Premise 3: ?
Therefore, you ought to wear a seat belt whenever you drive.

We have one more sentence left in the argument, which reads: "Besides, in our state you can get fined $100 if you are caught not wearing one." Is this a premise? Well, it is uncertain, since the sentence is not presented in the form of a claim. So let us paraphrase it as a claim as follows: "Not wearing a seat belt can result in a $100 fine." This is now a claim, and the paraphrasing has not altered the meaning, so we can proceed to our question: Is this a premise for the argument that we are examining? Once again, let us put the conclusion into a question:

Question: Why must you wear a seat belt whenever you drive?

Answer: Because not wearing a seat belt can result in a $100 fine.

This is a claim that can be a support for the conclusion, and thus, we have another premise. We can now see the argument presented more formally as follows:

Driving without a seat belt is dangerous.
Statistics show that you are 10 times more likely to be injured in an accident if you are not wearing one.

Not wearing a seat belt can result in a $100 fine.
Therefore, you ought to wear a seat belt whenever you drive.

The Necessity of Paraphrasing

As we have discussed, extracting the fundamental claims from a written or a spoken argument often involves paraphrasing. Paraphrasing is not merely an option but rather a necessity in order to uncover the intended argument in the best way possible. Most other arguments presented to you (especially those in the media) will not consist of only premises and the conclusion in clearly identifiable language. Furthermore, many arguments will be much longer and complicated than the seat belt argument example. Often, arguments are presented with many other sentences that do not serve the purposes of an argument, such as empty rhetorical devices, filler sentences that aim to manipulate your emotions, and so on. So your task in extracting an argument from such sources is akin to that of a surgeon—removing all those linguistic tumors that obscure the argument in order to reveal the basic claims presented and their supporting evidence. In other words, you should expect to do paraphrasing as a necessary task when you attempt to draw an argument in the standard form from almost any source.

It is important to recognize that not everyone who advances an argument does so clearly or even coherently. This is precisely why the structure of the standard argument form is such a powerful tool to command. It offers you the machinery to distinguish arguments from what are not arguments. It also helps you unearth the elements of an argument that are buried under complicated prose and rhetoric. And it helps you evaluate the worthiness of the argument presented once it has been fully clarified. You should paraphrase all claims when presenting them in the standard argument form, whether the claims are implied in a long argumentative essay or speech or in shorter arguments that may be ambiguous or unclear. (To understand the added benefits, see *Everyday Logic: Modesty and Charity*.)

Everyday Logic: Modesty and Charity

The goal of paraphrasing is to find the best presentation of the premises and conclusions intended. By presenting the argument offered in its best possible light, this will help you see not only how far off the argument is from an optimal defense, but also how good it is despite its bad presentation. Why should you be so charitable?

First we must keep in mind that ideas are important, even if the ideas are not ours. So we must always give our utmost due diligence to the examination of ideas. Sometimes even the roughest presentation of ideas can contain the most impressive pearls of insight. If we are not charitable to the ideas of others, then we might miss out on hidden wisdom.

Second, modesty is a good intellectual habit to develop. It is very easy to fall into the trap of thinking that our thoughts are the best ones around. This is generally far from the truth. The most fruitful innovations of mankind have been quite unexpected, often as the result of someone paying attention to others' ideas and coming up with a new way of putting them to use. This applies to all sorts of things, including everything from the ways in which cooking methods turned into regional cuisines, to scientific discoveries, product innovations, and the emergence of the Internet.

That modesty has advantages is not a new idea. In the 1980s Peter Drucker wrote the book *Innovation and Entrepreneurship*, in which he recounts, among many other stories, the story of how Ray Kroc founded the burger chain McDonald's®. As the well-known story goes, Kroc bought a hamburger stand from the McDonald brothers, along with their invention of a milkshake machine. Although Kroc never invented anything, his entrepreneurial genius was in seeing the potential of a hamburger, fries, and milkshake business that catered to mothers with little children and turning this vision into a billion-dollar standardized operation (Drucker, 1985/2007).

Even if you dislike McDonald's, the point is that Kroc noticed the potential for something that many, including the McDonald brothers themselves, had overlooked. Gems are everywhere in the world of ideas, but we often have to dust them off, remove all the excess baggage, and extract what is good in them. Intellectual modesty allows us to do this; we don't blind ourselves by assuming our own ideas are best. Once we seek to fully understand others' ideas and allow them to challenge our own, we can do all sorts of good things: understand an idea more clearly, understand someone better, and understand ourselves (our values, what we find important, and so on) better as well.

Given that our human social world is characterized by diversity of ideas, modesty also marks the path of cooperation, harmony, and respect among human beings. This is one of the many small ways in which the application of logical reasoning can help us all have better lives and better relations with other people. If we could all use logical reasoning on a regular basis, perhaps we would not have as many wars and atrocities as we have today.

Thinking Analytically

Identifying an argument's components as we have just done is an example of analytical thinking. When we analyze something, we examine its architectural structure—that is, the relation of the whole to its parts—to identify its parts and to see how the parts fit together as a whole.

Let us examine an excerpt from President Barack Obama's (2014) speech on Ebola as a way of bringing the new skills from this section all together:

> In West Africa, Ebola is now an epidemic of the likes that we have not seen before. It's spiraling out of control. It is getting worse. It's spreading faster and exponentially. Today, thousands of people in West Africa are infected. That number could rapidly grow to tens of thousands. And if the outbreak is not stopped now, we could be looking at hundreds of thousands of people infected, with profound political and economic and security implications for all of us. So this is an epidemic that is not just a threat to regional security—it's a potential threat to global security if these countries break down, if their economies break down, if people panic. That has profound effects on all of us, even if we are not directly contracting the disease. (para. 8)

We have identified "The West African Ebola epidemic is a potential threat to global security" as the conclusion. What are the premises? Read the passage a few times while asking yourself, "Why should I think the epidemic is a global threat?" Obama says that the epidemic is not like others, that it is growing faster and exponentially. He moves from there being thousands of people infected, to tens of thousands, to the possibility of hundreds of thousands. So far, everything is about how fast the epidemic is growing.

In the middle of the seventh sentence, the president switches from talking about the growth of the epidemic to claiming that it has profound economic and security implications. What is the basis for the claim that the growth will have these effects? Notice that it is not in the seventh sentence, at least not explicitly. However, the last part of the eighth sentence does address this. In that sentence, Obama suggests three conditions that might lead to a global security threat: "if these countries break down, if their economies break down, if people panic." So the extreme growth of the epidemic may lead to the breakdown of economies or countries, or it may lead to widespread panic. If any of these things happen, there are "profound effects on all of us." Therefore, the epidemic is a potential threat to global security. We can now list the premises, and indeed the entire argument, in standard form as follows:

> The West African Ebola epidemic is growing extremely fast.
> If the growth isn't stopped, the countries may break down.
> If the growth isn't stopped, the economies may break down.
> If the growth isn't stopped, people may panic.
> Any of these things would have profound effects on people outside of the region.
> Therefore, the West African Ebola epidemic is a potential threat to global security.

Notice that putting the argument in standard form may lose some of the fluidity of the original, but it more than makes up for it in increased clarity.

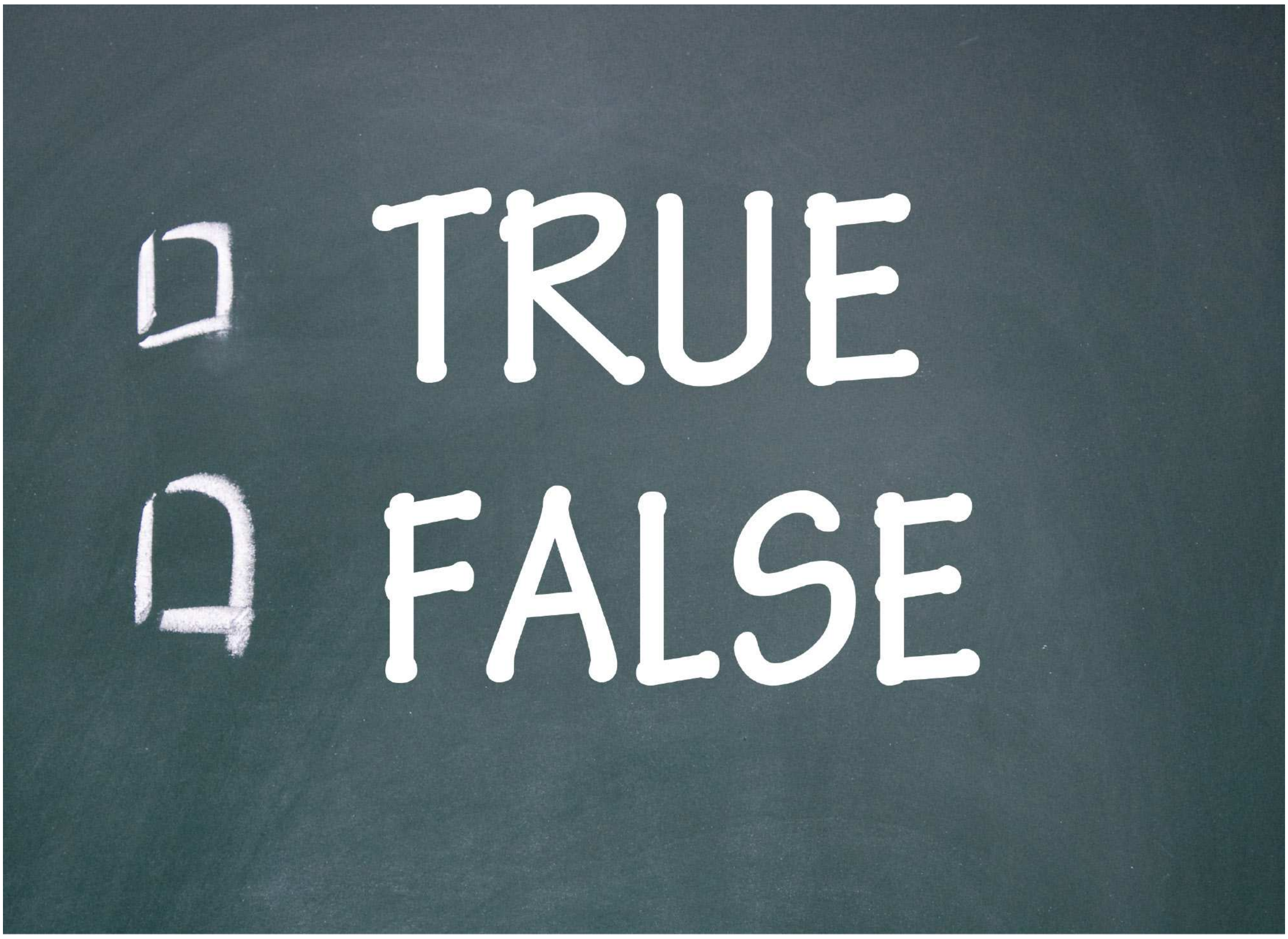
Practice Problems 2.2

Identify the premises and conclusions in the following arguments. Click [here](https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems2.2.pdf) (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems2.2.pdf) to check your answers.

1. Every time I turn on the radio, all I hear is vulgar language about sex, violence, and drugs. Whether it's rock and roll or rap, it's all the same. The trend toward vulgarity has to change. If it doesn't, younger children will begin speaking in these ways, and this will spoil their innocence.
2. Letting your kids play around on the Internet all day is like dropping them off in downtown Chicago to spend the day by themselves. They will find something that gets them into trouble.

3. Too many intravenous drug users continue to risk their lives by sharing dirty needles. This situation could be changed if we were to supply drug addicts with a way to get clean needles. This would lower the rate of AIDS in this high-risk population as well as allow for the opportunity to educate and attempt to aid those who are addicted to heroin and other intravenous drugs.
4. I know that Stephen has a lot of money. His parents drive a Mercedes. His dogs wear cashmere sweaters, and he paid cash for his Hummer.
5. Dogs are better than cats, since they always listen to what their masters say. They also are more fun and energetic.
6. All dogs are warm-blooded. All warm-blooded creatures are mammals. Hence, all dogs are mammals.
7. Chances are that I will not be able to get in to see Slipknot since it is an over-21 show, and Jeffrey, James, and Sloan were all carded when they tried to get in to the club.
8. This is not the best of all possible worlds, because the best of all possible worlds would not contain suffering, and this world contains much suffering.
9. Some apples are not bananas. Some bananas are things that are yellow. Therefore, some things that are yellow are not apples.
10. Since all philosophers are seekers of truth, it follows that no evil human is a seeker after truth, since no philosophers are evil humans.
11. All squares are triangles, and all triangles are rectangles. So all squares are rectangles.
12. Deciduous trees are trees that shed their leaves. Maple trees are deciduous trees. Thus, maple trees will shed their leaves at some point during the growing season.
13. Joe must make a lot of money teaching philosophy, since most philosophy professors are rich.
14. Since all mammals are cold-blooded, and all cold-blooded creatures are aquatic, all mammals must be aquatic.
15. If you drive too fast, you will get into an accident. If you get into an accident, your insurance premiums will increase. Therefore, if you drive too fast, your insurance premiums will increase.
16. The economy continues to descend into chaos. The stock market still moves down after it makes progress forward, and unemployment still hovers around 10%. It is going to be a while before things get better in the United States.
17. Football is the best sport. The athletes are amazing, and it is extremely complex.
18. We should go to see *Avatar* tonight. I hear that it has amazing special effects.
19. All doctors are people who are committed to enhancing the health of their patients. No people who purposely harm others can consider themselves to be doctors. It follows that some people who harm others do not enhance the health of their patients.
20. Guns are necessary. Guns protect people. They give people confidence that they can defend themselves. Guns also ensure that the government will not be able to take over its citizenry.

Propositional Logic 4



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Learning Objectives

After reading this chapter, you should be able to:

1. Explain key words and concepts from propositional logic.
2. Describe the basic logical operators and how they function in a statement.
3. Symbolize complex statements using logical operators.
4. Generate truth tables to evaluate the validity of truth-functional arguments.
5. Evaluate common logical forms.

Chapter 3 discussed categorical logic and touched on how analyzing an argument's logical form helps determine its validity. The usefulness of form in determining validity will become even clearer in this chapter's discussion of what is known as *propositional logic*, another type of deductive logic. Whereas categorical logic analyzes arguments whose validity is based on quantitative terms like *all* and *some*, propositional logic looks at arguments whose validity is based on the way they combine smaller sentences to make larger ones, using connectives like *or*, *and*, and *not*.

In this chapter, we will learn about the symbols and tools that help us analyze arguments and test for validity; we will also examine several common deductive argument forms. Whereas Chapter 3 introduced the idea of form—and thereby, formal logic—this chapter will more thoroughly consider the study of validity based on logical form. We shall see that by adding a couple more symbols to propositional logic, it is also possible to represent the types of statements represented in categorical logic, creating the robust and highly applicable discipline known today as *predicate logic*. (See *A Closer Look: Translating Categorical Logic* for more on predicate logic.)

4.1 Basic Concepts in Propositional Logic

Propositional logic aims to make the concept of *validity* formal and precise. Remember from Chapter 3 that an argument is valid when the truth of its premises guarantees the truth of its conclusion. Propositional logic demonstrates exactly why certain types of premises guarantee the truth of certain types of conclusions. It does this by breaking down the forms of complex claims into their simple component parts. For example, consider the following argument:

Either the maid or the butler did it.
The maid did not do it.
Therefore, the butler did it.

This argument is valid, but not because of anything about the maid or butler. It is valid because of the way that the sentences combine words like *or* and *not* to make a logically valid form. Formal logic is not concerned about the *content* of arguments but with their *form*. Recall from Chapter 3, Section 3.2, that an argument's *form* is the way it combines its component parts to make an overall pattern of reasoning. In this argument, the component parts are the small sentences "the butler did it" and "the maid did it." If we give those parts the names P and Q, then our argument has the form:

P or Q.
Not P.
Therefore, Q.

Note that the expression "not P" means "P is not true." In this case, since P is "the butler did it," it follows that "not P" means "the butler did not do it." An inspection of this form should reveal it is logically valid reasoning.

As the name suggests, propositional logic deals with arguments made up of *propositions*, just as categorical logic deals with arguments made up of *categories* (see Chapter 3). In philosophy, a **proposition** is the *meaning* of a claim about the world; it is what that claim *asserts*. We will refer to the subject of this chapter as "propositional logic" because that is the most common terminology in the field. However, it is sometimes called "sentence logic." The principles are the same no matter which terminology we use, and in the rest of the chapter we will frequently talk about P and Q as representing sentences (or "statements") as well.

The Value of Formal Logic

This process of making our reasoning more precise by focusing on an argument's form has proved to be enormously useful. In fact, formal logic provides the theoretical underpinnings for computers. Computers operate on what are called "logic circuits," and computer programs are based on propositional logic. Computers are able to understand our commands and always do exactly what they are programmed to do because they use formal logic. In *A Closer Look: Alan Turing and How Formal Logic Won the War*, you will see how the practical applications of logic changed the course of history.

Another value of formal logic is that it adds efficiency, precision, and clarity to our language. Being able to examine the structure of people's statements allows us to clarify the meanings of complex sentences. In doing so, it creates an exact, structured way to assess reasoning and to discern between formally valid and invalid arguments.

A Closer Look: Alan Turing and How Formal Logic Won the War

Science and Society/SuperStock

The idea of a computing machine was conceived over the last few centuries by great thinkers such as Gottfried Leibniz, Blaise Pascal, and Charles Babbage. However, it was not until the first half of the 20th century that philosophers, logicians, mathematicians, and engineers were actually able to create "thinking machines" or "electronic brains" (Davis, 2000).

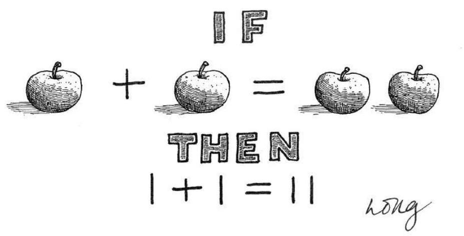
One pioneer of the computer age was British mathematician, philosopher, and logician Alan Turing. He came up with the concept of a Turing machine, an electronic device that takes input in the form of zeroes and ones, manipulates it according to an algorithm, and creates a new output (BBC News, 1999).

Computers themselves were invented by creating electric circuits that do basic logical operations that you will learn about in this chapter. These electric circuits are called “logic gates” (see Figure 4.2 later in the chapter). By turning logic into circuits, basic “thinking” could be done with a series of fast electrical impulses.

Using logical brilliance, Turing was able to design early computers for use during World War II. The British used these early computers to crack the Nazis’ very complex Enigma code. The ability to know the German plans in advance gave the Allies a huge advantage. Prime Minister Winston Churchill even said to King George VI, “It was thanks to Ultra [one of the computers used] that we won the war” (as cited in Shaer, 2012).

An Enigma cipher machine, which was widely used by the Nazi Party to encipher and decipher secret military messages during World War II.

Statement Forms



Bill Long/Cartoonstock

Formal logic uses symbols and statement forms to clarify an argument’s reasoning.

As we have discussed, propositional logic clarifies formal reasoning by breaking down the forms of complex claims into the simple parts of which they are composed. It does this by using symbols to represent the smaller parts of complex sentences and showing how the larger sentence results from combining those parts in a certain way. By doing so, formal logic clarifies the argument’s form, or the pattern of reasoning it uses.

Consider what this looks like in mathematics. If you have taken a course in algebra, you will remember statements such as the following:

$$x + y = y + x$$

This statement is true no matter what we put for x and for y. That is why we call x and y *variables*; they do not represent just one number but *all* numbers. No matter what specific numbers we put in, we will still get a true statement, like the following:

$$5 + 3 = 3 + 5$$

$$7 + 2 = 2 + 7$$

$$1{,}235 + 943 = 943 + 1{,}235$$

By replacing the variables in the general equation with these specific values, we get *instances* (as discussed in Chapter 3) of that general truth. In other words, 5 + 3 = 3 + 5 is an instance of the general statement x + y = y + x. One does not even need to use a calculator to know that the last statement of the three is true, for its truth is not based on the specific numbers used but on the general form of the equation. Formal logic works in the exact same way.

Take the statement “If you have a dog, then you have a dog or you have a cat.” This statement is true, but its truth does not depend on anything about dogs or cats; its truth is based on its logical form—the way the sentence is structured. Here are two other statements with the same logical form: “If you are a miner, then you are a miner or you are a trapper” and “If you are a man, then you are a man or a woman.” These statements are all true not because of their content, but because of their shared logical form.

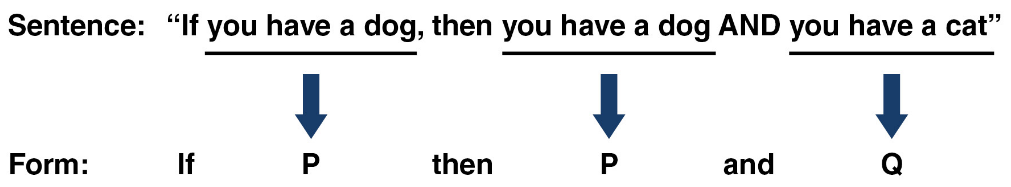
To help us see exactly what this form is, propositional logic uses variables to represent the different sentences within this form. Just as algebra uses letters like *x* and *y* to represent *numbers*, logicians use letters like P and Q to represent *sentences.* These letters are therefore called **sentence variables**.

The chief difference between propositional and categorical logic is that, in categorical logic (Chapter 3), variables (like M and S) are used to represent categories of things (like *dogs* and *mammals*), whereas variables in propositional logic (like P and Q) represent whole sentences (or propositions).

In our current example, propositional logic enables us to take the statement “If you have a dog, then you have a dog or you have a cat” and replace the simple sentences “You have a dog” and “You have a cat,” with the variables P and Q, respectively (see Figure 4.2). The result, “If P, then P or Q,” is known as the general **statement form**. Our specific sentence, “If you have a dog, then you have a dog or you have a cat,” is an *instance* of this general form. Our other example statements—”If you are a miner, then you are a miner or you are a trapper” and “If you are a man, then you are a man or a woman"—are other instances of that same statement form, "If P, then P or Q." We will talk about more specific forms in the next section.

Figure 4.1: Finding the form

In this instance of the statement form, you can see that P and Q relate to the prepositions "you have a dog" and "you have a cat," respectively.



At first glance, propositional logic can seem intimidating because it can be very mathematical in appearance, and some students have negative associations with math. We encourage you to take each section one step at a time and see the symbols as tools you can use to your advantage. Many students actually find that logic helps them because it presents symbols in a friendlier manner than in math, which can then help them warm up to the use of symbols in general.

The Origins of Modern Logic

Before moving on, watch this video to learn about how modern propositional logic came to be, thanks to the contributions of George Boole and Gottlob Frege.

19th Century Modern Logic
From Title: *Logic: The Structure of Reason*
(https://fod.infobase.com/PortalPlaylists.aspx?wID=100753&xtid=32714)

Critical Thinking Questions

1. How did Boole refine the manner in which logic could be applied? What did Frege do to build on Boole's developments?
2. Why did logicians take words and turn them into variables and constants? What does this provide them as they examine propositions and arguments?

4.2 Logical Operators

In the prior section, we learned about what constitutes a statement form in propositional logic: a complex sentence structure with propositional variables like P and Q. In addition to the variables, however, there are other words that we used in representing forms, words like *and* and *or*. These terms, which connect the variables together, are called logical **operators**, also known as **connectives** or *logical terms*.

Logicians like to express formal precision by replacing English words with symbols that represent them. Therefore, in a statement form, logical operators are represented by symbols. The resulting symbolic statement forms are precise, brief, and clear. Expressing sentences in terms of such forms allows logic students more easily to determine the validity of arguments that include them. This section will analyze some of the most common symbols used for logical operators.

Conjunction

Those of you who have heard the *Schoolhouse Rock!* song "Conjunction Junction" (what's your function?)—or recall past English grammar lessons—will recognize that a *conjunction* is a word used to connect, or conjoin, sentences or concepts. By that definition, it refers to words like *and*, *but*, and *or*. Logic, however, uses the word *conjunction* to refer only to *and* sentences. Accordingly, a **conjunction** is a compound statement in which the smaller component statements are joined by *and*.

For example, the conjunction of "roses are red" and "violets are blue" is the sentence "roses are red *and* violets are blue." In logic, the symbol for *and* is an ampersand (&). Thus, the general form of a conjunction is P & Q. To get a specific instance of a conjunction, all you have to do is replace the P and the Q with any specific sentences. Here are some examples:

P	Q	P & Q
Joe is nice.	Joe is tall.	Joe is nice, and Joe is tall.
Mike is sad.	Mike is lonely.	Mike is sad, and Mike is lonely.
Winston is gone.	Winston is not forgotten.	Winston is gone and not forgotten.

Notice that the last sentence in the example does not repeat "Winston is" before "forgotten." That is because people tend to abbreviate things. Thus, if we say "Jim and Mike are on the team," this is actually an abbreviation for "Jim is on the team, and Mike is on the team."

The use of the word *and* has an effect on the truth of the sentence. If we say that P & Q is true, it means that *both* P *and* Q are true. For example, suppose we say, "Joe is nice *and* Joe is tall." This means that he is *both* nice *and* tall. If he is not tall, then the statement is false. If he is not nice, then the statement is false as well. He has to be *both* for the conjunction to be true. The truth of a complex statement thus depends on the truth of its parts. Whether a proposition is true or false is known as its **truth value**: The truth value of a true sentence is simply the word *true*, while the truth value of a false sentence is the word *false*.

To examine how the truth of a statement's parts affects the truth of the whole statement, we can use a **truth table**. In a truth table, each variable (in this case, P and Q) has its own column, in which all possible truth values for those variables are listed. On the right side of the truth table is a column for the complex sentence(s) (in this case the conjunction P & Q) whose truth we want to test. This last column shows the truth value of the statement in question based on the assigned truth values listed for the variables on the left. In other words, each row of the truth table shows that *if* the letters (like P and Q) on the left have these assigned truth values, *then* the complex statements on the right will have these resulting truth values (in the complex column).

Here is the truth table for conjunction:

P	Q	P & Q
T (Joe is nice.)	T (Joe is tall.)	T (Joe is nice, and Joe is tall.)
T (Joe is nice.)	F (Joe is not tall.)	F (It is not true that Joe is nice and tall.)
F (Joe is not nice.)	T (Joe is tall.)	F (It is not true that Joe is nice and tall.)
F (Joe is not nice.)	F (Joe is not tall.)	F (It is not true that Joe is nice and tall.)

What the first row means is that *if* the statements P and Q are both true, *then* the conjunction P & Q is true as well. The second row means that *if* P is true and Q is false, *then* P & Q is false (because P & Q means that *both* statements are true).

The third row means that *if* P is false and Q is true, *then* P & Q is false. The final row means that *if* both statements are false, *then* P & Q is false as well.

A shorter method for representing this truth table, in which T stands for "true" and F stands for "false," is as follows:

P	Q	P & Q
T	T	T
T	F	F
F	T	F
F	F	F

The P and Q columns represent all of the possible truth combinations, and the P & Q column represents the resulting truth value of the conjunction. Again, within each row, on the left we simply assume a set of truth values (for example, in the second row we assume that P is true and Q is false), then we determine what the truth value of P & Q should be to the right. Therefore, each row is like a formal "if–then" statement: *If* P is true and Q is false, *then* P & Q will be false.

Truth tables highlight why propositional logic is also called *truth-functional logic*. It is truth-functional because, as truth tables demonstrate, the truth of the complex statement (on the right) is a *function* of the truth values of its component statements (on the left).

Everyday Logic: The Meaning of *But*

Like the word *and*, the word *but* is also a conjunction. If we say, "Mike is rich, but he's mean," this seems to mean three things: (1) Mike is rich, (2) Mike is mean, and (3) these things are in contrast with each other. This third part, however, cannot be measured with simple truth values. Therefore, in terms of logic, we simply ignore such conversational elements (like point 3) and focus only on the truth conditions of the sentence. Therefore, strange as it may seem, in propositional logic the word *but* is taken to be a synonym for *and*.

Disjunction

Disjunction is just like conjunction except that it involves statements connected with an *or* (see Figure 4.2 for a helpful visualization of the difference). Thus, a statement like "You can either walk or ride the bus" is the disjunction of the statements "You can walk" and "you can ride the bus." In other words, a disjunction is an *or* statement: P *or* Q. In logic the symbol for *or* is ∨. An *or* statement, therefore, has the form P ∨ Q.

Here are some examples:

P	Q	P ∨ Q
Mike is tall.	Doug is rich.	Mike is tall, or Doug is rich.
You can complain.	You can change things.	You can complain, or you can change things.
The maid did it.	The butler did it.	Either the maid or the butler did it.

Notice that, as in the conjunction example, the last example abbreviates one of the clauses (in this case the first clause, "the maid did it"). It is common in natural (nonformal) languages to abbreviate sentences in such ways; the compound sentence actually has two complete component sentences, even if they are not stated completely. The nonabbreviated version would be "Either the maid did it, or the butler did it."

The truth table for disjunction is as follows:

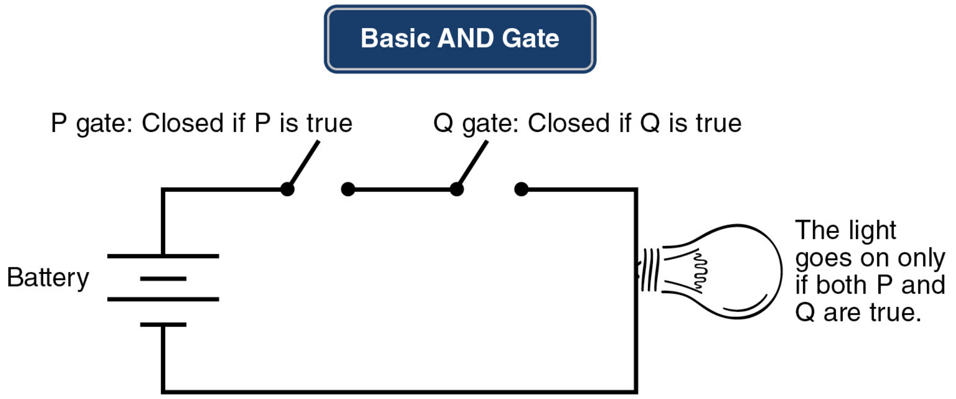
P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T

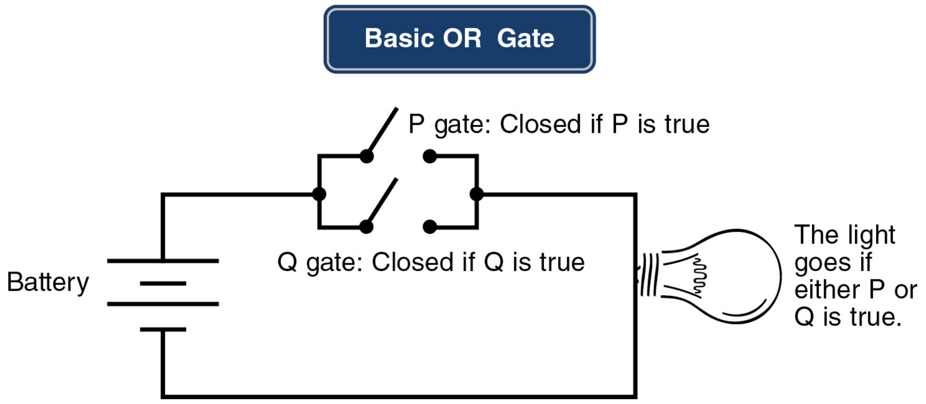
P	Q	P ∨ Q
F	F	F

Note that *or* statements are true whenever *at least one* of the component sentences (the "disjuncts") is true. The only time an *or* statement is false is when P and Q are *both false*.

Figure 4.2: Simple logic circuits

These diagrams of simple logic circuits (recall the reference to these circuits in *A Closer Look: Alan Turing and How Formal Logic Won the War*) help us visualize how the rules for conjunctions (AND gate) and disjunctions (OR gate) work. With the AND gate, there is only one path that will turn on the light, but with the OR gate, there are two paths to illumination.





Everyday Logic: Inclusive Versus Exclusive *Or*

The top line of the truth table for disjunctions may seem strange to some. Some think that the word *or* is intended to allow only *one* of the two sentences to be true. They therefore argue for an interpretation of disjunction called *exclusive or*. An *exclusive or* is just like the *or* in the truth table, except that it makes the top row (the one in which P and Q are both true) false.

One example given to justify this view is that of a waiter asking, "Do you want soup or salad?" If you want both, the answer should not be "yes." Some therefore suggest that the English *or* should be understood in the exclusive sense.

However, this example can be misleading. The waiter is not asking "Is the statement 'do you want soup or salad' *true*?" The waiter is asking you to choose between the two options. When we ask for the truth value of a sentence of the form P or Q, on the other hand, we are asking whether the sentence is *true*. Consider it this way: If you wanted both soup *and* salad, the answer to the waiter's question would not be "no," but it would be if you were using an exclusive *or*.

When we see the connective *or* used in English, it is generally being used in the *inclusive* sense (so called because it *includes* cases in which both disjuncts are true). Suppose that your tax form states, “If you made more than $20,000, or you are self-employed, then fill out form 201-Z.” Suppose that you made more than $20,000, *and* you are self-employed—would you fill out that form? You should, because the standard *or* that we use in English and in logic is the *inclusive* version. Therefore, in logic we understand the word *or* in its inclusive sense, as seen in the truth table.

Negation

The simplest logical symbol we use on sentences simply negates a claim. **Negation** is the act of asserting that a claim is false. For every statement P, the negation of P states that P is false. It is symbolized ~P and pronounced “not P.” Here are some examples:

P	~P
Snow is white.	Snow is not white.
I am happy.	I am not happy.
Either John or Mike got the job.	Neither John nor Mike got the job.

Since ~P states that P is not true, its truth value is the opposite of P’s truth value. In other words, if P is true, then ~P is false; if P is false then ~P is true. Here, then, is the truth table:

P	~P
T	F
F	T

Everyday Logic: The Word *Not*

Sometimes just putting the word *not* in front of the verb does not quite capture the meaning of negation. Take the statement “Jack and Jill went up the hill.” We could change it to “Jack and Jill did *not* go up the hill.” This, however, seems to mean that neither Jack nor Jill went up the hill, but the meaning of negation only requires that at least one did not go up the hill. The simplest way to correctly express the negation would be to write “It is not true that Jack and Jill went up the hill” or “It is not the case that Jack and Jill went up the hill.”

Similar problems affect the negation of claims such as “John likes you.” If John does not know you, then this statement is not true. However, if we put the word *not* in front of the verb, we get “John does not like you.” This seems to imply that John *dislikes* you, which is not what the negation means (especially if he does not know you). Therefore, logicians will instead write something like, “It is not the case that John likes you.”

Conditional

A **conditional** is an “if–then” statement. An example is “If it is raining, then the street is wet.” The general form is “If P, then Q,” where P and Q represent any two claims. Within a conditional, P—the part that comes between *if* and *then*—is called the **antecedent**; Q—the part after *then*—is called the **consequent**. A conditional statement is symbolized P → Q and pronounced “if P, then Q.”

Here are some examples:

P	Q	P → Q
You are rich.	You can buy a boat.	If you are rich, then you can buy a boat.
You are not satisfied.	You can return the product.	If you are not satisfied, then you can return the product.

You need bread or milk. You should go to the market. If you need bread or milk, then you should go to the market.

Everyday Logic: Other Instances of Conditionals

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People use conditionals frequently in real life. Think of all the times someone has said, "Get some rest if you are tired" or "You don't have to do something if you don't want to."

Sometimes conditionals are expressed in other ways. For example, sometimes people leave out the *then*. They say things like, "If you are hungry, you should eat." In many of these cases, we have to be clever in determining what P and Q are.

Sometimes people even put the consequent first: for example, "You should eat if you are hungry." This statement means the same thing as "If you are hungry, then you should eat"; it is just ordered differently. In both cases the antecedent is what comes after the *if* in the English sentence (and prior to the → in the logical form). Thus, "If P then Q" is translated "P → Q," and "P if Q" is translated "Q → P."

Formulating the truth table for conditional statements is somewhat tricky. What does it take for a conditional statement to be true? This is actually a controversial issue within philosophy. It is actually easier to think of it as: What does it mean for a conditional statement to be *false*?

Suppose Mike promises, "If you give me $5, then I will wash your car." What would it take for this statement to be false? Under what conditions, for example, could you accuse Mike of breaking his promise?

It seems that the only way for Mike to break his promise is if you give him the $5, but he does not wash the car. If you give him the money and he washes the car, then he kept his word. If you did not give him the money, then his word was simply not tested (with no payment on your part, he is under no obligation). If you do not pay him, he may choose to wash the car anyway (as a gift), or he may not; neither would make him a liar. His promise is only broken in the case in which you give him the money but he does not wash it. Therefore, in general, we call conditional statements false only in the case in which the antecedent is true and the consequent is false (in this case, if you give him the money, but he still does not wash the car). This results in the following truth table:

P	**Q**	**P → Q**
T	T	T
T	F	F
F	T	T
F	F	T

Some people question the bottom two lines. Some feel that the truth value of those rows should depend on whether he *would* have washed the car if you *had* paid him. However, this sophisticated hypothetical is beyond the power of truth-functional logic. The truth table is as close as we can get to the meaning of "if . . . then . . ." with a simple truth table; in other words, it is best we can do with the tool at hand.

Finally, some feel that the third row should be false. That, however, would mean that Mike choosing to wash the car of a person who had no money to give him would mean that he broke his promise. That does not appear, however, to be a broken promise, only an act of generosity on his part. It therefore does not appear that his initial statement "If you give me $5, then I will wash your car" commits to washing the car *only if* you give him $5. This is instead a variation on the conditional theme known as "only if."

Only If

So what does it mean to say “P *only if* Q”? Let us take a look at another example: “You can get into Harvard *only if* you have a high GPA.” This means that a high GPA is a requirement for getting in. Note, however, that that is *not* the same as saying, “You can get into Harvard *if* you have a high GPA,” for there might be other requirements as well, like having high test scores, good letters of recommendation, and a good essay.

Thus, the statement “You can get into Harvard *only if* you have a high GPA” means:

> You can get into Harvard → You have a high GPA

However, this does *not* mean the same thing as “You have a high GPA → You can get into Harvard.”

In general, “P *only if* Q” is translated P → Q. Notice that this is the same as the translation of “If P, then Q.” However, it is not the same as “P if Q,” which is translated Q → P. Here is a summary of the rules for these translations:

> P only if Q is translated: P → Q
>
> P if Q is translated: Q → P

Thus, “P if Q” and “P only if Q” are the converse of each other. Recall the discussion of *conversion* in Chapter 3; the **converse** is what you get when you switch the order of the elements within a conditional or categorical statement.

To say that P → Q is true is to assert that the truth of Q is *necessary* for the truth of P. In other words, Q *must* be true for P to be true. To say that P → Q is true is also to say that the truth of P is *sufficient* for the truth of Q. In other words, knowing that P is true is enough information to conclude that Q is also true.

In our earlier example, we saw that having a high GPA is necessary but not sufficient for getting into Harvard, because one must also have high test scores and good letters of recommendation. Further discussion of the concepts of necessary and sufficient conditions will occur in Chapter 5.

In some cases P is both a necessary *and* a sufficient condition for Q. This is called a *biconditional*.

Biconditional

A **biconditional** asserts an “if and only if” statement. It states that if P is true, then Q is true, *and* if Q is true, then P is true. For example, if I say, “I will go to the party if you will,” this means that if you go, then I will too (P → Q), but it does not rule out the possibility that I will go without you. To rule out that possibility, I could state “I will go to the party *only if* you will” (Q → P). If we want to assert *both* conditionals, I could say, “I will go to the party *if and only if* you will.” This is a biconditional.

The statement “P if and only if Q” literally means “P if Q *and* P only if Q.” Using the translation methods for *if* and *only if*, this is translated “(Q → P) & (P → Q).” Because the biconditional makes the arrow between P and Q go both ways, it is symbolized: P ↔ Q.

Here are some examples:

P	Q	P ↔ Q
You can go to the party.	You are invited.	You can go to the party if and only if you are invited.
You will get an A.	You get above a 92%.	You will get an A if and only if you get above a 92%.
You should propose.	You are ready to marry her.	You should propose if and only if you are ready to marry her.

There are other phrases that people sometimes use instead of “if and only if.” Some people say “just in case” or something else like it. Mathematicians and philosophers even use the abbreviation *iff* to stand for “if and only if.” Sometimes people even simply say “if” when they really mean “if and only if.” One must be clever to understand what people really mean when they speak in sloppy, everyday language. When it comes to precision, logic is perfect; English is fuzzy!

Here is how we do the truth table: For the biconditional P ↔ Q to be true, it must be the case that if P is true then Q is true and vice versa. Therefore, one cannot be true when the other one is false. In other words, they must both have the *same truth value*. That means the truth table looks as follows:

P	Q	P ↔ Q

T	T	T
T	F	F
F	T	F
F	F	T

The biconditional is true in exactly those cases in which P and Q have the same truth value.

Practice Problems 4.1

Complete the following identifications. Click [here](https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems4.1.pdf) (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems4.1.pdf) to check your answers.

1. "I am tired and hungry." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional
2. "If we learn logic, then we will be able to evaluate arguments." This statement is a _________.
   a. conjunction
   b. disjunction

c. conditional
d. biconditional

3. "We can learn logic if and only if we commit ourselves to intense study." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

4. "We either attack now, or we will lose the war." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

5. "The tide will rise only if the moon's gravitational pull acts on the ocean." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

6. "If I am sick or tired, then I will not go to the interpretive dance competition." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

7. "One can surf monster waves if and only if one has experience surfing smaller waves." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

8. "The economy is recovering, and people are starting to make more money." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

9. "If my computer crashes again, then I am going to buy a new one." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

10. "You can post responses on 2 days or choose to write a two-page paper." This statement is a _________.
   a. conjunction
   b. disjunction
   c. conditional
   d. biconditional

4.3 Symbolizing Complex Statements

We have learned the basic logical operators and their corresponding symbols and truth tables. However, these basic symbols also allow us analyze much more complicated statements. Within the statement form P → Q, what if either P or Q itself is a complex statement? For example:

P	Q	P → Q
You are hungry or thirsty.	We should go to the diner.	If you are hungry or thirsty, then we should go to the diner.

In this example, the antecedent, P, states, “You are hungry or thirsty,” which can be symbolized H ∨ T, using the letter H for “You are hungry” and T for “You are thirsty.” If we use the letter D for “We should go to the diner,” then the whole statement can be symbolized (H ∨ T) → D.

Notice the use of parentheses. Parentheses help specify the order of operations, just like in arithmetic. For example, how would you evaluate the quantity 3 + (2 × 5)? You would execute the mathematical operation *within* the parentheses first. In this case you would first multiply 2 and 5 and then add 3, getting 13. You would *not* add the 3 and the 2 first and then multiply by 5 to get 25. This is because you know to evaluate what is within the parentheses first.

It is the exact same way with logic. In the statement (H ∨ T) → D, because of the parentheses, we know that this statement is a conditional (not a disjunction). It is of the form P → Q, where P is replaced by H ∨ T and Q is replaced by D.

Here is another example:

N & S	G	(N & S) → G
He is nice and smart.	You should get to know him.	If he is nice and smart, then you should get to know him.

This example shows a complex way to make a sentence out of three component sentences. N is “He is nice,” S is “he is smart,” and G is “you should get to know him.” Here is another:

R	(S & C)	R → (S & C)
You want to be rich.	You should study hard and go to college.	If you want to be rich, then you should study hard and go to college.

If R is “You want to be rich,” S is “You should study hard,” and C is “You should go to college,” then the whole statement in this final example, symbolized R → (S & C), means “If you want to be rich, then you should study hard and go to college.”

Complex statements can be created in this manner for every form. Take the statement (~A & B) ∨ (C → ~D). This statement has the general form of a disjunction. It has the form P ∨ Q, where P is replaced with ~A & B, and Q is replaced with C → ~D.

Everyday Logic: Complex Statements in Ordinary Language

It is not always easy to determine how to translate complex, ordinary language statements into logic; one sometimes has to pick up on clues within the statement.

For instance, notice in general that *neithe*r P nor Q is translated ~(P ∨ Q). This is because P ∨ Q means that either one is true, so ~(P ∨ Q) means that neither one is true. It happens to be equivalent to saying ~P & ~Q (we will talk about logical equivalence later in this chapter).

Here are some more complex examples:

Statement	Translation
If you don’t eat spinach, then you will neither be big nor strong.	~S → ~(B ∨ S)
Either he is strong and brave, or he is both reckless and foolish.	(S & B) ∨ (R & F)
Come late and wear wrinkled clothes only if you don’t want the job.	(L & W) → ~J
He is strong and brave, and if he doesn’t like you, he will let you know.	(S & B) & (~L → K)

Truth Tables With Complex Statements

We have managed to symbolize complex statements by seeing how they are systematically constructed out of their parts. Here we use the same principle to create truth tables that allow us to find the truth values of complex statements based on the truth values of their parts. It will be helpful to start with a summary of the truth values of sentences constructed with the basic truth-functional operators:

P	**Q**	**~P**	**P & Q**	**P ∨ Q**	**P → Q**	**P ↔ Q**
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

The truth values of more complex statements can be discovered by applying these basic formulas one at a time. Take a complex statement like (A ∨ B) → (A & B). Do not be intimidated by its seemingly complex form; simply take it one operator at a time. First, notice the main form of the statement: It is a conditional (we know this because the other operators are within parentheses). It therefore has the form P → Q, where P is "A ∨ B" and Q is "A & B."

The antecedent of the conditional is A ∨ B; the consequent is A & B. The way to find the truth values of such statements is to start *inside* the parentheses and find those truth values first, and then work our way out to the main operator—in this case →.

Here is the truth table for these components:

A	**B**	**A ∨ B**	**A & B**
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

Now we take the truth tables for these components to create the truth table for the overall conditional:

A	**B**	**A ∨**	**A & B**	**(A ∨ B) → (A & B)**
T	T	T	T	**T**
T	F	T	F	**F**
F	T	T	F	**F**
F	F	F	F	**T**

In this way the truth values of very complex statements can be determined from the values of their parts. We may refer to these columns (in this case A ∨ B and A & B) as *helper columns*, because they are there just to assist us in determining the truth values for the more complex statement of which they are a part.

Here is another one: (A & ~B) → ~(A ∨ B). This one is also a conditional, where the antecedent is A & ~B and the consequent is ~(A ∨ B). We do these components first because they are inside parentheses. However, to find the truth table for A & ~B, we will have to fill out the truth table for ~B first (as a helper column).

A	**B**	**~B**	**A & ~B**
T	T	F	**F**
T	F	T	**T**
F	T	F	**F**
F	F	T	**F**

We found ~B by simply negating B. We then found A & ~B by applying the truth table for conjunctions to the column for A and the column for ~B.

Now we can fill out the truth table for A ∨ B and then use that to find the values of ~(A ∨ B):

A	B	A ∨ B	~(A ∨ B)
T	T	T	**F**
T	F	T	**F**
F	T	T	**F**
F	F	F	**T**

Finally, we can now put A & ~B and ~(A ∨ B) together with the conditional to get our truth table:

A	B	A & ~B	~(A ∨ B)	(A & ~B) → ~(A ∨ B)
T	T	F	F	**T**
T	F	T	F	**F**
F	T	F	F	**T**
F	F	F	T	**T**

Although complicated, it is not hard when one realizes that one has to apply only a series of simple steps in order to get the end result.

Here is another one: (A → ~B) ∨ ~(A & B). First we will do the truth table for the left part of the disjunction (called the *left disjunct*), A → ~B:

A	B	~B	A → ~B
T	T	F	**F**
T	F	T	**T**
F	T	F	**T**
F	F	T	**T**

Of course, the last column is based on combining the first column, A, with the third column, ~B, using the conditional. Now we can work on the right disjunct, ~(A & B):

A	B	A & B	~(A & B)
T	T	T	**F**
T	F	F	**T**
F	T	F	**T**
F	F	F	**T**

The final truth table, then, is:

A	B	A→~B	~(A & B)	(A→~B) ∨ ~(A & B)
T	T	F	F	**F**
T	F	T	T	**T**
F	T	T	T	**T**
F	F	T	T	**T**

You may have noticed that three formulas in the truth table have the exact same values on every row. That means that the formulas are logically equivalent. In propositional logic, two formulas are **logically equivalent** if they have the same truth values on every row of the truth table. Logically equivalent formulas are therefore true in the exact same circumstances. Logicians consider this important because two formulas that are logically equivalent, in the logical sense, mean the same thing, even though they may look quite different. The conditions for their truth and falsity are identical.

The fact that the truth value of a complex statement follows from the truth values of its component parts is why these operators are called truth-functional. The operators, &, ∨, ~, →, and ↔, are *truth-functions*, meaning that the truth of the whole sentence is a function of the truth of the parts.

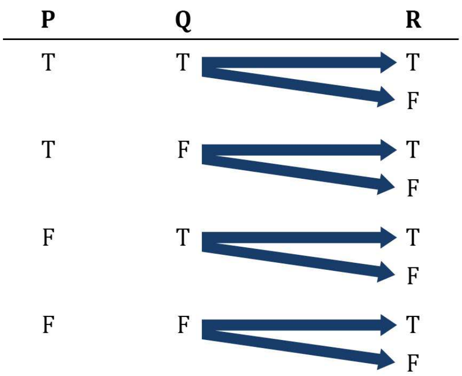
Because the validity of argument forms within propositional logic is based on the behavior of the truth-functional operators, another name for propositional logic is *truth-functional logic*.

Truth Tables With Three Letters

In each of the prior complex statement examples, there were only two letters (variables like P and Q or constants like A and B) in the top left of the truth table. Each truth table had only four rows because there are only four possible combinations of truth values for two variables (both are true, only the first is true, only the second is true, and both are false).

It is also possible to do a truth table for sentences that contain three or more variables (or constants). Recall one of the earlier examples: "Come late and wear wrinkled clothes only if you don't want the job," which we represented as (L & W) → ~J. Now that there are three letters, how many possible combinations of truth values are there for these letters?

The answer is that a truth table with three variables (or constants) will have *eight* lines. The general rule is that whenever you add another letter to a truth table, you double the number of possible combinations of truth values. For each earlier combination, there are now two: one in which the new letter is *true* and one in which it is *false*. Therefore, to make a truth table with three letters, imagine the truth table for two letters and imagine each row splitting in two, as follows:



The resulting truth table rows would look like this:

P	Q	R
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

The goal is to have a row for every possible truth value combination. Generally, to fill in the rows of any truth table, start with the last letter and simply alternate T, F, T, F, and so on, as in the R column. Then move one letter to the left and do twice as many Ts followed by twice as many Fs (two of each): T, T, F, F, and so on, as in the Q column. Then move another letter to the left and do twice as many of each again (four each), in this case T, T, T, T, F, F, F, F, as in the P column. If there are more letters, then we would repeat the process, adding twice as many Ts for each added letter to the left.

With three letters, there are eight rows; with four letters, there are sixteen rows, and so on. This chapter does not address statements with more than three letters, so another way to ensure you have enough rows is to memorize this pattern.

The column with the forms is filled out the same way as when there were two letters. The fact that they now have three letters makes little difference, because we work on only one operator, and therefore at most two columns of letters, at a time. Let us start with the example of P → (Q & R). We begin by solving inside the parentheses by determining the truth values for Q & R, then we create the conditional between P and that result. The table looks like this:

P	Q	R	Q & R	P → (Q & R)
T	T	T	T	**T**
T	T	F	F	**F**
T	F	T	F	**F**
T	F	F	F	**F**
F	T	T	T	**T**
F	T	F	F	**T**
F	F	T	F	**T**
F	F	F	F	**T**

The rules for determining the truth values of Q & R and then of P → (Q & R) are exactly the same as the rules for & and → that we used in the two-letter truth tables earlier; now we just use them for more rows. It is a formal process that generates truth values by the same strict algorithms as in the two-letter tables.

Practice Problems 4.2

Symbolize the following complex statements using the symbols that you have learned in this chapter. Click here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems4.2.pdf) to check your answers.

1. One should be neither a borrower nor a lender.
2. Atomic bombs are dangerous and destructive.
3. If we go to the store, then I need to buy apples and lettuce.
4. Either Microsoft enhances its product and Dell's sales decrease, or Gateway will start making computers again.
5. If Hondas have better gas mileage than Range Rovers and you are looking for something that is easy to park, I recommend that you buy the Honda.
6. Global warming will decrease if and only if emissions decrease in China and other major polluters around the world.
7. One cannot be both happy and successful in our society, but one can be happy or successful.
8. I will pass this course if and only if I study hard and practice regularly, if I have the time and energy to do so.
9. God can only exist if evil does not exist, if it is true that God is both all-powerful and all-good.
10. The conflict in Israel will end only if the Palestinians feel that they can live outside the supervision of the Israelis and the two sides stop attacking one another.

4.4 Using Truth Tables to Test for Validity

Truth tables serve many valuable purposes. One is to help us better understand how the logical operators work. Another is to help us understand how truth is determined within formally structured sentences. One of the most valuable things truth tables offer is the ability to test argument forms for validity. As mentioned at the beginning of this chapter, one of the main purposes of formal logic is to make the concept of validity precise. Truth tables help us do just that.

As mentioned in previous chapters, an argument is valid if and only if the truth of its premises guarantees the truth of its conclusion. This is equivalent to saying that there is no way that the premises can be true and the conclusion false.

Truth tables enable us to determine precisely if there is any way for all of the premises to be true and the conclusion false (and therefore whether the argument is valid): We simply create a truth table for the premises and conclusion and see if there is any row on which all of the premises are true and the conclusion is false. If there is, then the argument is invalid, because that row shows that it is possible for the premises to be true and the conclusion false. If there is no such line, then the argument is valid:

Since the rows of a truth table cover all possibilities, if there is no row on which all of the premises are true and the conclusion is false, then it is impossible, so the argument is valid.

Let us start with a simple example—note that the symbol means "therefore":

P ∨ Q
~Q
∴ P

This argument form is valid; if there are only two options, P and Q, and one of them is false, then it follows that the other one must be true. However, how can we formally demonstrate its validity? One way is to create a truth table to find out if there is any possible way to make all of the premises true and the conclusion false.

Here is how to set up the truth table, with a column for each premise (P1 and P2) and the conclusion (C):

		P1	**P2**	**C**
P	**Q**	**P ∨ Q**	**~Q**	**P**
T	T			
T	F			
F	T			
F	F			

We then fill in the columns, with the correct truth values:

		P1	**P2**	**C**
P	**Q**	**P ∨ Q**	**~Q**	**P**
T	T	**T**	**F**	**T**
T	F	**T**	**T**	**T**
F	T	**T**	**F**	**F**
F	F	**F**	**T**	**F**

We then check if there are any rows in which all of the premises are true and the conclusion is false. A brief scan shows that there are no such lines. The first two rows have true conclusions, and the remaining two rows each have at least one false premise. Since the rows of a truth table represent all possible combinations of truth values, this truth table therefore demonstrates that there is no possible way to make all of the premises true and the conclusion false. It follows, therefore, that the argument is logically valid.

To summarize, the steps for using the truth table method to determine an argument's validity are as follows:

1. Set up the truth table by creating rows for each possible combination of truth values for the basic letters and a column for each premise and the conclusion.
2. Fill out the truth table by filling out the truth values in each column according to the rules for the relevant operator (~, &, ∨, →, ↔).

3. Use the table to evaluate the argument's validity. If there is even one row on which all of the premises are true and the conclusion is false, then the argument is invalid; if there is no such row, then the argument is valid.

This truth table method works for all arguments in propositional logic: Any valid propositional logic argument will have a truth table that shows it is valid, and every invalid propositional logic argument will have a truth table that shows it is invalid. Therefore, this is a perfect test for validity: It works every time (as long as we use it accurately).

Examples With Arguments With Two Letters

Let us do another example with only two letters. This argument will be slightly more complex but will still involve only two letters, A and B.

Example 1

A → B
~(A & B)
∴ ~(B ∨ A)

To test this symbolized argument for validity, we first set up the truth table by creating rows with all of the possible truth values for the basic letters on the left and then create a column for each premise (P1 and P2) and conclusion (C), as follows:

		P1	**P2**	**C**
A	**B**	**A → B**	**~(A & B)**	**~(B ∨ A)**
T	T			
T	F			
F	T			
F	F			

Second, we fill out the truth table using the rules created by the basic truth tables for each operator. Remember to use helper columns where necessary as steps toward filling in the columns of complex formulas. Here is the truth table with only the helper columns filled in:

		P1		**P2**		**C**
A	**B**	**A → B**	**A & B**	**~(A & B)**	**B ∨ A**	**~(B ∨ A)**
T	T		T		T	
T	F		F		T	
F	T		F		T	
F	F		F		F	

Here is the truth table with the rest of the columns filled in:

		P1		**P2**		**C**
A	**B**	**A → B**	**A & B**	**~(A & B)**	**B ∨ A**	**~(B ∨ A)**
T	T	**T**	T	**F**	T	**F**
T	F	**F**	F	**T**	T	**F**
F	T	**T**	F	**T**	T	**F**
F	F	**T**	F	**T**	F	**T**

Finally, to evaluate the argument's validity, all we have to do is check to see if there are any lines in which all of the premises are true and the conclusion is false. Again, if there is such a line, since we know it is possible for all of the premises to be true and the conclusion false, the argument is invalid. If there is no such line, then the argument is valid.

It does not matter what other rows may exist in the table. There may be rows in which all of the premises are true and the conclusion is also true; there also may be rows with one or more false premises. Neither of those types of rows determine the argument's validity; our only concern is whether there is any possible row on which all of the premises are true and the conclusion false. Is there such a line in our truth table? (Remember: Ignore the helper columns and just focus on the premises and conclusion.)

The answer is *yes*, all of the premises are true and the conclusion is false in the third row. This row supplies a proof that this argument's form is invalid. Here is the line:

		P1	**P2**	**C**
A	**B**	**A → B**	**~(A & B)**	**~(B ∨ A)**
F	T	**T**	**T**	**F**

Again, it does not matter what is on the other row. As long as there is (at least) one row in which all of the premises are true and the conclusion false, the argument is invalid.

Example 2

A → (B & ~A)
A ∨ ~B
∴ ~(A ∨ B)

First we set up the truth table:

				P1		**P2**		**C**
A	**B**	**~A**	**B & ~A**	**A → (B & ~A)**	**~B**	**A ∨ ~B**	**A ∨ B**	**~(A ∨ B)**
T	T							
T	F							
F	T							
F	F							

Next we fill in the values, filling in the helper columns first:

				P1		**P2**		**C**
A	**B**	**~A**	**B & ~A**	**A → (B & ~A)**	**~B**	**A ∨ ~B**	**A ∨ B**	**~(A ∨ B)**
T	T	F	F		F		T	
T	F	F	F		T		T	
F	T	T	T		F		T	
F	F	T	F		T		F	

Now that the helper columns are done, we can fill in the rest of the table's values:

				P1		**P2**		**C**
A	**B**	**~A**	**B & ~A**	**A → (B & ~A)**	**~B**	**A ∨ ~B**	**A ∨ B**	**~(A ∨ B)**
T	T	F	F	F	F	T	T	F
T	F	F	F	F	T	T	T	F
F	T	T	T	T	F	F	T	F
F	F	T	F	T	T	T	F	T

Finally, we evaluate the table for validity. Here we see that there are no lines in which all of the premises are true and the conclusion is false. Therefore, there is no possible way to make all of the premises true and the conclusion false, so the argument is valid.

The earlier examples each had two premises. The following example has three premises. The steps of the truth table test are identical.

Example 3

~(M ∨ B)
M → ~B
B ∨ ~M
∴ ~M & B)

First we set up the truth table. This table already has the helper columns filled in.

			P1		**P2**		**P3**	**C**
M	**B**	**M >V B**	**~(M V B)**	**~B**	**M → ~B**	**~M**	**B V ~M**	**~M & B**
T	T	T		F		F		
T	F	T		T		F		
F	T	T		F		T		
F	F	F		T		T		

Now we fill in the rest of the columns, using the helper columns to determine the truth values of our premises and conclusion on each row:

			P1		**P2**		**P3**	**C**
M	**B**	**M V B**	**~(M V B)**	**~B**	**M → ~B**	**~M**	**B V ~M**	**~M & B**
T	T	T	F	F	F	F	T	F
T	F	T	F	T	T	F	F	F
F	T	T	F	F	T	T	T	T
F	F	F	T	T	T	T	T	F

Now we look for a line in which all of the premises are true and the conclusion false. The final row is just such a line. This demonstrates conclusively that the argument is invalid.

Examples With Arguments With Three Letters

The last example had three premises, but only two letters. These next examples will have three letters. As explained earlier in the chapter, the presence of the extra letter doubles the number of rows in the truth table.

Example 1

A → (B ∨ C)
~(C & B)
∴ ~(A & B)

First we set up the truth table. Note, as mentioned earlier, now there are eight possible combinations on the left.

				P1		**P2**		**C**
A	**B**	**C**	**B V C**	**A → (B V C)**	**C & B**	**~(C & B)**	**A & B**	**~(A & B)**
T	T	T						

T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Then we fill the table out. Here it is with just the helper columns:

				P1		P2		C
A	**B**	**C**	**B ∨ C**	**A → (B ∨ C)**	**C & B**	**~(C & B)**	**A & B**	**~(A & B)**
T	T	T	T		T		T	
T	T	F	T		F		T	
T	F	T	T		F		F	
T	F	F	F		F		F	
F	T	T	T		T		F	
F	T	F	T		F		F	
F	F	T	T		F		F	
F	F	F	F		F		F	

Here is the full truth table:

				P1		P2		C
A	**B**	**C**	**B ∨ C**	**A → (B ∨ C)**	**C & B**	**~ (C & B)**	**A & B**	**~(A & B)**
T	T	T	T	T	T	F	T	F
T	T	F	T	T	F	T	T	F
T	F	T	T	T	F	T	F	T
T	F	F	F	F	F	T	F	T
F	T	T	T	T	T	F	F	T
F	T	F	T	T	F	T	F	T
F	F	T	T	T	F	T	F	T
F	F	F	F	T	F	T	F	T

Finally, we evaluate; that is, we look for a line in which all of the premises are true and the conclusion false. This is the case with the second line. Once you find such a line, you do not need to look any further. The existence of even one line in which all of the premises are true and the conclusion is false is enough to declare the argument invalid.

Let us do another one with three letters:

Example 2

A → ~B
B ∨ C
∴ A → C

We begin by setting up the table:

				P1	P2	C
A	B	C	~B	A → ~B	B ∨ C	A → C
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

Now we can fill in the rows, beginning with the helper columns:

				P1	P2	C
A	B	C	~B	A → ~B	B ∨ C	A → C
T	T	T	F	F	T	T
T	T	F	F	F	T	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	T	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

Here, when we look for a line in which all of the premises are true and the conclusion false, we do not find one. There is no such line; therefore the argument is *valid*.

Practice Problems 4.3

Answer these questions about truth tables. Click here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems4.3_corrected.pdf) to check your answers.

1. A truth table with two variables has how many lines?
   a. 1
   b. 2
   c. 4
   d. 8

2. A truth table with three variables has how many lines?
   a. 1
   b. 2
   c. 4
   d. 8

3. In order to prove that an argument is invalid using a truth table, one must ________.
   a. find a line in which all premises and the conclusion are false
   b. find a line in which the premises are true and the conclusion is false
   c. find a line in which the premises are false and the conclusion is true
   d. find a line in which the premises and the conclusion are true

4. This is how one can tell if an argument is valid using a truth table:

a. There is a line in which the premises and the conclusion are true.
b. There is no line in which the premises are false.
c. There is no line in which the premises are true and the conclusion is false.
d. All of the above
e. None of the above

5. When two statements have the same truth values in all circumstances, they are said to be _________.
    a. logically contradictory
    b. logically equivalent
    c. logically cogent
    d. logically valid

6. An *if–then* statement is called a _________.
    a. conjunction
    b. disjunction
    c. conditional
    d. biconditional

7. An *if and only if* statement is called a _________.
    a. conjunction
    b. disjunction
    c. conditional
    d. biconditional

8. An *and* statement is called a _________.
    a. conjunction
    b. disjunction
    c. conditional
    d. biconditional

Utilize truth tables to determine the validity of the following arguments.

J → K
J
∴ K

H → G
G
∴ H

K→ K
∴ K

~(H & Y)
Y ∨~H
∴ ~H

W → Q
~W
∴ ~Q

A → B
B → C
∴ A → C

~(P ↔ U)
∴ ~(P → U)

~S ∨ H
~S
∴ ~H

$\sim K \rightarrow \sim L$
$J \rightarrow \sim K$
$\therefore J \rightarrow \sim L$

$Y \& P$
P
$\therefore \sim Y$

$A \rightarrow \sim G$
$V \rightarrow \sim G$
$\therefore A \rightarrow V$

$B \& K \& I$
$\therefore K$

4.5 Some Famous Propositional Argument Forms

Using the truth table test for validity, we have seen that we can determine the validity or invalidity of all propositional argument forms. However, there are some basic argument forms that are so common that it is worthwhile simply to memorize them and whether or not they are valid. We will begin with five very famous valid argument forms and then cover two of the most famous invalid argument forms.

Common Valid Forms

It is helpful to know some of the most commonly used valid argument forms. Those presented in this section are used so regularly that, once you learn them, you may notice people using them all the time. They are also used in what are known as deductive proofs (see *A Closer Look: Deductive Proofs*).

A Closer Look: Deductive Proofs

Mark Wragg/iStock/Thinkstock

Rather than base decisions on chance, people use the information around them to make deductive and inductive inferences with varying degrees of strength and validity. Logicians use proofs to show the validity of inferences.

A big part of formal logic is constructing *proofs*. Proofs in logic are a lot like proofs in mathematics. We start with certain premises and then use certain rules—called *rules of inference*—in a step-by-step way to arrive at the conclusion. By using only valid rules of inference and applying them carefully, we make certain that every step of the proof is valid. Therefore, if there is a logical proof of the conclusion from the premises, then we can be certain that the argument itself is valid.

The rules of inference used in deductive proofs are actually just simple valid argument forms. In fact, the valid argument forms covered here—including *modus ponens*, hypothetical syllogisms, and disjunctive syllogisms—are examples of argument forms that are used as inference rules in logical proofs. Using these and other formal rules, it is possible to give a logical proof for every valid argument in propositional logic (Kennedy, 2012).

Logicians, mathematicians, philosophers, and computer scientists use logical proofs to show that the validity of certain inferences is absolutely certain and founded on the most basic principles. Many of the inferences we make in daily life are of limited certainty; however, the validity of inferences that have been logically proved is considered to be the most certain and uncontroversial of all knowledge because it is derivable from pure logic.

Covering how to do deductive proofs is beyond the scope of this book, but readers are invited to peruse a book or take a course on formal logic to learn more about how deductive proofs work.

Modus Ponens

Perhaps the most famous propositional argument form of all is known as ***modus ponens***—Latin for "the way of putting." (You may recognize this form from the earlier section on the truth table method.) *Modus ponens* has the following form:

$P \rightarrow Q$
P
$\therefore Q$

You can see that the argument is valid just from the meaning of the conditional. The first premise states, "If P is true, then Q is true." It would logically follow that if P *is* true, as the second premise states, then Q *must* be true. Here are some examples:

If you want to get an A, you have to study.
You want to get an A.
Therefore, you have to study.

If it is raining, then the street is wet.
It is raining.

Therefore, the street is wet.

If it is wrong, then you shouldn't do it.
It is wrong.
Therefore, you shouldn't do it.

A truth table will verify its validity.

		P1	P2	C
P	**Q**	**P → Q**	**P**	**Q**
T	T	**T**	**T**	**T**
T	F	**F**	**T**	**F**
F	T	**T**	**F**	**T**
F	F	**T**	**F**	**F**

There is no line in which all of the premises are true and the conclusion false, verifying the validity of this important logical form.

Modus Tollens

A closely related form has a closely related name. ***Modus tollens***—Latin for "the way of taking"—has the following form:

P → Q
~Q
∴ ~P

A truth table can be used to verify the validity of this form as well. However, we can also see its validity by simply thinking it through. Suppose it is true that "If P, then Q." Then, if P were true, it would follow that Q would be true as well. But, according to the second premise, Q is not true. It follows, therefore, that P must not be true; otherwise, Q would have been true. Here are some examples of arguments that fit this logical form:

In order to get an A, I must study.
I will not study.
Therefore, I will not get an A.

If it rained, then the street would be wet.
The street is not wet.
Therefore, it must not have rained.

If the ball hit the window, then I would hear glass shattering.
I did not hear glass shattering.
Therefore, the ball must not have hit the window.

For practice, construct a truth table to demonstrate the validity of this form.

Ruth Black/iStock/Thinkstock

Evaluate this argument form for validity: If the cake is made with sugar, then the cake is sweet. The cake is not sweet. Therefore, the cake is not made with sugar.

Disjunctive Syllogism

A **disjunctive syllogism** is a valid argument form in which one premise states that you have two options, and another premise allows you to rule one of them out. From such premises, it follows that the other option must be true. Here are two versions of it formally (both are valid):

P ∨ Q	P ∨ Q
~P	~Q
∴ Q	∴ P

In other words, if you have "P or Q" and *not* Q, then you may infer P. Here is another example: "Either the butler or the maid did it. It could not have been the butler. Therefore, it must have been the maid." This argument form is quite handy in real life. It is frequently useful to consider alternatives and to rule one out so that the options are narrowed down to one.

Hypothetical Syllogism

One of the goals of a logically valid argument is for the premises to link together so that the conclusion follows smoothly, with each premise providing a link in the chain. **Hypothetical syllogism** provides a nice demonstration of just such premise linking. Hypothetical syllogism takes the following form:

P → Q
Q → R
∴ P → R

For example, "If you lose your job, then you will have no income. If you have no income, then you will starve. Therefore, if you lose your job, then you will starve!"

Double Negation

Negating a sentence (putting a ~ in front of it) makes it say the opposite of what it originally said. However, if we negate it again, we end up with a sentence that means the same thing as our original sentence; this is called **double negation**.

Imagine that our friend Johnny was in a race, and you ask me, "Did he win?" and I respond, "He did not fail to win." Did he win? It would appear so. Though some languages allow double negations to count as negative statements, in logic a double negation is logically equivalent to the original statement. Both of these forms, therefore, are valid:

P	~~P
∴ ~~P	∴ P

A truth table will verify that each of these forms is valid; both P and ~~P have the same truth values on every row of the truth table.

Common Invalid Forms

Both *modus ponens* and *modus tollens* are logically valid forms, but not all famous logical forms are valid. The last two forms we will discuss—denying the antecedent and affirming the consequent—are famous invalid forms that are the evil twins of the previous two.

Denying the Antecedent

Take a look at the following argument:

If you give lots of money to charity, then you are nice.
You do not give lots of money to charity.
Therefore, you must not be nice.

This might initially seem like a valid argument. However, it is actually invalid in its form. To see that this argument is logically invalid, take a look at the following argument with the same form:

If my cat is a dog, then it is a mammal.
My cat is not a dog.
Therefore, my cat is not a mammal.

This second example is clearly invalid since the premises are true and the conclusion is false. Therefore, there must be something wrong with the form. Here is the form of the argument:

P → Q
~P
∴ ~Q

Because this argument form's second premise rejects the antecedent, P, of the conditional in the first premise, this argument form is referred to as **denying the antecedent**. We can conclusively demonstrate that the form is invalid using the truth table method.

Here is the truth table:

			P1	**P2**	**C**

P	Q	P → Q	~P	~Q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

We see on the third line that it is possible to make both premises true and the conclusion false, so this argument form is definitely invalid. Despite its invalidity, we see this form all the time in real life. Here some examples:

> If you are religious, then you believe in living morally.
> Jim is not religious, so he must not believe in living morally.

Plenty of people who are not religious still believe in living morally. Here is another one:

> If you are training to be an athlete, then you should stay in shape.
> You are not training to be an athlete.
> Thus, you should not stay in shape.

There are plenty of other good reasons to stay in shape.

> If you are Republican, then you support small government.
> Jack is not Republican, so he must not support small government.

Libertarians, for example, are not Republicans, yet they support small government. These examples abound; we can generate them on any topic.

Because this argument form is so common and yet so clearly invalid, denying the antecedent is a famous fallacy of formal logic.

Affirming the Consequent

Another famous formal logical fallacy also begins with a conditional. However, the other two lines are slightly different. Here is the form:

> P → Q
> Q
> ∴ P

Because the second premise states the consequent of the conditional, this form is called **affirming the consequent**. Here is an example:

> If you get mono, you will be very tired.
> You are very tired.
> Therefore, you have mono.

The invalidity of this argument can be seen in the following argument of the same form:

> If my cat is a dog, then it is a mammal.
> My cat is a mammal.
> Therefore, my cat is a dog.

Clearly, this argument is invalid because it has true premises and a false conclusion. Therefore, this must be an invalid form. A truth table will further demonstrate this fact:

		P1	P2	C
P	Q	P → Q	Q	P
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

The third row again demonstrates the possibility of true premises and a false conclusion, so the argument form is invalid. Here are some examples of how this argument form shows up in real life:

In order to get an A, I have to study.
I am going to study.
Therefore, I will get an A.

There might be other requirements to get an A, like showing up for the test.

If it rained, then the street would be wet.
The street is wet.
Therefore, it must have rained.

Sprinklers may have done the job instead.

If he committed the murder, then he would have had to have motive and opportunity.
He had motive and opportunity.
Therefore, he committed the murder.

This argument gives some evidence for the conclusion, but it does not give proof. It is possible that someone else also had motive and opportunity.

The reader may have noticed that in some instances of affirming the consequent, the premises do give us *some* reason to accept the conclusion. This is because of the similarity of this form to the inductive form known as *inference to the best explanation*, which is covered in more detail in Chapter 6. In such inferences we create an "if–then" statement that expresses something that would be the case if a certain assumption were true. These things then act as symptoms of the truth of the assumption. When those symptoms are observed, we have some evidence that the assumption is true. Here are some examples:

If you have measles, then you would present the following symptoms. . . .
You have all of those symptoms.
Therefore, it looks like you have measles.

If he is a faithful Catholic, then he would go to Mass.
I saw him at Mass last Sunday.
Therefore, he is probably a faithful Catholic.

All of these seem to supply decent evidence for the conclusion; however, the argument form is not logically valid. It is logically possible that another medical condition could have the same symptoms or that a person could go to Mass out of curiosity. To determine the (inductive) inferential strength of an argument of that form, we need to think about how likely Q is under different assumptions.

A Closer Look: Translating Categorical Logic

The chapter about categorical logic seems to cover a completely different type of reasoning than this chapter on propositional logic. However, logical advancements made just over a century ago by a man named Gottlob Frege showed that the two types of logic can be combined in what has come to be known as *quantificational logic* (also known as *predicate logic*) (Frege, 1879).

In addition to truth-functional logic, quantificational logic allows us to talk about *quantities* by including logical terms for *all* and *some*. The addition of these terms dramatically increases the power of our logical language and allows us to represent all of categorical logic and much more. Here is a brief overview of how the basic sentences of categorical logic can be represented within quantificational logic.

The statement "All dogs are mammals" can be understood to mean "*If* you are a dog, *then* you are a mammal." The word *you* in this sentence applies to any individual. In other words, the sentence states, "For *all* individuals, *if* that individual is a dog, *then* it is a mammal." In general, statements of the form "All S is M" can be represented as "For all things, *if* that thing is S, *then* it is M."

The statement "Some dogs are brown" means that there *exist* dogs that are brown. In other words, there exist things that are both dogs *and* brown. Therefore, statements of the form "Some S is M" can be represented as

"There exists a thing that is both S *and* M" (propositions of the form "Some S are *not* M" can be represented by simply adding a negation in front of the *M*).

Statements like "No dogs are reptiles" can be understood to mean that all dogs are not reptiles. In general, statements of the form "No S are M" can be represented as "For all things, *if* that thing is an S, *then* it is *not* M."

Quantificational logic allows us to additionally represent the meanings of statements that go well beyond the AEIO propositions of categorical logic. For example, complex statements like "All dogs that are not brown are taller than some cats" can also be represented with the power of quantificational logic though they are well beyond the capacity of categorical logic. The additional power of quantificational logic enables us to represent the meaning of vast stretches of the English language as well as statements used in formal disciplines like mathematics. More instruction in this interesting area can be found in a course on formal logic.

Practice Problems 4.4

Each of the following arguments is a deductive form. Identify the valid form under which the example falls. If the example is not a valid form, select "not a valid form." Click here (https://ne.edgecastcdn.net/0004BA/constellation/PDFs/PHI103_2e/Answers_PracticeProblems4.4.pdf) to check your answers.

1. If we do not decrease poverty in society, then our society will not be an equal one. We are not going to decrease poverty in society. Therefore, our society will not be an equal one.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form
2. If we do not decrease poverty in society, then our society will not be an equal one. Our society will be an equal one. Therefore, we will decrease poverty in society.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form
3. If the moon is full, then it is a good time for night fishing. If it's a good time for night fishing, then we should go out tonight. Therefore, if the moon is full, then we should go out tonight.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form
4. Either the Bulls or the Knicks will lose tonight. The Bulls are not going to lose. Therefore, the Knicks will lose.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form
5. If the battery is dead, then the car won't start. The car won't start. Therefore, the battery is dead.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form

6. If I take this new job, then we will have to move to Alaska. I am not going to take the new job. Therefore, we will not have to move to Alaska.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form

7. If human perception conditions reality, then humans cannot know things in themselves. If humans cannot know things in themselves, then they cannot know the truth. Therefore, if human perceptions conditions reality, then humans cannot know the truth.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form

8. We either adopt the plan or we will be in danger of losing our jobs. We are not going to adopt the plan. Therefore, we will be in danger of losing our jobs.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form

9. If media outlets are owned by corporations with advertising interests, then it will be difficult for them to be objective. Media outlets are owned by corporations with advertising interests. Therefore, it will be difficult for them to be objective.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form

10. If you eat too much aspartame, you will get a headache. You do not have a headache. Therefore, you did not eat too much aspartame.
    a. *modus ponens*
    b. *modus tollens*
    c. disjunctive syllogism
    d. hypothetical syllogism
    e. not a valid form

Summary and Resources

Chapter Summary

Propositional logic shows how the truth values of complex statements can be systematically derived from the truth values of their parts. Words like *and*, *or*, *not*, and *if . . . then . . .* each have truth tables that demonstrate the algorithms for determining these truth values. Once we have found the logical form of an argument, we can determine whether it is logically valid by using the truth table method. This method involves creating a truth table that represents all possible truth values of the component parts and the resulting values for the premises and conclusion of the argument. If there is even one row of the truth table in which all of the premises are true and the conclusion is false, then the argument is invalid; if there is no such row, then it is valid.

Knowledge of propositional logic has proved very valuable to humankind: It allows us to formally demonstrate the validity of different types of reasoning; it helps us precisely understand the meaning of certain types of terms in our language; it enables us to determine the truth conditions of formally complex statements; and it forms the basis for computing.

Critical Thinking Questions

1. Symbolizing arguments makes them easier to visualize and examine in the realm of propositional logic. Do you find that the symbols make things easier to visualize or more confusing? If logicians use these methods to make things easier, then what does that mean if you think that using these symbols is confusing?
2. In your own words, what is the difference between categorical logic and propositional logic? How do they relate to one another? How do they differ?

3. How does understanding how to symbolize statements and complete truth tables relate to your everyday life? What is the practical importance of understanding how to use these methods to determine validity?
4. If you were at work or with your friends and someone presented an argument, do you think you could evaluate it using the methods you have learned thus far in this book? Is it important to evaluate arguments, or is this just something academics do in their spare time? Why do you believe this is (or is not) the case?
5. How would you now explain the concept of *validity* to someone with whom you interact on a daily basis who might not have an understanding of logic? How would you explain how validity differs from truth?

Web Resources

http://www.manyworldsoflogic.com/exercises/quizTruthFunctional.html (http://www.manyworldsoflogic.com/exercises/quizTruthFunctional.html)
Test your understanding of propositional, or truth-functional, logic by taking the quizzes available at philosophy professor Paul Herrick's Many Worlds of Logic website.

https://www.youtube.com/watch?v=moHkk_89UZE (https://www.youtube.com/watch?v=moHkk_89UZE)
Watch a video that walks you through how to construct a truth table.

https://www.youtube.com/watch?feature=player_embedded&v=83xPkTqoulE (https://www.youtube.com/watch?feature=player_embedded&v=83xPkTqoulE)
Watch Ashford University professor Justin Harrison explain how to construct a conjunction truth table.

Key Terms

affirming the consequent

antecedent

biconditional

conditional

conjunction

connectives

consequent

converse

denying the antecedent

disjunction

disjunctive syllogism

double negation

hypothetical syllogism

logically equivalent

modus ponens

modus tollens

negation

operators

proposition

propositional logic

sentence variables

statement form

truth table

truth value

An argument with two premises, one of which is a conditional and the other of which is the consequent of that conditional. It has the form $P \rightarrow Q$, Q, therefore P. It is invalid.

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