Q1. Let $\mathbf{u}, \mathbf{v}$ be the vectors as given

$$\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

Find

a. A vector $\mathbf{w}$ so that $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ are linearly independent

when; $u x_1 + v x_2 = 0$; $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$

$w = \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$; while $Ax = 0$ has a trivial solution, $\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = u$; $\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} = v$ ~~while all povit position~~ are linearly independent.

b. Vectors $\mathbf{w}, \mathbf{x}$ so that the vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ and $\mathbf{x}$ are linearly independent

Q2. Let $T : \mathbb{R}^2 \to R^2$ be a linear transformation given by the matrix

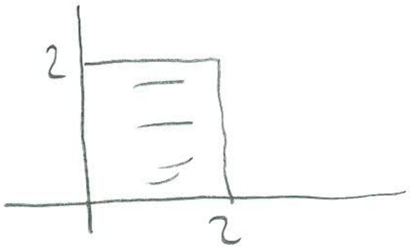
$$A = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

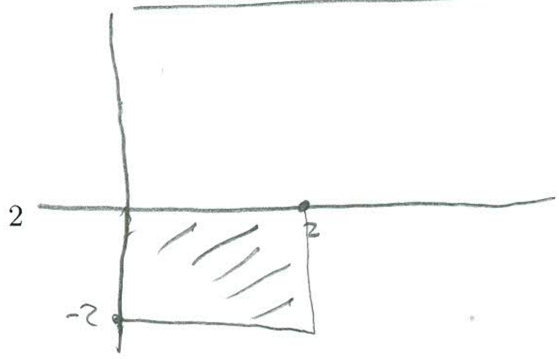
a. Find the image of a unit box and show it graphically.

Let assume $u = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$; $T(u) = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$; the image of $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ is ; $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \boxed{\begin{pmatrix} 2 \\ -2 \end{pmatrix}}$

b. Show that the transformation above is one to one.

transformation;





Q3. Consider the following matrix

$$A = \begin{bmatrix} 3 & -1 & 4 \\ 1 & -1 & 0 \\ -1 & 0 & -2 \end{bmatrix} \quad R$$

a. Reduce the matrix into Echelon form

$$A = \begin{bmatrix} 1 & -1/3 & 4/3 \\ 1 & -1 & 0 \\ -1 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1/3 & 4/3 \\ 0 & -2/3 & -4/3 \\ 0 & -1/3 & -2/3 \end{bmatrix} \times -\tfrac{3}{2} \sim \begin{bmatrix} 1 & -1/3 & 4/3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -1/3 & 4/3 \\ 0 & \boxed{1} & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

b. Solve $A\mathbf{x} = \mathbf{0}$.

$$\begin{bmatrix} 1 & -1/3 & 4/3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0 \quad ; \quad \begin{aligned} x_1 - \tfrac{1}{3}x_2 + \tfrac{4}{3}x_3 &= 0 \\ x_2 + 2x_3 &= 0 \end{aligned}$$

Q4. Consider the following matrix

$$B = \begin{bmatrix} \boxed{1} & 2 & 2 & 0 & 0 \\ 0 & 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & 0 & \boxed{1} & 1 \\ 0 & 0 & 0 & 0 & p \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & p \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a. What is the dimension of $Null(B)$, assume $p \neq 0$?

$x_1 + 2x_2 = 0$

$x_3 + x_5 = 0$

$x_4 + x_5 = 0$

$p x_5 = 0$; $x_5 = x_5$

$$\text{Col } B = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} ; \therefore \text{Rank } B = 2$$

$$\dim \text{Nul } B = n - \text{Rank } B = 5 - 2 = \boxed{3}$$

b. Find the basis for $Null(B)$, assume $p \neq 0$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ x_3 + 5x_5 \\ x_4 + x_5 \\ p x_5 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 5 \\ 1 \\ p \\ 1 \end{bmatrix} x_5$$

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basis of Null B $= \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\5\\1\\p\\1\end{bmatrix}$

c. If $p = 0$, what vectors form the basis for $Null(B)$

Basis of Null B $= \begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\5\\1\\0\\1\end{bmatrix}$

Q5. Construct a 3×3 matrix such that

a. The columns form a basis for $\mathbb{R}^3$

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

the column of A from basis for R^3.

b. The rank of the matrix is 1.

$\begin{bmatrix}1 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$; ~~white col~~ it has 1 rank

Q6. Let A be the following matrix, and let B be it's Echelon form.

$$A = \begin{pmatrix} 1 & 2 & -5 & 11 & -3\\ 2 & 4 & -5 & 15 & 2\\ 1 & 2 & 0 & 4 & 5\\ 3 & 6 & -5 & 19 & -2 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 2 & 0 & 4 & 5\\ 0 & 0 & 5 & -7 & 8\\ 0 & 0 & 0 & 0 & -9\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

a. Solve the system of equations $A\mathbf{x} = \mathbf{0}$

$$\begin{pmatrix} 1 & 2 & 0 & 4 & 5\\ 0 & 0 & 5 & -7 & 8\\ 0 & 0 & 0 & 0 & -9\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix} ; \begin{pmatrix} 1 & 2 & 0 & 4 & 5\\ 0 & 0 & 5 & -7 & 8\\ 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1\\x_2\\x_3\\x_4\\x_5\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$x_1 + 2x_2 + 4x_4 + 5x_5 = 0$

$5x_3 - 7x_4 + 8x_5 = 0$

$x_5 = 0$

$x_5 = x_5$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{bmatrix} x_1 + 2x_2 + 4x_4 + 5x_5 \\ 5x_3 - 7x_4 + 8x_5 \\ x_5 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_1 + \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 0 \\ 5 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 4 \\ -7 \\ 0 \\ 0 \end{bmatrix} x_4 + \begin{bmatrix} 5 \\ 8 \\ 1 \\ 1 \end{bmatrix} x_5$$

X b. Is the $Col(A) = \mathbb{R}^4$, why? dim Null A = n - Rank A ; 4×5

$n = 5$; dim Null A = 4 ;

X Q7. Let $S \subset \mathbb{R}^3$ consisting of vectors $\mathbf{u}$ such that $x + y + z = 1$, where

$$\mathbf{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Is the set S a subspace of $\mathbb{R}^3$. why?

The set S is not a subspace of $\mathbb{R}^3$, because $x + y + z \neq 0$

Q8. Use Cramer's rule to compute the solution of the system of equations

$$3x - 2y = 4, \quad -5x + 6y = -2$$

$A\bar{x} = \bar{b}$; $A = \begin{bmatrix} 3 & -2 \\ -5 & 6 \end{bmatrix}$; $A_1\vec{b} = \begin{bmatrix} 4 & -2 \\ -2 & 6 \end{bmatrix}$; $A_2\vec{b} = \begin{bmatrix} 3 & 4 \\ -5 & -2 \end{bmatrix}$

$\det A = 8, -10 = 8$; $\det A_1\vec{b} = 20$; $\det A_2\vec{b} = 14$; $x_1 = \frac{\det A_1 b}{\det A} = \frac{20}{8} = \boxed{\frac{5}{2}}$

Q9. Find the determinant of the following matrix. $x_2 = \frac{\det A_2 b}{\det A} = \frac{14}{8} = \boxed{\frac{7}{4}}$

$$A = \begin{pmatrix} 3 & 2 & 5 & 2 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$A = 3\begin{pmatrix} 2 & 5 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

$A = 3 \times 2 \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = 6[-1] = \boxed{-6}$ 5