

Next, we consider activities A and B, both of which have only Start as an immediate predecessor. Using the earliest start time rule, the ES for both activities A and B equals zero, which is the EF of activity Start. Now, using the earliest finish time rule, the EF for A is 2 ($= 0 + 2$), and the EF for B is 3 ($= 0 + 3$).

Since activity A precedes activity C, the ES of C equals the EF of A ($= 2$). The EF of C is therefore 4 ($= 2 + 2$).

We now come to activity D. Both activities A and B are immediate predecessors for D. Whereas A has an EF of 2, activity B has an EF of 3. Using the earliest start time rule, we compute the ES of activity D as follows:

$$\text{ES of D} = \text{Max}(\text{EF of A, EF of B}) = \text{Max}(2, 3) = 3$$

The EF of D equals 7 ($= 3 + 4$). Next, both activities E and F have activity C as their only immediate predecessor. Therefore, the ES for both E and F equals 4 ($= \text{EF of C}$). The EF of E is 8 ($= 4 + 4$), and the EF of F is 7 ($= 4 + 3$).

Activity G has both activities D and E as predecessors. Using the earliest start time rule, its ES is therefore the maximum of the EF of D and the EF of E. Hence, the ES of activity G equals 8 ($= \text{maximum of 7 and 8}$), and its EF equals 13 ($= 8 + 5$).

Finally, we come to activity H. Since it also has two predecessors, F and G, the ES of H is the maximum EF of these two activities. That is, the ES of H equals 13 ($= \text{maximum of 13 and 7}$). This implies that the EF of H is 15 ($= 13 + 2$). Since H is the last activity in the project, this also implies that the earliest time in which the entire project can be completed is 15 weeks.

INSIGHT ► The ES of an activity that has only one predecessor is simply the EF of that predecessor. For an activity with more than one predecessor, we must carefully examine the EFs of all immediate predecessors and choose the largest one.

LEARNING EXERCISE ► A new activity I, *EPA Approval*, takes 1 week. Its predecessor is activity H. What are I's ES and EF? [Answer: 15, 16]

RELATED PROBLEMS ► 3.11, 3.14c

EXCEL OM Data File Ch03Ex4.xls can be found at www.pearsonhighered.com/heizer.

Although the forward pass allows us to determine the earliest project completion time, it does not identify the critical path. To identify this path, we need to now conduct the backward pass to determine the LS and LF values for all activities.

Backward Pass

Just as the forward pass began with the first activity in the project, the **backward pass** begins with the last activity in the project. For each activity, we first determine its LF value, followed by its LS value. The following two rules are used in this process.

Backward pass

An activity that finds all the late start and late finish times.

Latest Finish Time Rule This rule is again based on the fact that before an activity can start, all its immediate predecessors must be finished:

- If an activity is an immediate predecessor for just a single activity, its LF equals the LS of the activity that immediately follows it.
- If an activity is an immediate predecessor to more than one activity, its LF is the minimum of all LS values of all activities that immediately follow it. That is:

$$\text{LF} = \text{Min}\{\text{LS of all immediate following activities}\} \quad (3-3)$$

Latest Start Time Rule The latest start time (LS) of an activity is the difference of its latest finish time (LF) and its activity time. That is:

$$\text{LS} = \text{LF} - \text{Activity time} \quad (3-4)$$

Example 5

COMPUTING LATEST START AND FINISH TIMES FOR MILWAUKEE PAPER

Calculate the latest start and finish times for each activity in Milwaukee Paper's pollution project.

APPROACH ► Use Figure 3.10 as a beginning point. Overlay 1 of Figure 3.10 shows the complete project network for Milwaukee Paper, along with added LS and LF values for all activities. In what follows, we see how these values were calculated.

SOLUTION ► We begin by assigning an LF value of 15 weeks for activity H. That is, we specify that the latest finish time for the entire project is the same as its earliest finish time. Using the latest start time rule, the LS of activity H is equal to 13 ($= 15 - 2$).

Since activity H is the lone succeeding activity for both activities F and G, the LF for both F and G equals 13. This implies that the LS of G is 8 ($= 13 - 5$), and the LS of F is 10 ($= 13 - 3$).

Proceeding in this fashion, we see that the LF of E is 8 ($=$ LS of G), and its LS is 4 ($= 8 - 4$). Likewise, the LF of D is 8 ($=$ LS of G), and its LS is 4 ($= 8 - 4$).

We now consider activity C, which is an immediate predecessor to two activities: E and F. Using the latest finish time rule, we compute the LF of activity C as follows:

$$\text{LF of C} = \text{Min}(\text{LS of E, LS of F}) = \text{Min}(4, 10) = 4$$

The LS of C is computed as 2 ($= 4 - 2$). Next, we compute the LF of B as 4 ($=$ LS of D) and its LS as 1 ($= 4 - 3$).

We now consider activity A. We compute its LF as 2 ($=$ minimum of LS of C and LS of D). Hence, the LS of activity A is 0 ($= 2 - 2$). Finally, both the LF and LS of activity Start are equal to 0.

INSIGHT ► The LF of an activity that is the predecessor of only one activity is just the LS of that following activity. If the activity is the predecessor to more than one activity, its LF is the smallest LS value of all activities that follow immediately.

LEARNING EXERCISE ► A new activity I, *EPA Approval*, takes 1 week. Its predecessor is activity H. What are I's LS and LF? [Answer: 15, 16]

RELATED PROBLEMS ► 3.11, 3.14c.

Slack time

Free time for an activity. Also referred to as free float or free slack.

Calculating Slack Time and Identifying the Critical Path(s)

After we have computed the earliest and latest times for all activities, it is a simple matter to find the amount of **slack time** that each activity has. Slack is the length of time an activity can be delayed without delaying the entire project. Mathematically:

$$\text{Slack} = \text{LS} - \text{ES} \quad \text{or} \quad \text{Slack} = \text{LF} - \text{EF} \quad (3-5)$$

Example 6

CALCULATING SLACK TIMES FOR MILWAUKEE PAPER

Calculate the slack for the activities in the Milwaukee Paper project.

APPROACH ► Start with the data in Overlay 1 of Figure 3.10 in Example 5 and develop Table 3.3 one line at a time.

SOLUTION ► Table 3.3 summarizes the ES, EF, LS, LF, and slack time for all of the firm's activities. Activity B, for example, has 1 week of slack time since its LS is 1 and its ES is 0 (alternatively, its LF is 4 and its EF is 3). This means that activity B can be delayed by up to 1 week, and the whole project can still be finished in 15 weeks.

On the other hand, activities A, C, E, G, and H have *no* slack time. This means that none of them can be delayed without delaying the entire project. Conversely, if plant manager Julie Ann Williams wants to reduce the total project times, she will have to reduce the length of one of these activities.

Overlay 2 of Figure 3.10 shows the slack computed for each activity.

INSIGHT ► Slack may be computed from either early/late starts or early/late finishes. The key is to find which activities have zero slack.

TABLE 3.3 Milwaukee Paper's Schedule and Slack Times

ACTIVITY	EARLIEST START ES	EARLIEST FINISH EF	LATEST START LS	LATEST FINISH LF	SLACK LS - ES	ON CRITICAL PATH
A	0	2	0	2	0	Yes
B	0	3	1	4	1	No
C	2	4	2	4	0	Yes
D	3	7	4	8	1	No
E	4	8	4	8	0	Yes
F	4	7	10	13	6	No
G	8	13	8	13	0	Yes
H	13	15	13	15	0	Yes

LEARNING EXERCISE ▶ A new activity I, *EPA Approval*, follows activity H and takes 1 week. Is it on the critical path? [Answer: Yes, it's $LS - ES = 0$]

RELATED PROBLEMS ▶ 3.6, 3.11, 3.27

ACTIVE MODEL 3.1 This example is further illustrated in Active Model 3.1 at www.pearsonhighered.com/heizer.

The activities with zero slack are called *critical activities* and are said to be on the critical path. The critical path is a continuous path through the project network that:

LO4 Determine a critical path

- ▶ Starts at the first activity in the project (Start in our example).
- ▶ Terminates at the last activity in the project (H in our example).
- ▶ Includes only critical activities (i.e., activities with no slack time).

Example 7

SHOWING CRITICAL PATH WITH BLUE ARROWS

Show Milwaukee Paper's critical path and find the project completion time.

APPROACH ▶ We use Table 3.3 and Overlay 3 of Figure 3.10. Overlay 3 of Figure 3.10 indicates that the total project completion time of 15 weeks corresponds to the longest path in the network. That path is Start-A-C-E-G-H in network form. It is shown with thick blue arrows.

INSIGHT ▶ The critical path follows the activities with slack = 0. This is considered the longest path through the network.

LEARNING EXERCISE ▶ Why are activities B, D, and F not on the path with the thick blue line? [Answer: They are not critical and have slack values of 1, 1, and 6 weeks, respectively.]

RELATED PROBLEMS ▶ 3.3, 3.4, 3.5, 3.6, 3.7, 3.12, 3.14b, 3.15, 3.17, 3.20a, 3.22a, 3.23, 3.26

Total Slack Time Look again at the project network in Overlay 3 of Figure 3.10. Consider activities B and D, which have slack of 1 week each. Does it mean that we can delay *each* activity by 1 week, and still complete the project in 15 weeks? The answer is no.

Let's assume that activity B is delayed by 1 week. It has used up its slack of 1 week and now has an EF of 4. This implies that activity D now has an ES of 4 and an EF of 8. Note that these are also its LS and LF values, respectively. That is, activity D also has no slack time now. Essentially, the slack of 1 week that activities B and D had is, for that path, *shared* between them. Delaying either activity by 1 week causes not only that activity, but also the other activity, to lose its slack. This type of a slack time is referred to as *total slack*. Typically, when two or more noncritical activities appear successively in a path, they share total slack.

To plan, monitor, and control the huge number of details involved in sponsoring a rock festival attended by more than 100,000 fans, managers use Microsoft Project and the tools discussed in this chapter. The *Video Case Study* “Managing Hard Rock’s Rockfest,” at the end of the chapter, provides more details of the management task.



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Variability in Activity Times

In identifying all earliest and latest times so far, and the associated critical path(s), we have adopted the CPM approach of assuming that all activity times are known and fixed constants. That is, there is no variability in activity times. However, in practice, it is likely that activity completion times vary depending on various factors.

For example, building internal components (activity A) for Milwaukee Paper Manufacturing is estimated to finish in 2 weeks. Clearly, supply-chain issues such as late arrival of materials, absence of key personnel, and so on could delay this activity. Suppose activity A actually ends up taking 3 weeks. Since A is on the critical path, the entire project will now be delayed by 1 week to 16 weeks. If we had anticipated completion of this project in 15 weeks, we would obviously miss our Earth Day deadline.

Although some activities may be relatively less prone to delays, others could be extremely susceptible to delays. For example, activity B (modify roof and floor) could be heavily dependent on weather conditions. A spell of bad weather could significantly affect its completion time.

This means that we cannot ignore the impact of variability in activity times when deciding the schedule for a project. PERT addresses this issue.

STUDENT TIP ★

PERT's ability to handle three time estimates for each activity enables us to compute the probability that we can complete the project by a target date.

Three Time Estimates in PERT

In PERT, we employ a probability distribution based on three time estimates for each activity, as follows:

Optimistic time

The “best” activity completion time that could be obtained in a PERT network.

Pessimistic time

The “worst” activity time that could be expected in a PERT network.

Most likely time

The most probable time to complete an activity in a PERT network.

Optimistic time (a) = time an activity will take if everything goes as planned. In estimating this value, there should be only a small probability (say, 1/100) that the activity time will be $< a$.

Pessimistic time (b) = time an activity will take assuming very unfavorable conditions. In estimating this value, there should also be only a small probability (also 1/100) that the activity time will be $> b$.

Most likely time (m) = most realistic estimate of the time required to complete an activity.

When using PERT, we often assume that activity time estimates follow the beta probability distribution (see Figure 3.11). This continuous distribution is often appropriate for determining the expected value and variance for activity completion times.

To find the *expected activity time*, t , the beta distribution weights the three time estimates as follows:

$$t = (a + 4m + b)/6 \quad (3-6)$$

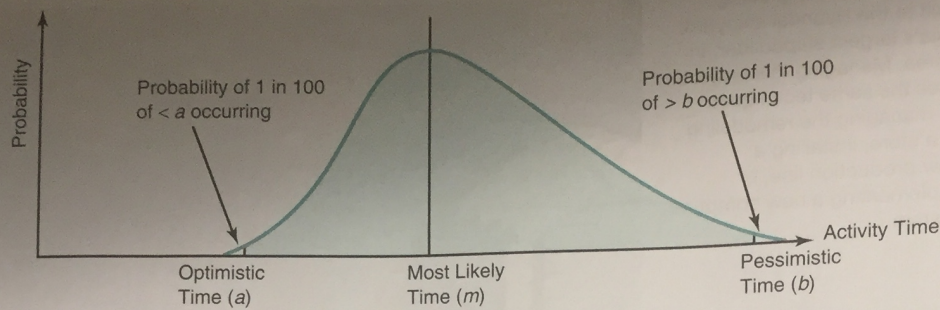


Figure 3.11
Beta Probability Distribution
with Three Time Estimates

That is, the most likely time (m) is given four times the weight as the optimistic time (a) and pessimistic time (b). The time estimate t computed using Equation (3-6) for each activity is used in the project network to compute all earliest and latest times.

To compute the dispersion or variance of activity completion time, we use the formula:¹

$$\text{Variance} = [(b - a)/6]^2 \quad (3-7)$$

Example 8

EXPECTED TIMES AND VARIANCES FOR MILWAUKEE PAPER

Julie Ann Williams and the project management team at Milwaukee Paper want an expected time and variance for Activity F (Installing the Pollution Control System) where:

$$a = 1 \text{ week}, m = 2 \text{ weeks}, b = 9 \text{ weeks}$$

APPROACH ► Use Equations (3-6) and (3-7) to compute the expected time and variance for F.

SOLUTION ► The expected time for Activity F is:

$$t = \frac{a + 4m + b}{6} = \frac{1 + 4(2) + 9}{6} = \frac{18}{6} = 3 \text{ weeks}$$

The variance for Activity F is:

$$\text{Variance} = \left[\frac{(b - a)}{6} \right]^2 = \left[\frac{(9 - 1)}{6} \right]^2 = \left(\frac{8}{6} \right)^2 = \frac{64}{36} = 1.78$$

INSIGHT ► Williams now has information that allows her to understand and manage Activity F. The expected time is, in fact, the activity time used in our earlier computation and identification of the critical path.

LEARNING EXERCISE ► Review the expected times and variances for all of the other activities in the project. These are shown in Table 3.4.

TABLE 3.4 Time Estimates (in weeks) for Milwaukee Paper's Project

ACTIVITY	OPTIMISTIC a	MOST LIKELY m	PESSIMISTIC b	EXPECTED TIME $t = (a + 4m + b)/6$	VARIANCE $[(b - a)/6]^2$
A	1	2	3	2	$[(3 - 1)/6]^2 = 4/36 = .11$
B	2	3	4	3	$[(4 - 2)/6]^2 = 4/36 = .11$
C	1	2	3	2	$[(3 - 1)/6]^2 = 4/36 = .11$
D	2	4	6	4	$[(6 - 2)/6]^2 = 16/36 = .44$
E	1	4	7	4	$[(7 - 1)/6]^2 = 36/36 = 1.00$
F	1	2	9	3	$[(9 - 1)/6]^2 = 64/36 = 1.78$
G	3	4	11	5	$[(11 - 3)/6]^2 = 64/36 = 1.78$
H	1	2	3	2	$[(3 - 1)/6]^2 = 4/36 = .11$

STUDENT TIP

Can you see why the variance is higher in some activities than in others? Note the spread between the optimistic and pessimistic times.

RELATED PROBLEMS ► 3.13, 3.14a, 3.17a,b, 3.21a

EXCEL OM Data File Ch03Ex8.xls can be found at www.pearsonhighered.com/heizer.

¹This formula is based on the statistical concept that from one end of the beta distribution to the other is 6 standard deviations (± 3 standard deviations from the mean). Since $(b - a)$ is 6 standard deviations, the variance is $[(b - a)/6]^2$.

We see here a ship being built at the Hyundai shipyard, in Asia's largest shipbuilder, in Korea. Managing this project uses the same techniques as managing the remodeling of a store, installing a new production line, or implementing a new computer system.



Paul Chesley/Getty Images

Probability of Project Completion

The critical path analysis helped us determine that Milwaukee Paper's expected project completion time is 15 weeks. Julie Ann Williams knows, however, that there is significant variation in the time estimates for several activities. Variation in activities that are on the critical path can affect the overall project completion time—possibly delaying it. This is one occurrence that worries the plant manager considerably.

PERT uses the variance of critical path activities to help determine the variance of the overall project. Project variance is computed by summing variances of *critical* activities:

$$\sigma_p^2 = \text{Project variance} = \Sigma(\text{variances of activities on critical path}) \quad (3-8)$$

Example 9

COMPUTING PROJECT VARIANCE AND STANDARD DEVIATION FOR MILWAUKEE PAPER

Milwaukee Paper's managers now wish to know the project's variance and standard deviation.

APPROACH ► Because the activities are independent, we can add the variances of the activities on the critical path and then take the square root to determine the project's standard deviation.

SOLUTION ► From Example 8 (Table 3.4), we have the variances of all of the activities on the critical path. Specifically, we know that the variance of activity A is 0.11, variance of activity C is 0.11, variance of activity E is 1.00, variance of activity G is 1.78, and variance of activity H is 0.11.

Compute the total project variance and project standard deviation:

$$\text{Project variance } (\sigma_p^2) = 0.11 + 0.11 + 1.00 + 1.78 + 0.11 = 3.11$$

which implies:

$$\text{Project standard deviation } (\sigma_p) = \sqrt{\text{Project variance}} = \sqrt{3.11} = 1.76 \text{ weeks}$$

INSIGHT ► Management now has an estimate not only of expected completion time for the project but also of the standard deviation of that estimate.

LEARNING EXERCISE ► If the variance for activity A is actually 0.30 (instead of 0.11), what is the new project standard deviation? [Answer: 1.817.]

RELATED PROBLEM ► 3.17e

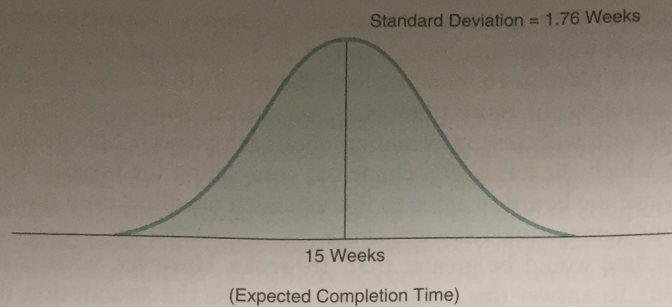


Figure 3.12

Probability Distribution for
Project Completion Times at
Milwaukee Paper

How can this information be used to help answer questions regarding the probability of finishing the project on time? PERT makes two more assumptions: (1) total project completion times follow a normal probability distribution, and (2) activity times are statistically independent. With these assumptions, the bell-shaped normal curve shown in Figure 3.12 can be used to represent project completion dates. This normal curve implies that there is a 50% chance that the manufacturer's project completion time will be less than 15 weeks and a 50% chance that it will exceed 15 weeks.

Example 10

PROBABILITY OF COMPLETING A PROJECT ON TIME

Julie Ann Williams would like to find the probability that her project will be finished on or before the 16-week Earth Day deadline.

APPROACH ► To do so, she needs to determine the appropriate area under the normal curve. This is the area to the left of the 16th week.

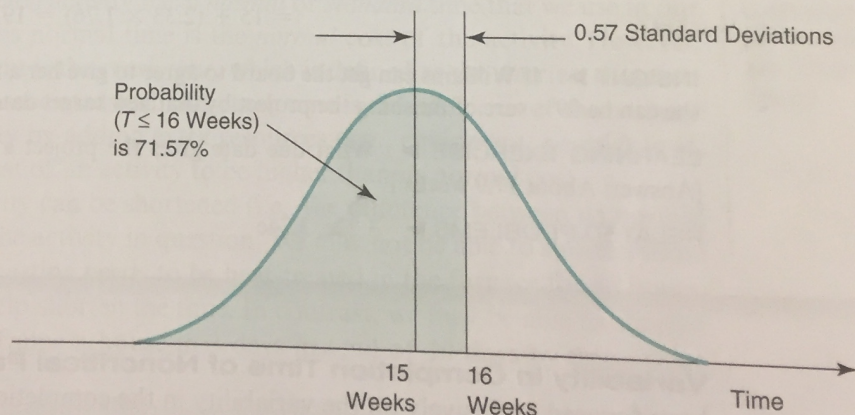
SOLUTION ► The standard normal equation can be applied as follows:

$$Z = (\text{Due date} - \text{Expected date of completion}) / \sigma_p \quad (3-9)$$

$$= (16 \text{ weeks} - 15 \text{ weeks}) / 1.76 \text{ weeks} = 0.57$$

where Z is the number of standard deviations the due date or target date lies from the mean or expected date.

Referring to the Normal Table in Appendix I, we find a Z value of 0.57 to the right of the mean indicates a probability of 0.7157. Thus, there is a 71.57% chance that the pollution control equipment can be put in place in 16 weeks or less. This is shown in Figure 3.13.



INSIGHT ► The shaded area to the left of the 16th week (71.57%) represents the probability that the project will be completed in less than 16 weeks.

LEARNING EXERCISE ► What is the probability that the project will be completed on or before the 17th week? [Answer: About 87.2%.]

RELATED PROBLEMS ► 3.14d, 3.17f, 3.21d, e, 3.22b, 3.24

STUDENT TIP

Here is a chance to review your statistical skills and use of a normal distribution table (Appendix I).

Figure 3.13

Probability That Milwaukee
Paper Will Meet the 16-Week
Deadline

Determining Project Completion Time for a Given Confidence Level Let's say Julie Ann Williams is worried that there is only a 71.57% chance that the pollution control equipment can be put in place in 16 weeks or less. She thinks that it may be possible to plead with the board of directors for more time. However, before she approaches the board, she wants to arm herself with sufficient information about the project. Specifically, she wants to find the deadline by which she has a 99% chance of completing the project. She hopes to use her analysis to convince the board to agree to this extended deadline, even though she is aware of the public relations damage the delay will cause.

Clearly, this due date would be greater than 16 weeks. However, what is the exact value of this new due date? To answer this question, we again use the assumption that Milwaukee Paper's project completion time follows a normal probability distribution with a mean of 15 weeks and a standard deviation of 1.76 weeks.

Example 11

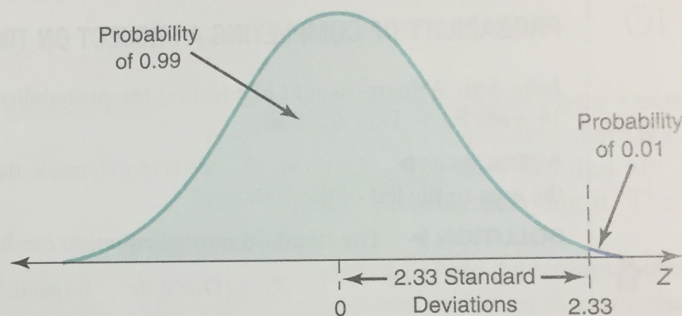
COMPUTING PROBABILITY FOR ANY COMPLETION DATE

Julie Ann Williams wants to find the due date that gives her company's project a 99% chance of *on-time* completion.

APPROACH ▶ She first needs to compute the Z-value corresponding to 99%, as shown in Figure 3.14. Mathematically, this is similar to Example 10, except the unknown is now the *due date* rather than Z.

Figure 3.14

Z-Value for 99% Probability of Project Completion at Milwaukee Paper



SOLUTION ▶ Referring again to the Normal Table in Appendix I, we identify a Z-value of 2.33 as being closest to the probability of 0.99. That is, Julie Ann Williams's due date should be 2.33 standard deviations above the mean project completion time. Starting with the standard normal equation [see Equation (3-9)], we can solve for the due date and rewrite the equation as:

$$\begin{aligned} \text{Due date} &= \text{Expected completion time} + (Z \times \sigma_p) \\ &= 15 + (2.33 \times 1.76) = 19.1 \text{ weeks} \end{aligned} \quad (3-10)$$

INSIGHT ▶ If Williams can get the board to agree to give her a new deadline of 19.1 weeks (or more), she can be 99% sure of finishing the project by that new target date.

LEARNING EXERCISE ▶ What due date gives the project a 95% chance of on-time completion? [Answer: About 17.9 weeks.]

RELATED PROBLEMS ▶ 3.22c, 3.24e

Variability in Completion Time of Noncritical Paths In our discussion so far, we have focused exclusively on the variability in the completion times of activities on the critical path. This seems logical since these activities are, by definition, the more important activities in a project network. However, when there is variability in activity times, it is important that we also investigate the variability in the completion times of activities on *noncritical* paths.

Consider, for example, activity D in Milwaukee Paper's project. Recall from Overlay 3 in Figure 3.10 (in Example 7) that this is a noncritical activity, with a slack time of 1 week. We have therefore not considered the variability in D's time in computing the probabilities of project completion times. We observe, however, that D has a variance of 0.44 (see Table 3.4 in

Example 8). In fact, the pessimistic completion time for D is 6 weeks. This means that if D ends up taking its pessimistic time to finish, the project will not finish in 15 weeks, even though D is not a critical activity.

For this reason, when we find probabilities of project completion times, it may be necessary for us to not focus only on the critical path(s). Indeed, some research has suggested that expending project resources to reduce the variability of activities not on the critical path can be an effective element in project management. We may need also to compute these probabilities for noncritical paths, especially those that have relatively large variances. It is possible for a noncritical path to have a smaller probability of completion within a due date, when compared with the critical path. Determining the variance and probability of completion for a noncritical path is done in the same manner as Examples 9 and 10.

What Project Management Has Provided So Far Project management techniques have thus far been able to provide Julie Ann Williams with several valuable pieces of management information:

1. The project's expected completion date is 15 weeks.
2. There is a 71.57% chance that the equipment will be in place within the 16-week deadline. PERT analysis can easily find the probability of finishing by any date Williams is interested in.
3. Five activities (A, C, E, G, and H) are on the critical path. If any one of these is delayed for any reason, the entire project will be delayed.
4. Three activities (B, D, F) are not critical and have some slack time built in. This means that Williams can borrow from their resources, and, if necessary, she may be able to speed up the whole project.
5. A detailed schedule of activity starting and ending dates, slack, and critical path activities has been made available (see Table 3.3 in Example 6).

Cost-Time Trade-Offs and Project Crashing

While managing a project, it is not uncommon for a project manager to be faced with either (or both) of the following situations: (1) the project is behind schedule, and (2) the scheduled project completion time has been moved forward. In either situation, some or all of the remaining activities need to be speeded up (usually by adding resources) to finish the project by the desired due date. The process by which we shorten the duration of a project in the cheapest manner possible is called project **crashing**.

CPM is a technique in which each activity has a *normal* or *standard* time that we use in our computations. Associated with this normal time is the *normal* cost of the activity. However, another time in project management is the *crash time*, which is defined as the shortest duration required to complete an activity. Associated with this crash time is the *crash cost* of the activity. Usually, we can shorten an activity by adding extra resources (e.g., equipment, people) to it. Hence, it is logical for the crash cost of an activity to be higher than its normal cost.

The amount by which an activity can be shortened (i.e., the difference between its normal time and crash time) depends on the activity in question. We may not be able to shorten some activities at all. For example, if a casting needs to be heat-treated in the furnace for 48 hours, adding more resources does not help shorten the time. In contrast, we may be able to shorten some activities significantly (e.g., frame a house in 3 days instead of 10 days by using three times as many workers).

Likewise, the cost of crashing (or shortening) an activity depends on the nature of the activity. Managers are usually interested in speeding up a project at the least additional cost. Hence, when choosing which activities to crash, and by how much, we need to ensure the following:

- ▶ The amount by which an activity is crashed is, in fact, permissible
- ▶ Taken together, the shortened activity durations will enable us to finish the project by the due date
- ▶ The total cost of crashing is as small as possible

Crashing

Shortening activity time in a network to reduce time on the critical path so total completion time is reduced.

LO6 Crash a project

Crashing a project involves four steps:

STEP 1: Compute the crash cost per week (or other time period) for each activity in the network. If crash costs are linear over time, the following formula can be used:

$$\text{Crash cost per period} = \frac{(\text{Crash cost} - \text{Normal cost})}{(\text{Normal time} - \text{Crash time})} \quad (3-11)$$

STEP 2: Using the current activity times, find the critical path(s) in the project network. Identify the critical activities.

STEP 3: If there is only one critical path, then select the activity on this critical path that (a) can still be crashed and (b) has the smallest crash cost per period. Crash this activity by one period.

If there is more than one critical path, then select one activity from each critical path such that (a) each selected activity can still be crashed and (b) the total crash cost per period of *all* selected activities is the smallest. Crash each activity by one period. Note that the same activity may be common to more than one critical path.

STEP 4: Update all activity times. If the desired due date has been reached, stop. If not, return to Step 2.

We illustrate project crashing in Example 12.

Example 12

PROJECT CRASHING TO MEET A DEADLINE AT MILWAUKEE PAPER

Suppose the plant manager at Milwaukee Paper Manufacturing has been given only 13 weeks (instead of 16 weeks) to install the new pollution control equipment. As you recall, the length of Julie Ann Williams's critical path was 15 weeks, but she must now complete the project in 13 weeks.

APPROACH ► Williams needs to determine which activities to crash, and by how much, to meet this 13-week due date. Naturally, Williams is interested in speeding up the project by 2 weeks, at the least additional cost.

SOLUTION ► The company's normal and crash times, and normal and crash costs, are shown in Table 3.5. Note, for example, that activity B's normal time is 3 weeks (the estimate used in computing the critical path), and its crash time is 1 week. This means that activity B can be shortened by up to 2 weeks if extra resources are provided. The cost of these additional resources is \$4,000 (= difference between the crash cost of \$34,000 and the normal cost of \$30,000). If we assume that the crashing cost is linear over time (i.e., the cost is the same each week), activity B's crash cost per week is \$2,000 (= \$4,000/2).

TABLE 3.5

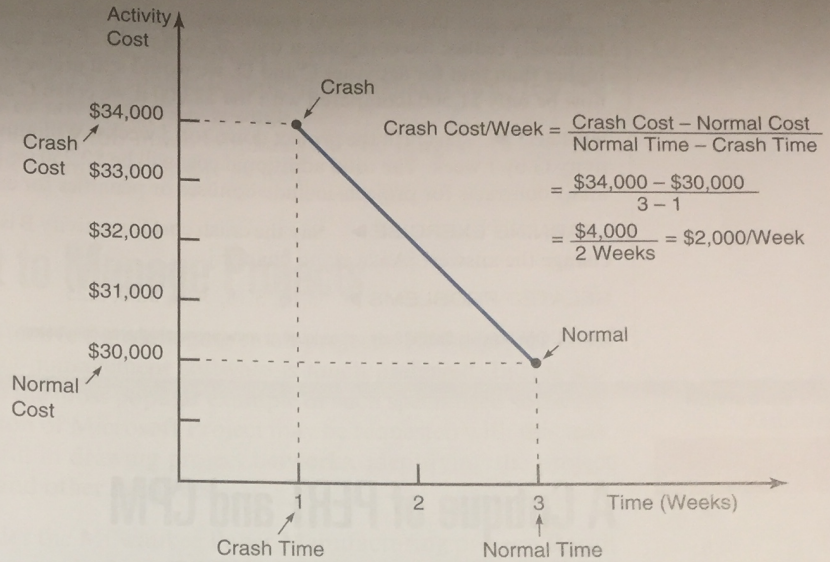
Normal and Crash Data for Milwaukee Paper Manufacturing

ACTIVITY	TIME (WEEKS)		COST (\$)		CRASH COST PER WEEK (\$)	CRITICAL PATH?
	NORMAL	CRASH	NORMAL	CRASH		
A	2	1	22,000	22,750	750	Yes
B	3	1	30,000	34,000	2,000	No
C	2	1	26,000	27,000	1,000	Yes
D	4	3	48,000	49,000	1,000	No
E	4	2	56,000	58,000	1,000	Yes
F	3	2	30,000	30,500	500	No
G	5	2	80,000	84,500	1,500	Yes
H	2	1	16,000	19,000	3,000	Yes

This calculation for Activity B is shown in Figure 3.15. Crash costs for all other activities can be computed in a similar fashion.

Figure 3.15

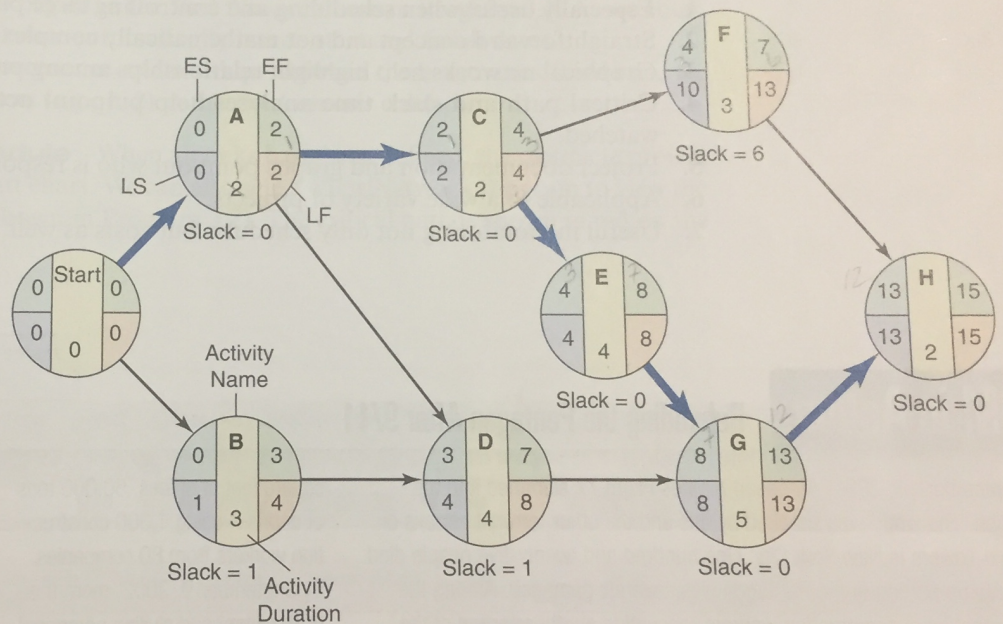
Crash and Normal Times and Costs for Activity B



Steps 2, 3, and 4 can now be applied to reduce Milwaukee Paper's project completion time at a minimum cost. We show the project network for Milwaukee Paper again in Figure 3.16.

Figure 3.16

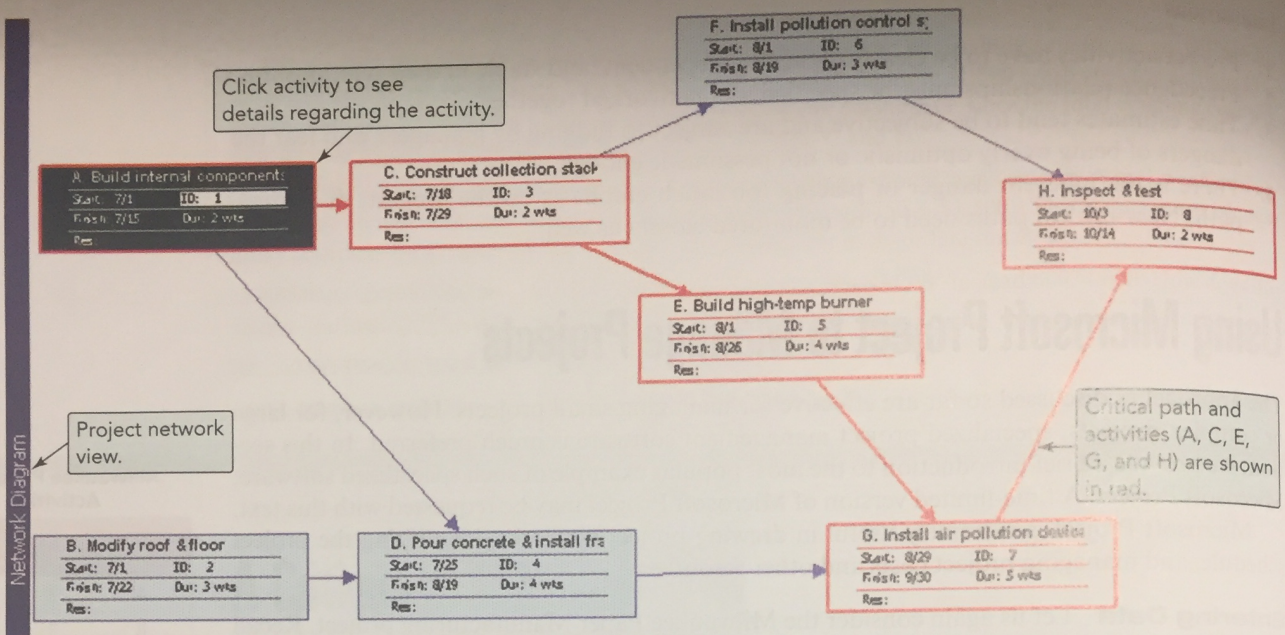
Critical Path and Slack Times for Milwaukee Paper



The current critical path (using normal times) is Start–A–C–E–G–H, in which Start is just a dummy starting activity. Of these critical activities, activity A has the lowest crash cost per week of \$750. Ju Ann Williams should therefore crash activity A by 1 week to reduce the project completion time to 13 weeks. The cost is an additional \$750. Note that activity A cannot be crashed any further, since it has reached its crash limit of 1 week.

At this stage, the original path Start–A–C–E–G–H remains critical with a completion time of 13 weeks. However, a new path Start–B–D–G–H is also critical now, with a completion time of 14 weeks. Hence, any further crashing must be done to both critical paths.

On each of these critical paths, we need to identify one activity that can still be crashed. We also want the total cost of crashing an activity on each path to be the smallest. We might be tempted to simply select the activities with the smallest crash cost per period in each path. If we did this, we would select activity C from the first path and activity D from the second path. The total crash cost would then be \$2,000 (= \$1,000 + \$1,000).



Program 3.2

Project Network in Microsoft Project for Milwaukee Paper Manufacturing

STUDENT TIP

Now that you understand the workings of PERT and CPM, you are ready to master this useful program. Knowing such software gives you an edge over others in the job market.

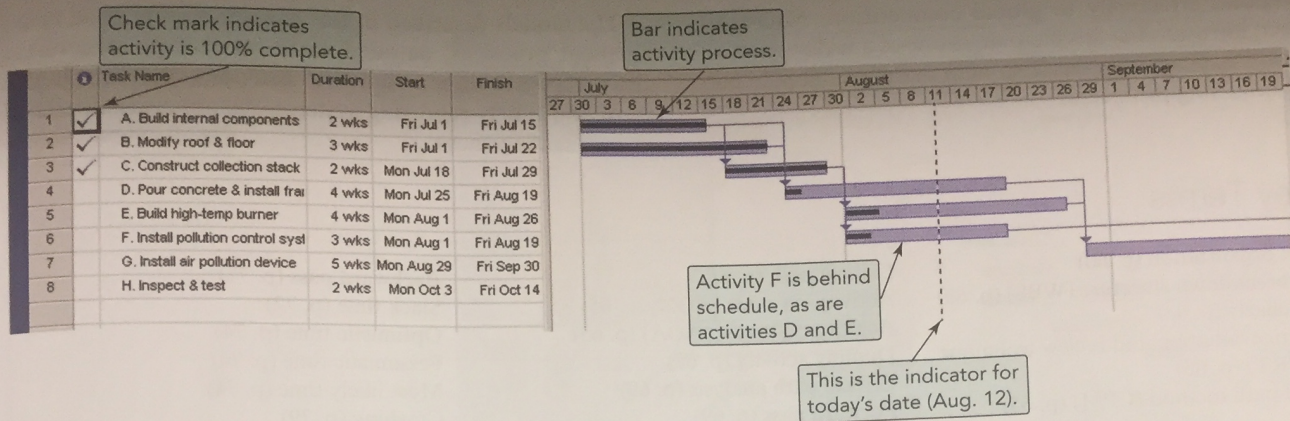
screen in the network diagram. We can click on any of the activities in the project network to view details of the activities. Likewise, we can easily add or remove activities from the project network. Each time we do so, Microsoft Project automatically updates all start dates, finish dates, and the critical path(s). If desired, we can manually change the layout of the network (e.g., reposition activities) by changing the options in **Format/Layout**.

Programs 3.1 and 3.2 show that if Milwaukee Paper's project starts July 1, it can be finished on October 14. The start and finish dates for all activities are also clearly identified. Project management software, we see, can greatly simplify the scheduling procedures discussed earlier in this chapter.

PERT Analysis Microsoft Project does not perform the PERT probability calculations discussed in Examples 10 and 11. However, by clicking **View/Toolbars/PERT Analysis**, we can get Microsoft Project to allow us to enter optimistic, most likely, and pessimistic times for each activity. We can then choose to view Gantt charts based on any of these three times for each activity.

Using PERT/CPM, Taco Bell built and opened this fast-food restaurant in Compton, California, in just 2 days! Typically, 2 months are needed to accomplish such a task. Good project management means a faster revenue stream instead of money tied up in construction.





Program 3.3

Tracking Project Progress in Microsoft Project

Tracking the Time Status of a Project Perhaps the biggest advantage of using software to manage projects is that it can track the progress of the project. In this regard, Microsoft Project has many features available to track individual activities in terms of time, cost, resource usage, and so on.

An easy way to track the time progress of tasks is to enter the percent of work completed for each task. One way to do so is to double-click on any activity in the *Task Name* column in Program 3.1. A window is displayed that allows us to enter the percent of work completed for each task.

The table in the margin provides data regarding the percent of each of Milwaukee Paper's activities that are completed as of today. (Assume that today is Friday, August 12, i.e., the end of the sixth week of the project schedule.)

As shown in Program 3.3, the Gantt chart immediately reflects this updated information by drawing a thick line within each activity's bar. The length of this line is proportional to the percent of that activity's work that has been completed.

How do we know if we are on schedule? Notice that there is a vertical line shown on the Gantt chart corresponding to today's date. Microsoft Project will automatically move this line to correspond with the current date. If the project is on schedule, we should see all bars to the left of today's line indicate that they have been completed. For example, Program 3.3 shows that activities A, B, and C are on schedule. In contrast, activities D, E, and F appear to be behind schedule. These activities need to be investigated further to determine the reason for the delay. This type of easy *visual* information is what makes such software so useful in practice for project management.

We encourage you to load the copy of Microsoft Project that may be ordered with your text and to create a project network for work you are currently doing.

Pollution Project Percentage Completed on Aug. 12	
ACTIVITY	COMPLETED
A	100
B	100
C	100
D	10
E	20
F	20
G	0
H	0

Summary

PERT, CPM, and other scheduling techniques have proven to be valuable tools in controlling large and complex projects. Managers use such techniques to segment projects into discrete activities (work breakdown structures), identifying specific resources and time requirements for each. With PERT and CPM, managers can understand the status of each activity, including its earliest start, latest start, earliest finish, and latest finish (ES, LS, EF, and LF) times. By controlling the trade-off between ES and

LS, managers can identify the activities that have slack and can address resource allocation, perhaps by smoothing resources. Effective project management also allows managers to focus on the activities that are critical to timely project completion. By understanding the project's critical path, they know where crashing makes the most economic sense.

Good project management also allows firms to efficiently create products and services for global markets and

to respond effectively to global competition. Microsoft Project, illustrated in this chapter, is one of a wide variety of software packages available to help managers handle network modeling problems.

The models described in this chapter require good management practices, detailed work breakdown structures, clear responsibilities assigned to activities, and straightforward and timely reporting systems. All are critical parts of project management.

Key Terms

Project organization (p. 60)

Work breakdown structure (WBS) (p. 62)

Gantt charts (p. 63)

Program evaluation and review technique (PERT) (p. 65)

Critical path method (CPM) (p. 65)

Critical path (p. 65)

Activity-on-node (AON) (p. 65)

Activity-on-arrow (AOA) (p. 65)

Dummy activity (p. 67)

Critical path analysis (p. 69)

Forward pass (p. 69)

Backward pass (p. 71)

Slack time (p. 72)

Optimistic time (p. 74)

Pessimistic time (p. 74)

Most likely time (p. 74)

Crashing (p. 79)

Ethical Dilemma

Two examples of massively mismanaged projects are TAURUS and the "Big Dig." The first, formally called the London Stock Exchange Automation Project, cost \$575 million before it was finally abandoned. Although most IT projects have a reputation for cost overruns, delays, and underperformance, TAURUS set a new standard.

But even TAURUS paled next to the biggest, most expensive public works project in U.S. history—Boston's 15-year-long Central Artery/Tunnel Project. Called the Big Dig, this

was perhaps the poorest and most felonious case of project mismanagement in decades. From a starting \$2 billion budget to a final price tag of \$15 billion, the Big Dig cost more than the Panama Canal, Hoover Dam, or Interstate 95, the 1,919-mile highway between Maine and Florida.

Read about one of these two projects (or another of your choice) and explain why it faced such problems. How and why do project managers allow such massive endeavors to fall into such a state? What do you think are the causes?

Discussion Questions

1. Give an example of a situation in which project management is needed.
2. Explain the purpose of project organization.
3. What are the three phases involved in the management of a large project?
4. What are some of the questions that can be answered with PERT and CPM?
5. Define *work breakdown structure*. How is it used?
6. What is the use of Gantt charts in project management?
7. What is the difference between an activity-on-arrow (AOA) network and an activity-on-node (AON) network? Which is primarily used in this chapter?
8. What is the significance of the critical path?
9. What would a project manager have to do to crash an activity?
10. Describe how expected activity times and variances can be computed in a PERT network.
11. Define *earliest start*, *earliest finish*, *latest finish*, and *latest start* times.
12. Students are sometimes confused by the concept of critical path, and want to believe that it is the *shortest* path through a network. Convincingly explain why this is not so.
13. What are dummy activities? Why are they used in activity-on-arrow (AOA) project networks?
14. What are the three time estimates used with PERT?
15. Would a project manager ever consider crashing a noncritical activity in a project network? Explain convincingly.
16. How is the variance of the total project computed in PERT?
17. Describe the meaning of slack, and discuss how it can be determined.
18. How can we determine the probability that a project will be completed by a certain date? What assumptions are made in this computation?
19. Name some of the widely used project management software programs.

Using Software to Solve Project Management Problems

In addition to the Microsoft Project software illustrated earlier, both Excel OM and POM for Windows are available to readers of this text as project management tools.

✕ USING EXCEL OM

Excel OM has a Project Scheduling module. Program 3.4 uses the data from the Milwaukee Paper Manufacturing example in this chapter (see Examples 4 and 5). The PERT/CPM analysis also handles activities with three time estimates.