P 3-4

3-4

A *square-law modulator* for generating an AM wave relies on the use of a nonlinear device (e.g., diode); Fig. 3.8 depicts the simplest form of such a modulator. Ignoring higher order terms, the input-output characteristic of the diode-load resistor combination in this figure is represented by the *square law*:

$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

where

$$v_1(t) = A_c \cos(2\pi f_c t) + m(t)$$

is the input signal, $v_2(t)$ is the output signal developed across the load resistor, and a_1 and a_2 are constants.

(a) Determine the spectral content of the output signal $v_2(t)$.

(b) To extract the desired AM wave from $v_2(t)$, we need a band-pass filter (not shown in Fig. 3.8). Determine the cutoff frequencies of the required filter, assuming that the message signal is limited to the band $-W \le f \le W$.

(c) To avoid *spectral distortion* by the presence of undesired modulation products in $v_2(t)$, the condition $f_c > 2W$ must be satisfied; validate this condition.

Solution

The output signal is

$$\begin{aligned} v_2(t) &= a_1 v_1(t) + a_2 v_1^2(t) \\ &= a_1(A_c\cos(2\pi f_c t) + m(t)) + a_2(A_c\cos(2\pi f_c t) + m(t))^2 \\ &= [a_1 + 2a_2 m(t)]A_c\cos(2\pi f_c t) \\ &\quad + [a_1 m(t) + a_2 A_c^2\cos^2(2\pi f_c t) + a_2 m^2(t)] \end{aligned} \qquad (1)$$

(a) The expression inside the first set of square brackets defines the desired AM wave:

$$\begin{aligned} s(t) &= A_c[a_1 + 2a_2 m(t)]\cos(2\pi f_c t) \\ &= a_1 A_c\left[1 + \frac{2a_2}{a_1}m(t)\right]\cos(2\pi f_c t) \end{aligned}$$

which represents an AM wave with

$$k_a = \frac{2a_2}{a_1}$$

defining the amplitude sensitivity of the modulator.

(b) The required band-pass filter must have a passband centered on f_c and a bandwidth equal to $2W$.

(c) The expression inside the second set of square brackets of Eq. (1) defines the undesired modulated products. The terms that matter are:

- The term $a_2 m^2(t)$, whose highest frequency component is 2W.
- The term $a_2 A_c^2\cos^2(2\pi f_c t)$, whose frequency is $2f_c$.

To extract the desired AM wave we therefore require:

Condition 1:

$(f_c + W) < 2f_c$

or $f_c > W$

Condition 2:

$(f_c - W) > 2W$

or $f_c > 3W$

If therefore we satisfy condition 2, then condition 1 is automatically satisfied.

HW#5 P3-17

Problem 3.17

(a) We are given

$c(t) = A_c \sin(2\pi f_c t)$

and

$m(t) = A_m \sin(2\pi f_m t)$

Invoking the definition of AM wave

$s(t) = [1 + k_a m(t)]c(t)$

we now write

$$s(t) = A_c[1 + k_a A_m \sin(2\pi f_m t)]\sin(2\pi f_c t)$$
$$= A_c \sin(2\pi f_c t) + \mu A_c \sin(2\pi f_m t)\sin(2\pi f_c t) \quad (1)$$

where

$\mu = k_a A_m$

is the modulation factor. Next, we use the trigonometric identity

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

Hence, we may rewrite Eq. (1) as

$$s(t) = A_c \sin(2\pi f_c t) + \frac{1}{2}\mu A_c[\cos(2\pi(f_c - f_m)t) - \cos(2\pi(f_c + f_m)t)] \quad (2)$$

The spectrum of the AM wave s(t) consists of three components:

(i) Carrier: $A_c \sin(2\pi f_c t)$

(ii) Lower side-frequency: $\frac{1}{2}\mu A_c \cos(2\pi(f_c - f_m)t)$

(iii) Upper side-frequency: $-\frac{1}{2}\mu A_c \cos(2\pi(f_c + f_m)t)$

This spectrum is depicted in Fig. 1.

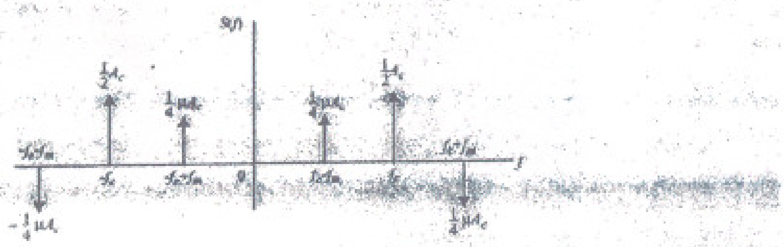


Figure 1

(b) Comparing the AM spectrum of Fig. 1 with the corresponding AM spectrum of Fig. 3.3(c) on page 105 of the text, we may make two observations:

- The frequency locations of the spectral components of these two AM waves are identical.
- The only difference between them is that the upper side-frequency $f_c + f_m$ in Fig. 1 is the negative of the upper side-frequency $f_c + f_m$ in Fig. 3.3(c).

HW#5 P3-18

Problem 3-18 Consider the message signal

$$m(t) = 20\cos(2\pi t) \text{ volts}$$

and the carrier wave

$$c(t) = 50\cos(100\pi t) \text{ volts}$$

(a) Sketch (to scale) the resulting AM wave for 75% modulation.
(b) Find the power developed across a load of 100 ohms due to this AM wave.

(a) Let $S(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$

$= A_c[1 + k_a A_m \cos 2\pi f_m t]\cos 2\pi f_c t$ (Single Tone Modulation)

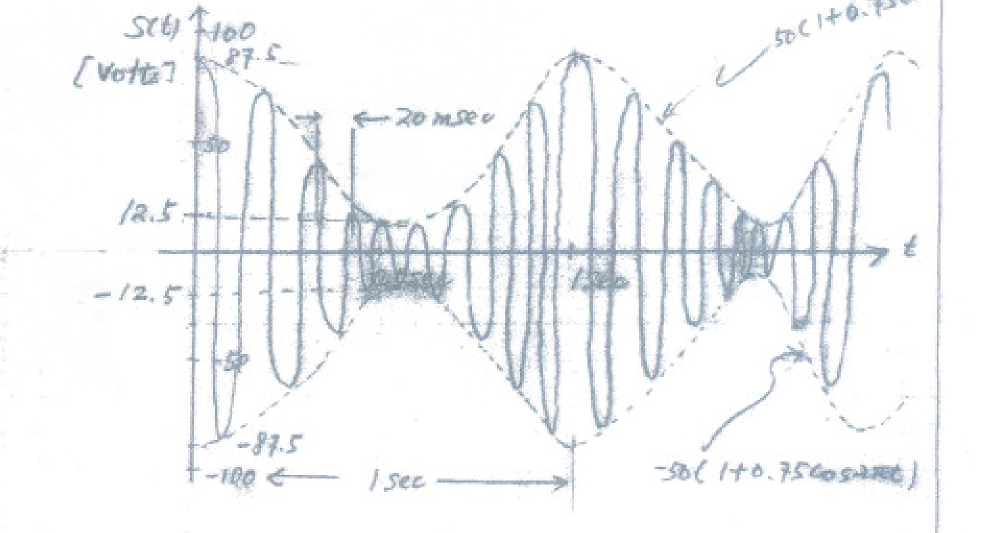
where $A_c = 50$ volts, $A_m = 20$ volts

$f_m = 1$ Hz $\quad f_c = 50$ Hz ($100\pi = 2\pi f_c$)

and $k_a A_m = 0.75 \quad k_a = \frac{0.75}{20} = 0.0375$ rad/volt

Hence

$S(t) = 50[1 + 0.75\cos 2\pi(1)t]\cos 2\pi(50)t$



(b) $S(t) = 50[1 + 0.75\cos 2\pi t]\cos 2\pi(50)t$

$= 50\cos 100\pi t + 37.5\cos 2\pi t\cos 100\pi t$

$= 50\cos 100\pi t + \frac{37.5}{2}\cos 102\pi t + \frac{37.5}{2}\cos 98\pi t$

$$P_{AM\text{-}100\Omega} = \frac{P_{AM\text{-}1\Omega}}{100} = \frac{1}{2}\frac{(50)^2}{100} + \frac{1}{2}\frac{(18.75)^2}{100} + \frac{1}{2}\frac{(18.75)^2}{100}$$

$$= \underbrace{12.5}_{\text{Carrier Component (78\%)}} + \underbrace{1.76 + 1.76}_{\text{modulation component (22\%)}} = 16.02 \text{ Watts}$$

Problem Using the message signal

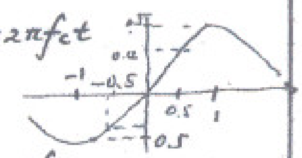
3-19

$$m(t) = \frac{t}{1 + t^2}$$

determine and sketch the modulated wave for amplitude modulation whose percentage modulation equals the following values:

(a) 50%
(b) 100%
(c) 125%

$\frac{t}{1+t^2}$



By defining AM wave ; $s(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$

for given $m(t) = \frac{t}{1+t^2}$

$= A_c[1 + \frac{k_a t}{1+t^2}]\cos 2\pi f_c t$

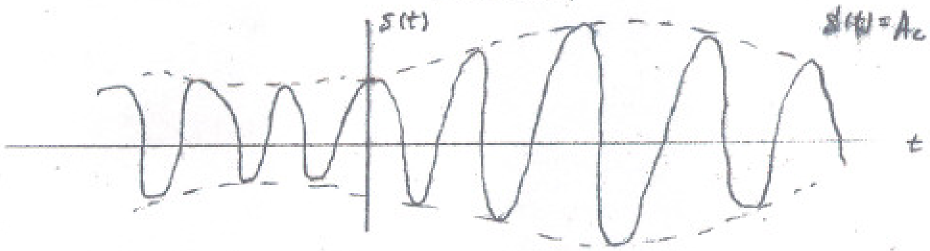
Percent Modulation: $|k_a m(t)|_{max} \times 100$

(a) For 50% modulation (under modulation ; $|k_a m(t)| \le 1$)

$|k_a m(t)| = 0.5 \quad \forall t$

hence $|k_a \cdot \frac{t}{1+t^2}|_{t=1} = k_a \cdot \frac{1}{2} = 0.5 \quad k_a = 1.0$

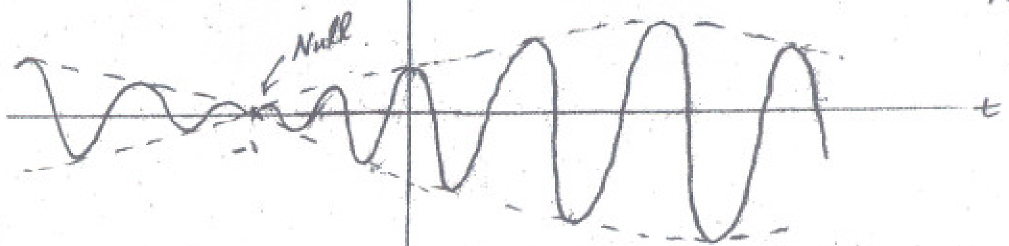
$s(t) = A_c(1 + \frac{t}{1+t^2})\cos 2\pi$



(b) For 100% modulation $|k_a m(t)| = 1.0 \sim |k_a \cdot \frac{t}{1+t^2}|_{t=1} = k_a \cdot \frac{1}{2} = 1.0$

$\sim k_a = 2.0$

$\sim s(t) = A_c[1 + \frac{2t}{1+t^2}]\cos 2\pi f_c t = A_c[\frac{(1+t)^2}{1+t^2}]\cos 2\pi f_c$



(c) For 125% modulation (over modulation) $|k_a m(t)| > 1$ for some t

$\sim |k_a \cdot \frac{t}{1+t^2}|_{t=1} = k_a \cdot \frac{1}{2} = 1.25 \sim k_a = 2.5$

$s(t) = A_c[1 + \frac{2.5t}{1+t^2}]\cos 2$