3.3. DSB-SC Modulation

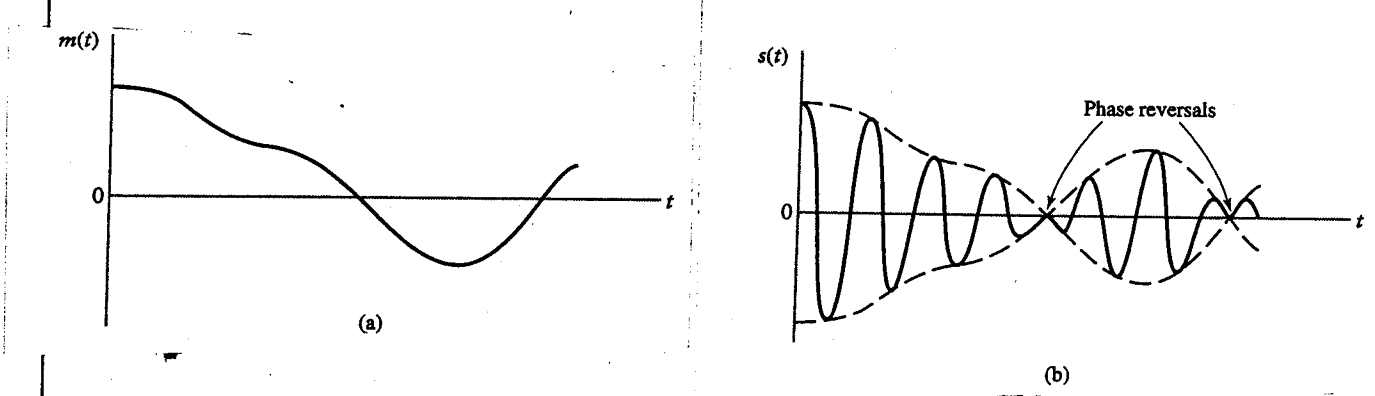
— As was described in Sec 3.2, Conventional AM is NOT efficient in that:
(1) $c(t)$ is independent from $m(t)$
(2) but much power is wasted to Tx $c(t)$ itself.

– Hence, let's simply Tx $m(t)$ only — DSB-SC
(still double sideband, but suppress Carrier)

$$S(t)_{DSB\text{-}SC} \triangleq m(t)\cdot c(t) = m(t)\cdot A_c \cos 2\pi f_c t$$

$$\mathcal{F}\downarrow \quad S(f)_{DSB\text{-}SC} = M(f) * C(f) = M(f) * \left[\tfrac{1}{2}\delta(f-f_c) + \tfrac{1}{2}\delta(f+f_c)\right]A_c$$

$$= \tfrac{1}{2}A_c\left[M(f-f_c) + M(f+f_c)\right]$$



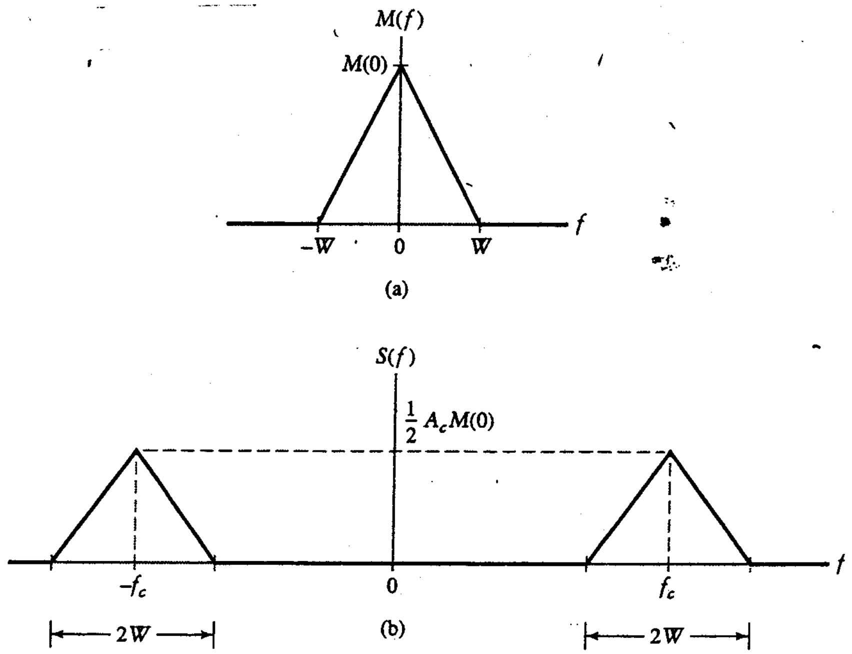
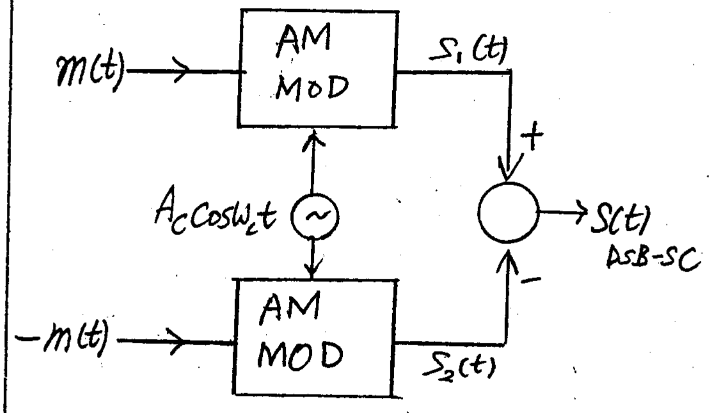


FIGURE 3.11 (a) Spectrum of message signal $m(t)$. (b) Spectrum of DSB-SC modulated wave $s(t)$.

Generation of DSB-SC

- In principle, $s(t) = m(t) \cdot c(t)$, hence Product Modulator (PM) is used
- Actual PM can be Implemented by "Balanced (AM) Modulator"



where

$$s_1(t) = A_c[1 + k_a m(t)] \cos \omega_c t$$

$$s_2(t) = A_c[1 - k_a m(t)] \cos \omega_c t$$

$$s(t) = s_1(t) - s_2(t)$$
$$= 2k_a A_c m(t) \cos \omega_c t$$
$$= A_c' m(t) \cdot \cos \omega_c t$$

Coherent Detection of DSB-SC

- Coherent Detection means the Knowledge of "f_c and ϕ"
  - Hence local OSC output ($A_c' \cos(\omega_c t + \phi)$) is coherent to $c(t)$

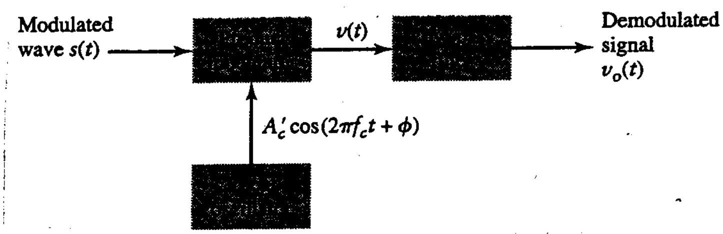


FIGURE 3.12 Block diagram of coherent detector, assuming that the local oscillator is out of phase by ϕ with respect to the sinusoidal carrier oscillator in the transmitter.

Let's define

$$\underset{DSB\text{-}SC}{s(t)} = A_c m(t) \cos \omega_c t$$

$$v(t) = s(t) \cdot A_c' \cos(\omega_c t + \phi) = A_c A_c' m(t) \cos \omega_c t \cdot \cos(\omega_c t + \phi)$$

$$= A_c A_c' m(t) \frac{1}{2}[\cos(2\omega_c t + \phi) + \cos(\phi)]$$

$$= \frac{1}{2} A_c A_c' m(t) \cos(2\omega_c t + \phi) + \frac{1}{2} A_c A_c' m(t) \cos \phi$$

Hence

if $\phi \neq 0$, after LPF

$$v_o(t) = \frac{1}{2} A_c A_c' m(t) \cos \phi$$

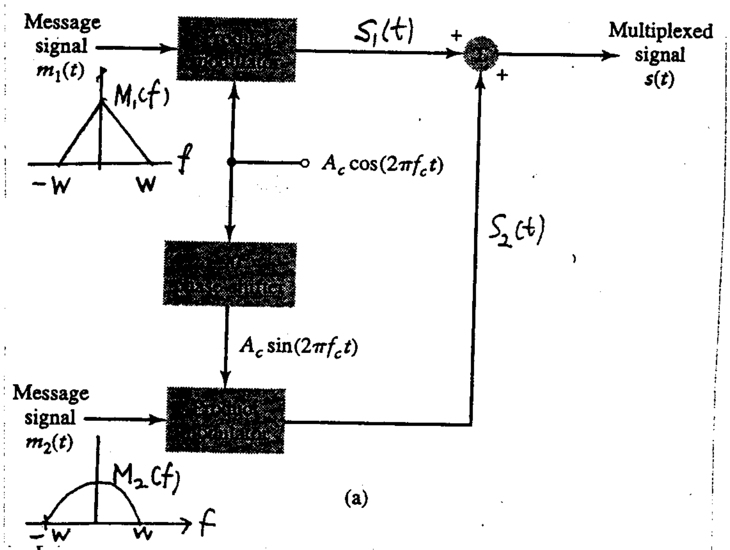
if $\phi = \pm \pi/2$ $\quad \cos\phi = 0, v_o(t) = 0$

(Quadrature Null Effect !)

Sec 3.4. Costas Rxer - Read it!

Sec 3.5. Quadrature-Carrier Multiplexing (of 2 DSB-SCs)

- Double the Bandwidth Efficiency
- Easy separation by Quadrature Signal (Orthogonality)



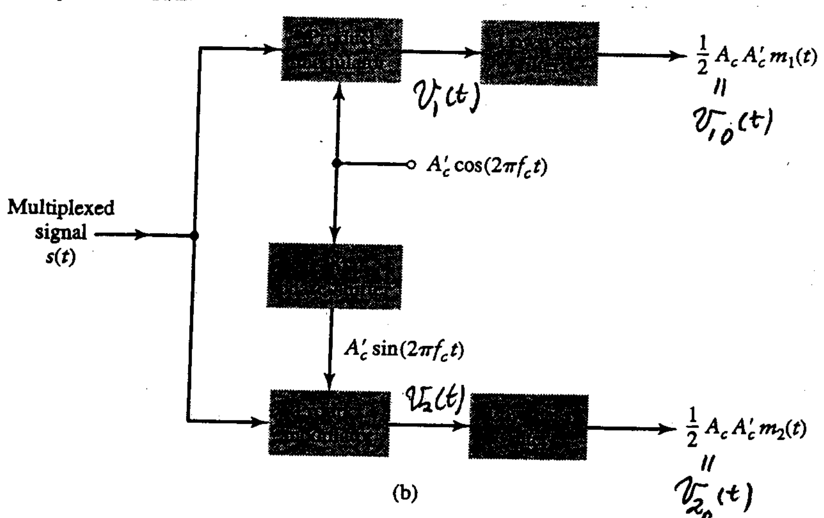
$S_1(t) =$

$S_2(t) =$

then

$S(t) =$

Relationship between $A_c \cos W_c t$ & $A_c \sin W_c t$?



$$V_1(t) = A_c A_c' m_1(t) \overbrace{\cos^2 W_c t}^{\frac{1}{2}(1+\cos 2W_c t)} + A_c A_c' m_2(t) \underbrace{\sin W_c t \cos W_c t}_{\frac{1}{2}\sin 2W_c t}$$

$$= \frac{A_c A_c'}{2} m_1(t) [1 + \cos 2W_c t] + \cancel{\frac{A_c A_c'}{2} m_2(t) \sin 2W_c t}$$

$$V_{1_{out}}(t) = \frac{1}{2} A_c A_c' m_1(t)$$ after LPF

FIGURE 3.17 Quadrature-carrier multiplexing system: (*a*) Transmitter, (*b*) receiver.

$$V_2(t) = \cancel{A_c A_c' m_1(t) \underbrace{\sin W_c t \cos W_c t}_{\frac{1}{2}\sin 2W_c t}} + A_c A_c' \underbrace{\sin^2 W_c t}_{\frac{1}{2}(1 - \cos 2W_c t)} m_2(t)$$

after LPF

$$= \frac{A_c A_c'}{2} m_2(t)$$

Max BW of LPF ?

On Frequency Translation (- Basis for SSB) (or Mixing)

- By freq translation, frequencies mixed (changed)
  $\Rightarrow$ Heterodyning.
- In principle, frequency can be changed to Higher OR Lower than original freq (carrier) f_1 to f_2
- If f_2 (Target freq) $> f_1$: Up-Conversion
  $f_2 < f_1$: Down-Conversion
- For both (freq) translation (or Shifting), PM with Local OSC freq f_l is used.

Note : It usually refers to Band-pass signal (i.e. Already modulated signal)

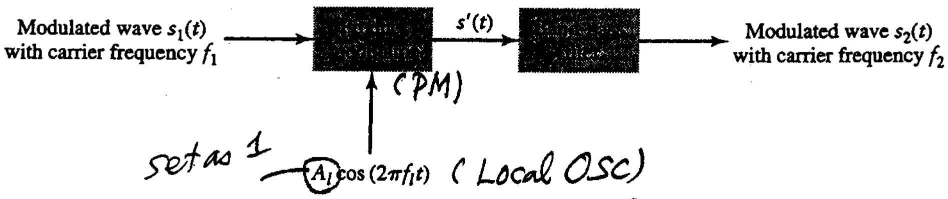
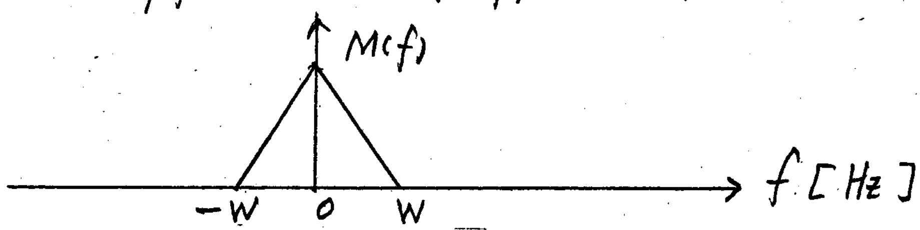


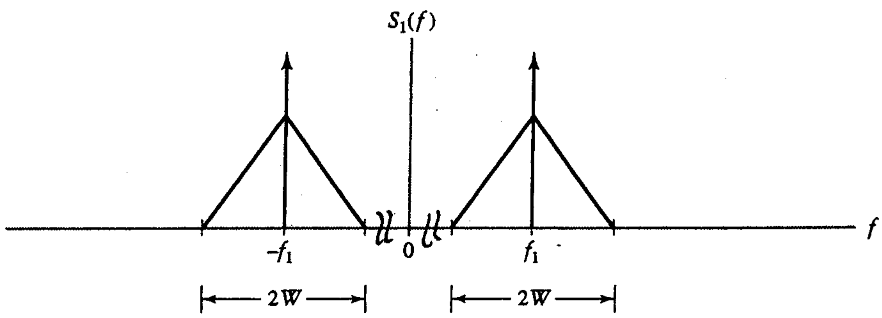
Figure 3.21 Block diagram of mixer. (Freq Translation)

Assume $S_1(t) = m(t) \cdot \cos \omega_1 t$, $\quad \omega_1 \triangleq 2\pi f_1$

$$\Downarrow$$

$$S_1(f) = \frac{1}{2}[M(f-f_1) + M(f+f_1)]$$





(a)

By PM, its output $s'(t)$ be

$$s'(t) = [\, m(t)\cos\omega_1 t \,]\cdot A_l \cos\omega_l t \qquad \text{Assume } A_l = 1$$

$$(= s_1(t)\cos\omega_l t\,)$$

$$\mathcal{F} \Big\downarrow \; = \frac{1}{2} m(t)\cos(\omega_1+\omega_l)t + \frac{1}{2} m(t)\cos(\omega_1-\omega_l)t$$

$$\omega_l = 2\pi f_l$$

$$S'(f) = \frac{1}{4}[M(f-f_1-f_l) + M(f+f_1+f_l)] \quad ①\ ②$$

$$+ \frac{1}{4}[M(f-f_1+f_l) + M(f+f_1-f_l)] \quad ③\ ④$$

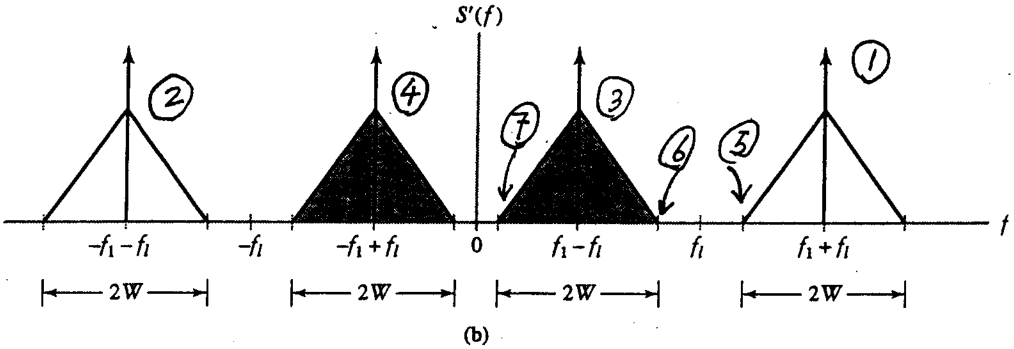
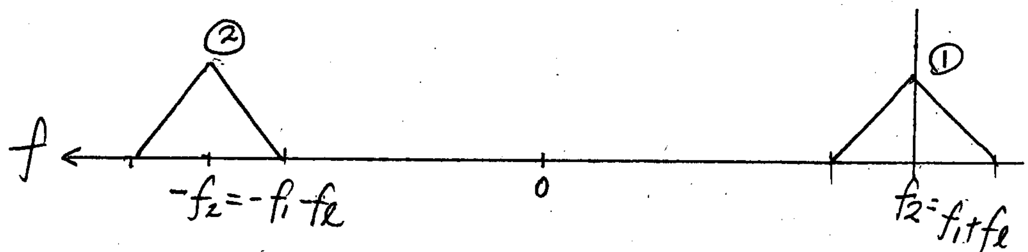


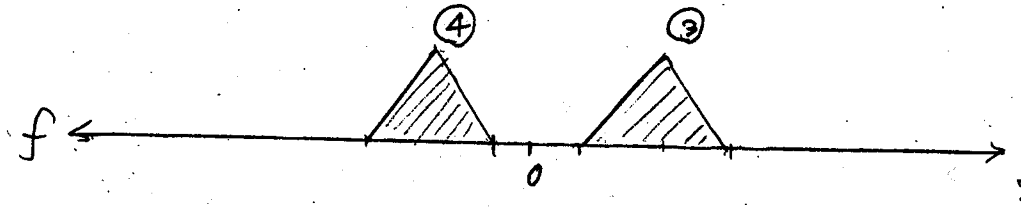
FIGURE 3.22 (*a*) Spectrum of modulated signal $s_1(t)$ at the mixer input. (*b*) Spectrum of the corresponding signal $s'(t)$ at the output of the product modulator in the mixer.



$f_2 = f_1 + f_l$

$\rightarrow f_l = f_2 - f_1$

Up-Conv



$f_2 = f_1 - f_l$

$\rightarrow f_l = f_1 - f_2$

Down-Conv

In addition to the conditions for f_l above

We need No spectral Overlap to avoid Distortion. Hence

$2f_l > 2W$

Freq of ⑤,⑥ $f_1 + f_l - W > f_1 - f_l + W$

$\rightarrow \underline{f_l > W}$

Also, we require (from ⑦)

$f_1 - f_l - W > 0 \Rightarrow \underline{f_l < f_1 - W}$

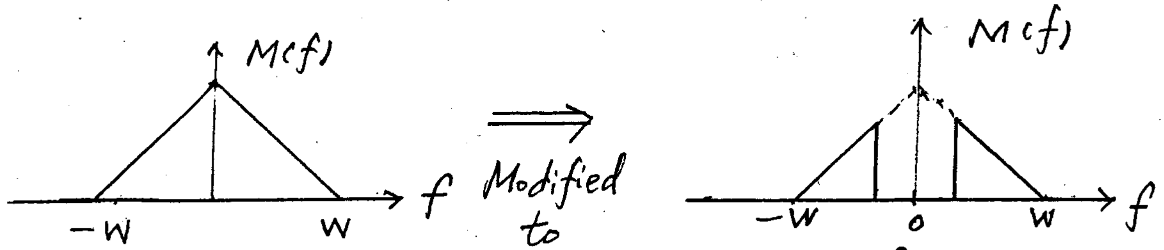
3.6. Single-Sideband (SSB) Modulation

(i) For Power Efficiency, DSB-SC was proposed earlier. One way to enhance Bandwidth Efficiency, Quad-Mux was also proposed

(ii) To the further enhancement of Bandwidth Efficiency, only one-side* of DSB spectrum can be used $\Rightarrow$ SSB

* Either the Upper-Sideband (USB) or Lower-Side band (LSB) can be used.

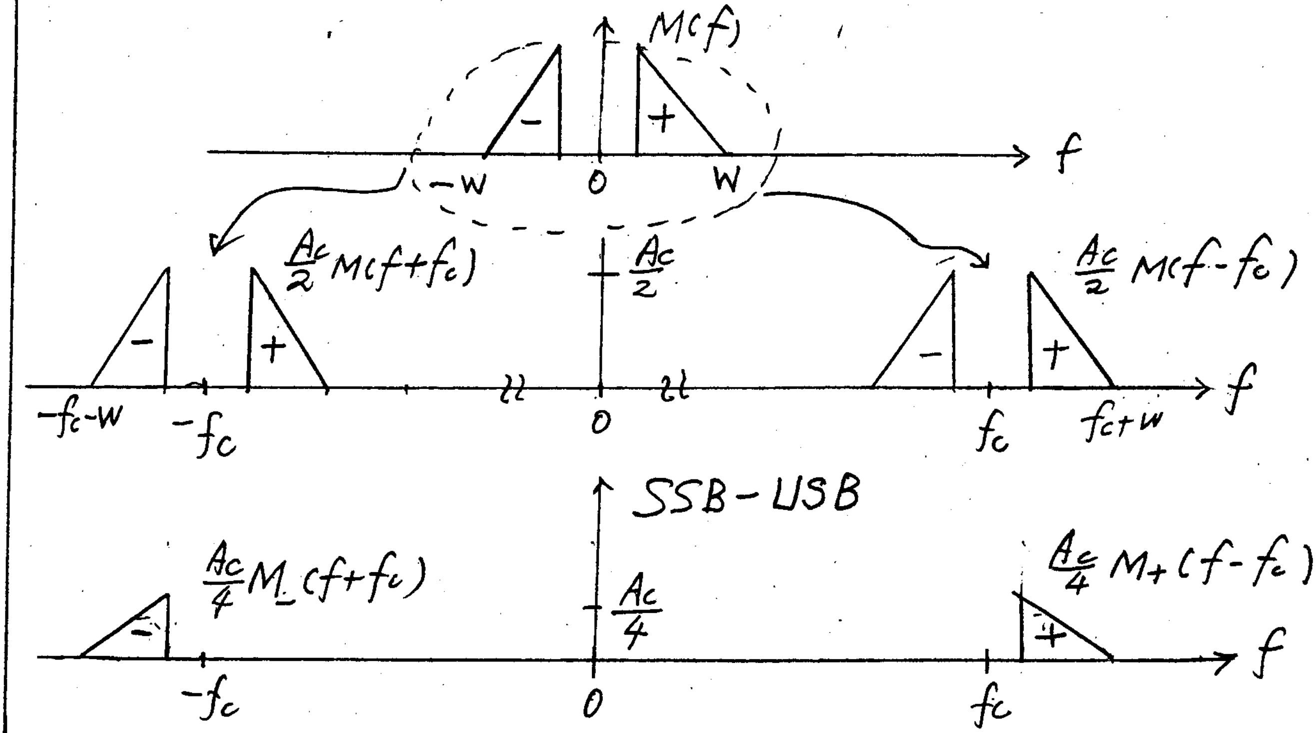
(iii). In fact, for the actual Tx of Audio/speech and using Freq Discriminator (Detection), usual message $m(t)$ can be modified as below:

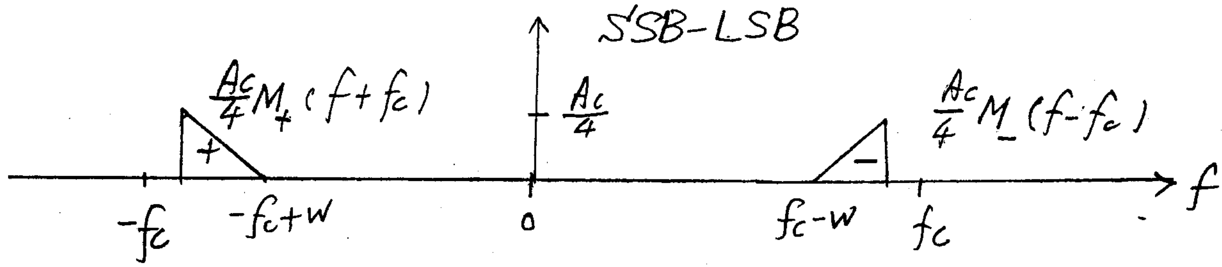


(Usual $M(f)$ for DSB, DSB-SC)

($M(f)$ for SSB) Hole or Gap in $M(f)$

(IV) SSB Signal Definition (assume $f_c \gg W$)





Refer to Sec 3.8 ⇒

Where $\begin{cases} m_+(t) = m(t) + j\hat{m}(t) \\ m_-(t) = m(t) - j\hat{m}(t) \end{cases}$

Hence, $\begin{cases} M_+(f) = M(f) + j\hat{M}(f) \\ M_-(f) = M(f) - j\hat{M}(f) \end{cases}$

where $\hat{m}(t) / \hat{M}(f)$ are Hilbert Transform of $m(t) / M(f)$ defined below.

Hilbert Transform

<Def> Given $m(t)$, $\hat{m}(t) \triangleq \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau}\, d\tau$: Hilbert Transform of $m(t)$

I.e, above integral is simply convolution integral of $m(t)$ & $\frac{1}{\pi t}$

↘ $\hat{m}(t) \triangleq m(t) * \frac{1}{\pi t} = \int_{-\infty}^{\infty} m(\tau) \frac{1}{\pi(t-\tau)}\, d\tau$

↘

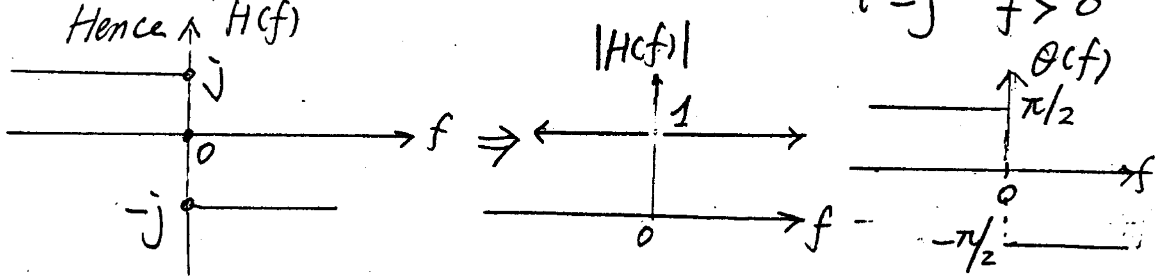
$$m(t) \longrightarrow \boxed{\begin{array}{c}\text{Hilbert Transformer} \\ h(t) = \frac{1}{\pi t}\end{array}} \longrightarrow \hat{m}(t)$$

By FT

$\hat{M}(f) \triangleq \mathcal{F}[\hat{m}(t)] = \mathcal{F}[m(t)] \cdot \underbrace{\mathcal{F}\left[\frac{1}{\pi t}\right]}_{\text{say } H(f)}$

Where $H(f) \triangleq -j\,\mathrm{sgn}(f) = \begin{cases} j & f < 0 \\ 0 & f = 0 \\ -j & f > 0 \end{cases}$

Hence



By the result

$$\hat{M}(f) = [-j\,\mathrm{sgn}(f)]\,M(f) = \begin{cases} jM(f) & f<0 \\ 0 & f=0 \\ -jM(f) & f>0 \end{cases}$$

i.e, Hilbert Transform works as phase shifter by $\pi/2$

Example: $m(t) = \cos \omega_c t \longleftrightarrow M(f) = \underbrace{\tfrac{1}{2}\delta(f-f_c)}_{f>0} + \underbrace{\tfrac{1}{2}\delta(f+f_c)}_{f<0}$

hence

$$\hat{M}(f) = j\,\tfrac{1}{2}\delta(f+f_c) - j\left(\tfrac{1}{2}\right)\delta(f-f_c)$$

$$= \frac{j}{2}\delta(f+f_c) - \frac{j}{2}\delta(f-f_c)$$

$$= \frac{-1}{2j}\delta(f+f_c) + \frac{1}{2j}\delta(f-f_c)$$

$$= \frac{1}{2j}[\delta(f-f_c) - \delta(f+f_c)]$$

by FT pair

$$\hat{m}(t) = \sin \omega_c t$$

Let's go back to sketch on SSB-USB

$$S_{SSB-USB}(f) = \frac{A_c}{4}M_+(f-f_c) + \frac{A_c}{4}M_-(f+f_c)$$

$\mathcal{F}^{-1}$

$$s(t) = \frac{A_c}{4}m_+(t)e^{+j\omega_c t} + \frac{A_c}{4}m_-(t)e^{-j\omega_c t}$$

$$= \frac{A_c}{4}[m(t)+j\hat{m}(t)][\cos\omega_c t + j\sin\omega_c t]$$

$$+ \frac{A_c}{4}[m(t)-j\hat{m}(t)][\cos\omega_c t - j\sin\omega_c t]$$

$$= \frac{A_c}{2}m(t)\cos\omega_c t - \frac{A_c}{2}\hat{m}(t)\sin\omega_c t \quad \begin{pmatrix} SSB \\ -USB \end{pmatrix}$$

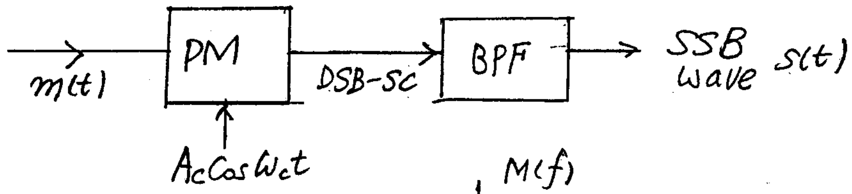
Similarily $$s_{SSB-LSB}(t) = \frac{A_c}{2}m(t)\cos\omega_c t + \frac{A_c}{2}\hat{m}(t)\sin\omega_c t \quad \begin{pmatrix} SSB \\ -LSB \end{pmatrix}$$

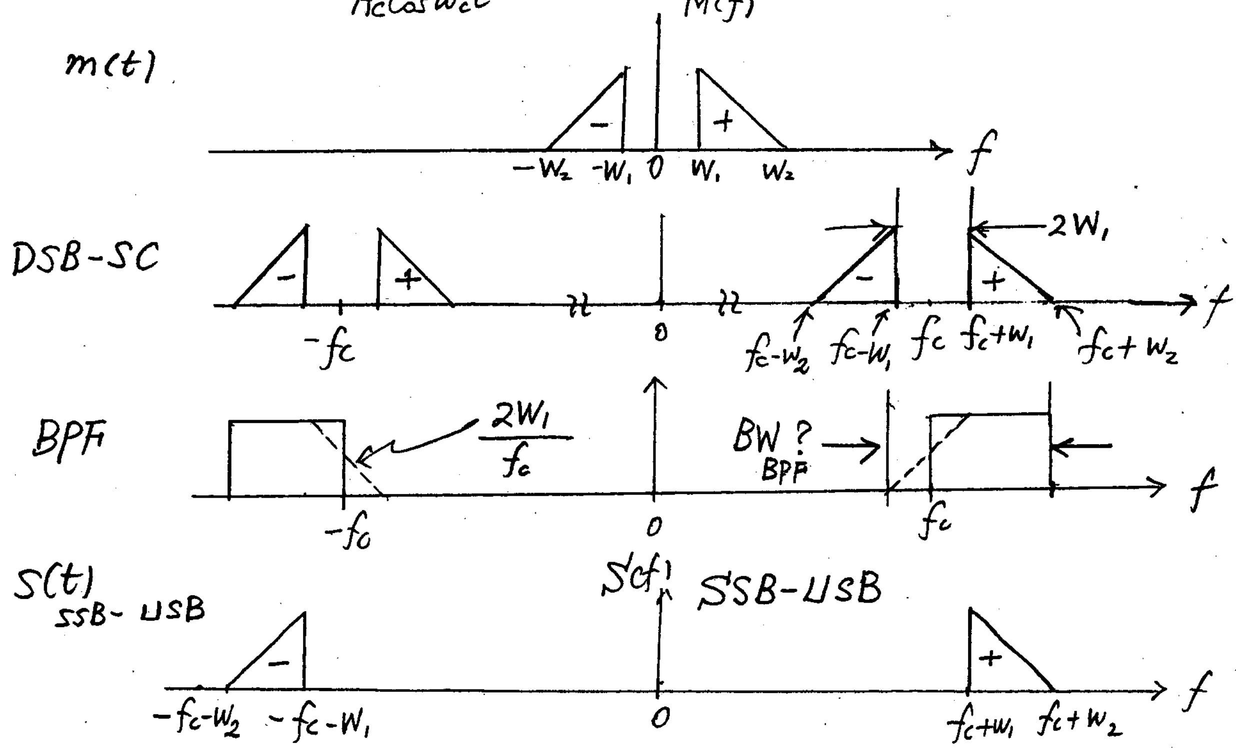
In Summary,

$$s_{SSB}(t) = \frac{A_c}{2}m(t)\cos\omega_c t \mp \frac{A_c}{2}\hat{m}(t)\sin\omega_c t \quad \begin{pmatrix} - : USB \\ + : LSB \end{pmatrix}$$

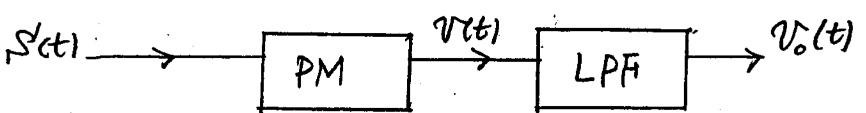
for message $m(t)$ and Carrier $C(t) = A_c\cos\omega_c t$

SSB Modulator – Freq Discrimination Method





Coherent Demodulation of SSB wave



Local osc : $A_c' \cos W_c t$

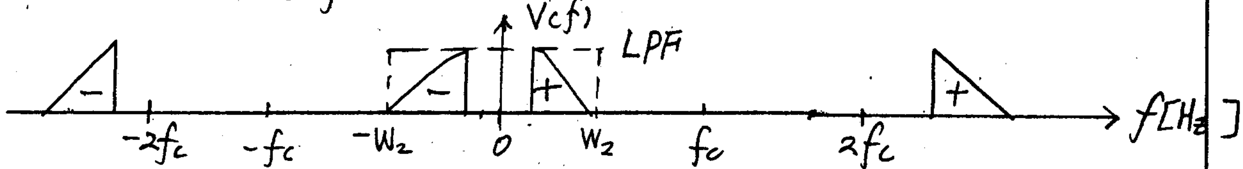
For $\underset{SSB-USB}{s(t)} = \frac{1}{2}A_c m(t) - \frac{1}{2}A_c \hat{m}(t) \sin W_c t$

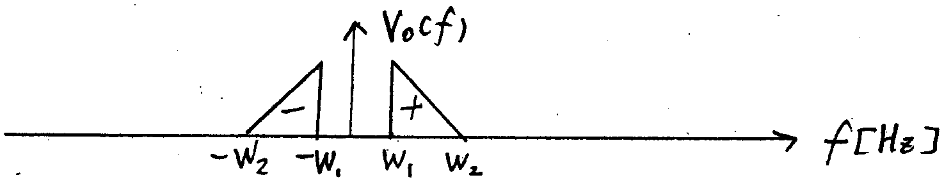
$$v(t) = \frac{1}{2}A_c A_c' m(t)\cos^2 W_c t - \frac{1}{2}A_c A_c' \hat{m}(t) \sin W_c t \cos W_c t$$

$$= \frac{1}{4}A_c A_c' m(t) + \cancel{\frac{1}{4}A_c A_c' m(t)\cos 2W_c t} - \cancel{\frac{1}{4}A_c A_c' \hat{m}(t) \sin 2W_c t}$$

$$v_o(t) = \frac{1}{4}A_c A_c' m(t)$$

For USB case (from above)



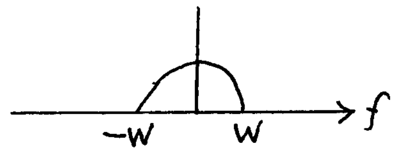


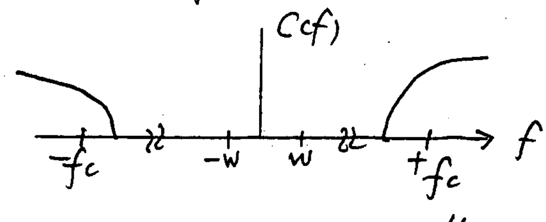
3.8 Baseband Representation of Band-Pass Signal: (Complex Envelope)

- From discussion on Hilbert Transform, we know

  if $\left.\begin{array}{l} c(t) = \cos \omega_c t \\ \hat{c}(t) = \sin \omega_c t \end{array}\right\}$ orthogonal signals (carriers)

- By applying above for the baseband message $m(t)$, assuming No spectral overlapping between $M(f)$ & $C(f)$ as below





- Under this condition, if $s(t) = m(t) \cdot c(t)$ [Hence Band pass Spectra]

  then $\hat{s}(t) = m(t)\,\hat{c}(t)$

Example : Let $c(t) = \cos \omega_c t$

$$s(t) = m(t) \cos \omega_c t \quad (f_c \gg W)$$

$$\mathcal{F} \downarrow \quad S(f) = \frac{1}{2} M(f - f_c) + \frac{1}{2} M(f + f_c)$$

then

$$\hat{S}(f) = \frac{1}{2j} M(f - f_c) - \frac{1}{2j} M(f + f_c)$$

$$\mathcal{F}^{-1} \downarrow \quad \hat{s}(t) = m(t) \cdot \sin \omega_c t$$

A. Pre-Envelope

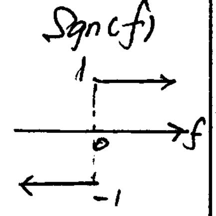
<Def> $m_+(t) = m(t) + j\hat{m}(t)$

$\mathcal{F} \downarrow$ then

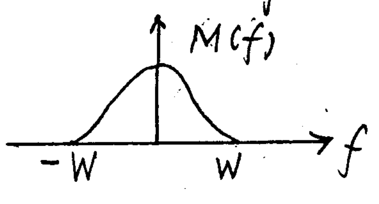
$$M_+(f) = M(f) + j\,[-j\,\mathrm{sgn}(f) M(f)]$$

$$= [1 + \mathrm{sgn}(f)] M(f)$$

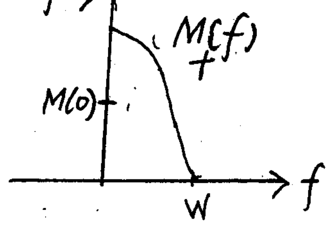
$$= \begin{cases} 0 & f < 0 \\ M(0) & f = 0 \quad (M(f)|_{f=0} \triangleq M(0)) \\ 2M(f) & f > 0 \end{cases}$$



hence if



$\Longrightarrow$

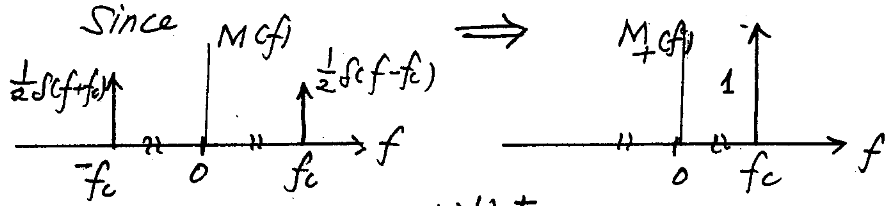


Using same principle,

If $m(t) = \cos \omega_c t$

$m_+(t) = \mathcal{F}^{-1}[M_+(f)] = e^{j\omega_c t} \; (\leftrightarrow 2 \cdot \tfrac{1}{2}\delta(f-f_c))$

Since



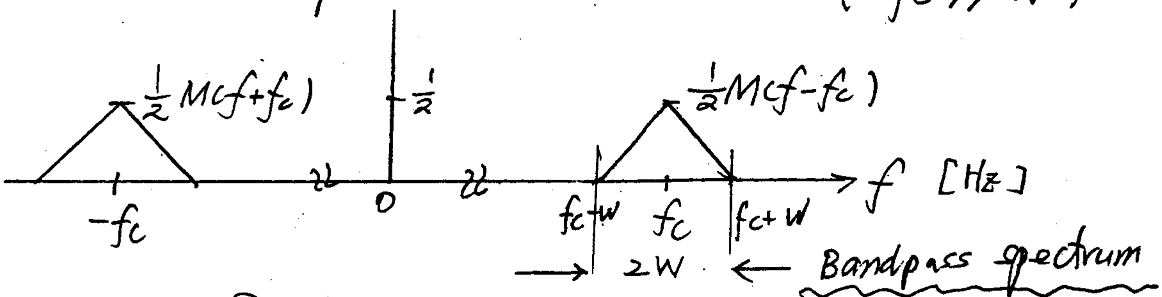
Hence $m_+(t) = e^{+j\omega_c t}$

$= \cos \omega_c t + j \sin \omega_c t$

A-1. Application to Band-Pass Signal (Narrow band Signal)

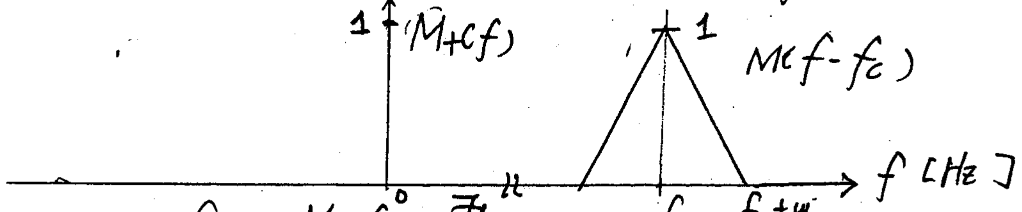
Let $s(t) = m(t) \cdot c(t)$

and its spectrum is shown $(f_c \gg W)$



By using Pre-Envelope

$\Rightarrow$ We can select Positive Freq Spectrum with 2 times of magnitude



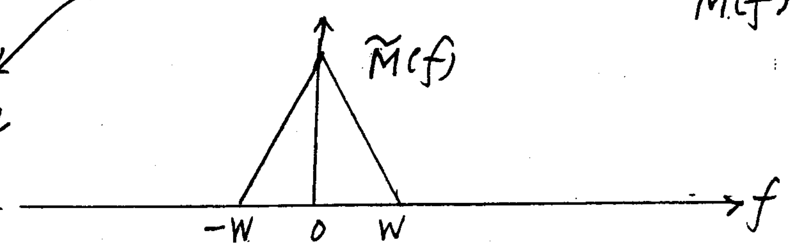
from $M_+(f) \xrightarrow{\mathcal{F}^{-1}} m_+(t)$

Still $M_+(f)$ is Band-pass Spectrum (Although it is only half in Freq)

By Using Complex Envelope

Freq-shifted version of Pre-Envelope

$\tilde{M}(f) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftarrows}} \tilde{m}(t)$



Ex. Assume $s(t) = m(t)\cos\omega_c t = m(t)\cdot c(t)$: Modulation

then $\hat{s}(t) = m(t)\sin\omega_c t = m(t)\,\hat{c}(t)$: Hilbert Transform

then $s_+(t) = m(t)\,e^{+j\omega_c t}$: Pre-Envelope

$\tilde{s}(t) = m(t)$: Complex Envelope

<Q> Is this Original Low-Pass (Baseband) Spectrum ?

A-2. Relationship between $s(t)$, $\hat{s}(t)$, $s_+(t)$ and $\tilde{s}(t)$

Let's define "Complex Envelope" of signal $s(t)$ (from $s_+(t)$ now)

(1) $\underline{\tilde{s}(t) \triangleq s_+(t)\,e^{-j\omega_c t}}$

Since $s_+(t) \triangleq \tilde{s}(t)\,e^{+j\omega_c t} = s(t) + j\hat{s}(t)$ — (1)'

and $s(t) \triangleq \mathrm{Re}[s_+(t)]$

$= \mathrm{Re}[\tilde{s}(t)\,e^{+j\omega_c t}]$ (2)

Since both $\tilde{s}(t)$ and $s_+(t)$ are Complex Variable

We can write

$$\tilde{s}(t) = s_c(t) + j s_s(t) \qquad (3)$$

Using Eqn (2), (3) above

$$s(t) = \mathrm{Re}[s_c(t) + j s_s(t)]\,e^{+j\omega_c t}$$

where $e^{j\omega_c t} \triangleq \cos\omega_c t + j\sin\omega_c t$

$$= \underbrace{s_c(t)}_{\text{In-phase Component}}\cos\omega_c t - \underbrace{s_s(t)}_{\text{Quadrature Component}}\sin\omega_c t \qquad (4)$$

$s_c(t) = s_I(t)$
$s_s(t) = s_Q(t)$
in Text

Furthermore, we can now define from Eq (3) and (4), (1) & (1)'

$$s_c(t) = \mathrm{Re}[\tilde{s}(t)] = \mathrm{Re}[s_+(t)\,e^{-j\omega_c t}]$$
$$= s(t)\cos\omega_c t + \hat{s}(t)\sin\omega_c t$$

$$s_s(t) = \mathrm{Im}[\tilde{s}(t)] = \mathrm{Im}[s_+(t)\,e^{-j\omega_c t}]$$
$$= \hat{s}(t)\cos\omega_c t - s(t)\sin\omega_c t$$

Hence, in general, for any complex phasor

$$S(t) = A(t)\cos[\omega_c t + \phi(t)]$$

$$\triangleq A(t)\,\mathrm{Re}[e^{j(\omega_c t + \phi(t))}] \qquad \omega_c \neq 0$$

By setting $\omega_c = 0$

$$\tilde{S}(t) = A(t)\,\mathrm{Re}[e^{j\omega_c t} \cdot e^{j\phi(t)}]\Big|_{\omega_c = 0}$$

$$= \underbrace{A(t)}_{\text{Envelope}}\,\mathrm{Re}[e^{j\overset{\text{phase}}{\phi(t)}}]$$

Using relationship defined earlier, we can say

$$A(t) \triangleq |\tilde{S}(t)| \triangleq |S_+(t)|$$

$$= \sqrt{S^2(t) + \hat{S}^2(t)}$$

this is angle!

$$\phi(t) = \angle\tilde{S}(t) = \angle S_+(t) - \widetilde{\omega_c t}$$

$$= \tan^{-1}\left|\frac{\hat{S}(t)}{S(t)}\right| - \omega_c t$$

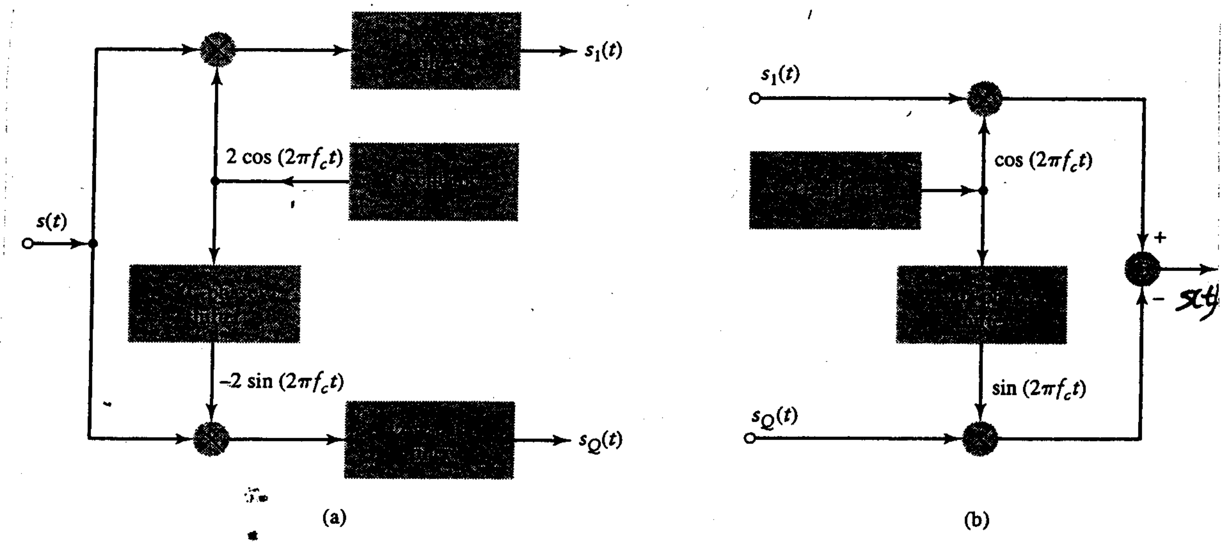


FIGURE 3.25 (*a*) Scheme for deriving the in-phase and quadrature components of a linearly modulated (i.e., band-pass) signal. (*b*) Scheme for reconstructing the modulated signal from its in-phase and quadrature components.

Do $S_1(t)$ and $S_2(t)$ have Carrier Components?