Chapter 3. Amplitude Modulation (AM)

* Modulation in General (Demodulation)

(1) Modulation $\triangleq$ The process by which some parameter(s) of a Carrier (wave) is varied in accordance with an information-bearing signal.

(2) Elements of Modulation

- Message Signal : $m(t)$, Analog OR Digital
- Carrier (Wave) : $C(t) = A_c \cos(2\pi f_c t + \theta)$

Continuous Wave! where A_c : Carrier Amplitude
f_c : Carrier Frequency
θ : Carrier phase.

* Usually, high (Compared to $m(t)$) freq Sinusoid
* Also, pulse (train) can be a Carrier.
* Note that $m(t)$ can be a Sinusoid (but lower freq than f_c)

(3) Type of Modulation

3-A. Classification by $m(t)$

Analog Modulation $\triangleq$ $m(t)$ is Analog ; AM, FM (PM)

Digital Modulation $\triangleq$ $m(t)$ is Digital (data) Sequence $\{1, 0, 1, 1, 1, 0 \cdots\}$

Ex. ASK, PSK, FSK, APK (QAM)

3-B. Classification by $C(t)$

Baseband Modulation : $C(t)$ is usually pulse

Ex. PCM, PWM, Δ-MOD

Band-pass Modulation : $C(t)$ is high freq

Ex. AM, FM (PM), ASK, FSK, PSK

3.1. Amplitude Modulation (AM)

AM $\triangleq$ The carrier amplitude A_c is linearly varied according to baseband message $m(t)$

(1) Message : Baseband signal $m(t)$: Modulating Signal
(can be low-freq sinusoid)

(2) Carrier : $C(t) = A_c \cos(\omega_c t + 0)$

(3) Modulated Signal [General form of AM]

Time Domain

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t) \quad (3.2)$$

where $k_a \triangleq$ Amplitude Sensitivity $[V^{-1}]$

$A_c[1 + k_a m(t)]$: Envelope

$|k_a m(t)|_{max} \triangleq$ Modulation Index (relative to 1.0)

If $|k_a m(t)|_{max} < 1$, Under-Modulation $\forall t$
(No Env Distortion)

$|k_a m(t)|_{max} = 1$, Critical Modulation

$|k_a m(t)|_{max} > 1$, Over-Modulation for some t
(Envelope Distortion)

$|k_a m(t)|_{max} \times 100 \triangleq$ % Modulation

Frequency Domain

From Eq (3.2) above

$$s(t)_{AM} = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t$$

Hence

$$S(f) = \frac{1}{2} A_c [\delta(f - f_c) + \delta(f + f_c)] + A_c k_a \{ \underbrace{\mathcal{F}[m(t)] * \mathcal{F}[\cos 2\pi f_c t]}_{M(f) * \frac{1}{2}[\delta(f-f_c) + \delta(f+f_c)]} \}$$

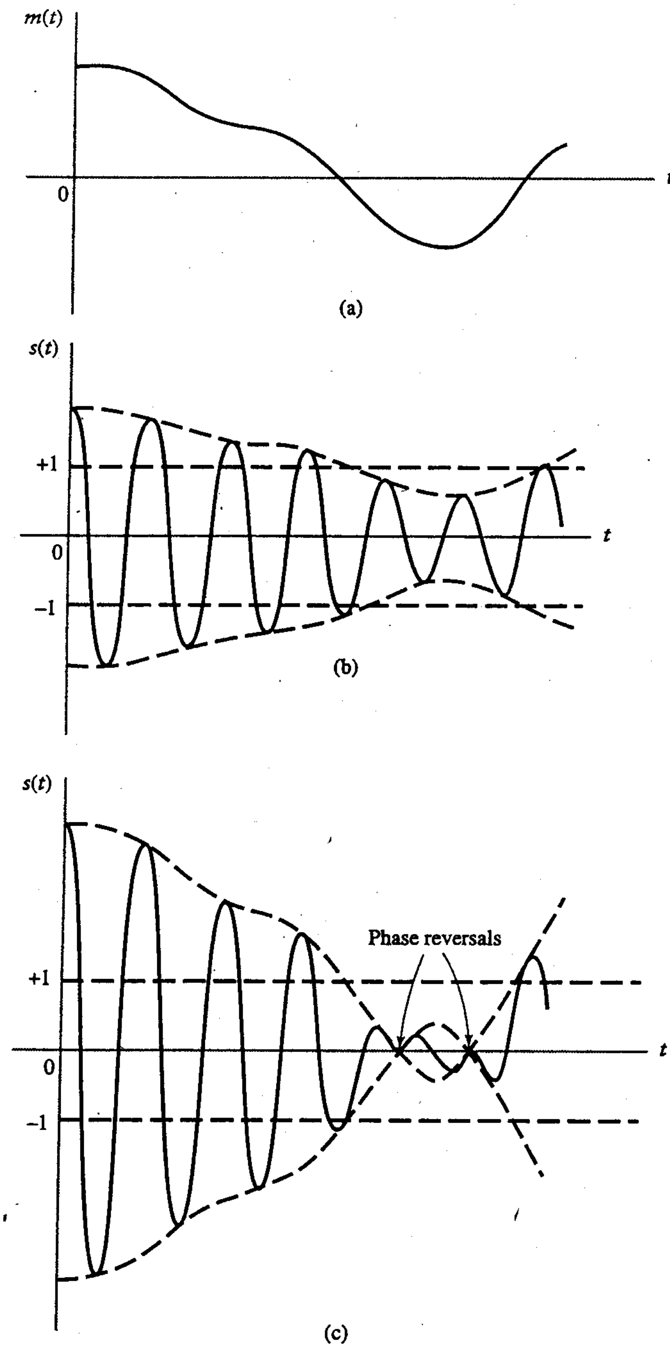


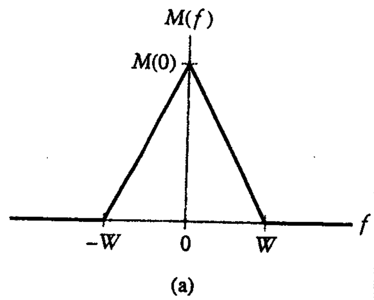
FIGURE 3.1 Illustration of the amplitude modulation process. (*a*) Message signal $m(t)$. (*b*) AM wave for $k_a m(t) < 1$ for all t. (*c*) AM wave for $|k_a m(t)| > 1$ for some t.

Can you define Carrier $c(t)$?

$$= \frac{1}{2}A_c[\delta(f-f_c)+\delta(f+f_c)] + \frac{1}{2}A_c k_a[M(f-f_c)+M(f+f_c)] \qquad (3.5)$$

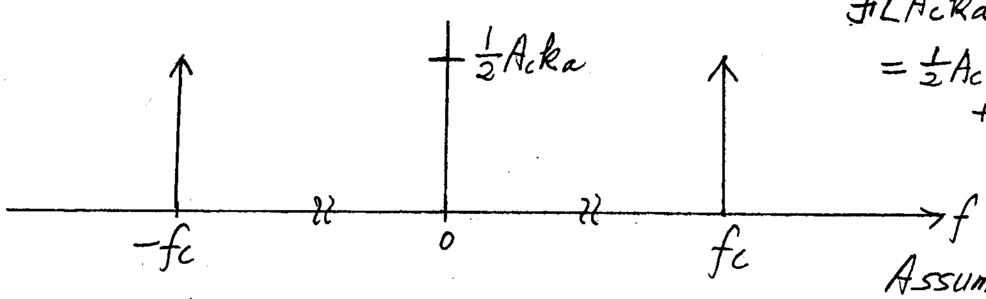
< Spectral Sketch >

Baseband Bandwidth W



- $M(f) = \mathcal{F}[m(t)]$
- It is strictly Band-limited to $-W \le f \le W$
- Low-pass signal
- $M(0) \triangleq M(f)|_{f=0}$ max peak

$*$



$\mathcal{F}[A_c k_a \cos 2\pi f_c t] = \frac{1}{2}A_c k_a[\delta(f-f_c) + \delta(f+f_c)]$

Assume $f_c > |W|$

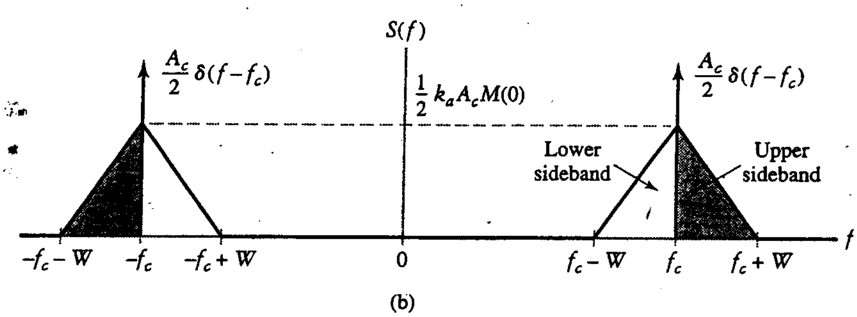


FIGURE 3.2 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of AM wave $s(t)$.

$S(f)$: Band-pass Spectrum

Bandwidth: $(f_c+W)-(f_c-W) = 2W$

From above sketch, We define

USB (Upper Side Band) : $f_c < |f| < f_c + W$

LSB (Lower Side Band) : $f_c - W < |f| < f_c$

Hence by AM, Bandwidth is expanded twice since

Baseband Bandwidth : W

AM Transmission Bandwidth : 2W

Example 3.1. Single-Tone AM (pp 104)

- message $m(t)$ be a sinusoid with A_m and f_m
  (* Usually $A_m < A_c$, $f_m \ll f_c$)
- This $m(t)$ modulate free-running Carrier $C(t) = A_c \cos 2\pi f_c t$
- Then $S(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t = A_c[1 + k_a A_m \cos 2\pi f_m t] \cdot \cos 2\pi f_c t$

$$= A_c[1 + \underset{\sim}{\mu} \cos 2\pi f_m t] \cos 2\pi f_c t$$

where $\mu \triangleq$ Modulation Factor and require $\mu < 1$ to avoid Env. Distortion

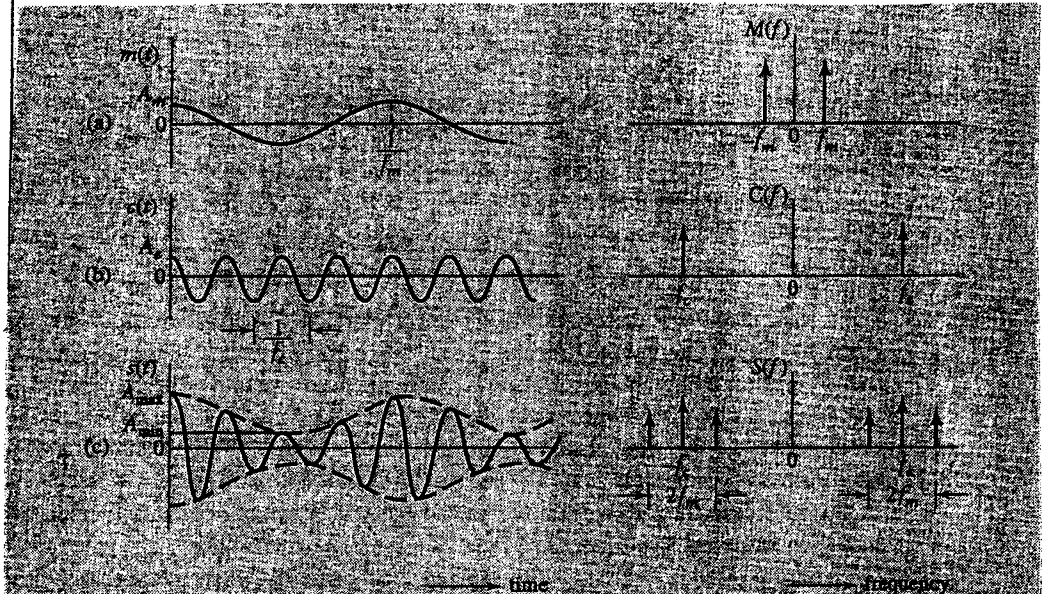


FIGURE 3.3 Illustration of the time-domain (on the left) and frequency-domain (on the right) characteristics of amplitude modulation produced by a single tone. (*a*) Modulating wave. (*b*) Carrier wave. (*c*) AM wave.

Figure 3.3(*c*) shows a sketch of $s(t)$ for μ less than unity. Let A_{max} and A_{min} denote the maximum and minimum values of the envelope of the modulated wave, respectively. Then, from Eq. (3.7) we get

$$\frac{A_{max}}{A_{min}} = \frac{A_c(1+\mu)}{A_c(1-\mu)}$$

<Q> What is Tx BW B_T in this case?

Rearranging this equation, we may express the modulation factor as

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Expressing the product of the two cosines in Eq. (3.7) as the sum of two sinusoidal waves, one having frequency $f_c + f_m$ and the other having frequency $f_c - f_m$, we get

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2}\mu A_c \cos[2\pi(f_c - f_m)t]$$

The Fourier transform of $s(t)$ is therefore

$$S(f) = \frac{1}{2}A_c[\delta(f - f_c) + \delta(f + f_c)]$$
$$+ \frac{1}{4}\mu A_c[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)]$$
$$+ \frac{1}{4}\mu A_c[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)]$$

Thus the spectrum of an AM wave, for the special case of sinusoidal modulation, consists of delta functions at $\pm f_c$, $f_c \pm f_m$, and $-f_c \pm f_m$, as shown in Fig. 3.3(*c*).

AM Signal Generation – Case of Square-Law Modulator

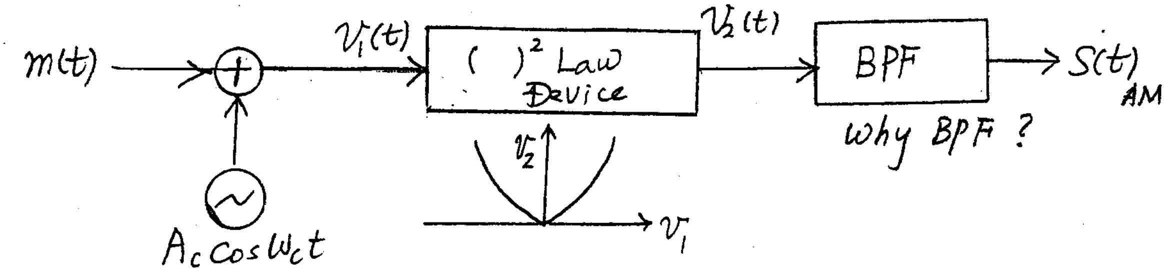
(Refer to Drill problem 3.4 (pp 110))

For the AM signal generation, $\exists$ 2 basic methods:

(1) Square-Law Modulator (Non-linear Device)

(2) Switching Modulator

In Method (1) above, $S(t)_{AM}$ can be generated by



Refer to Drill problem 3.4 (pp. 110)

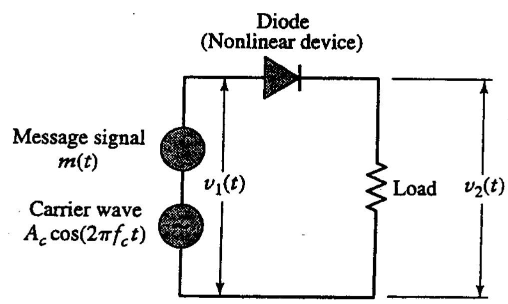


FIGURE 3.8 Nonlinear circuit using a diode.

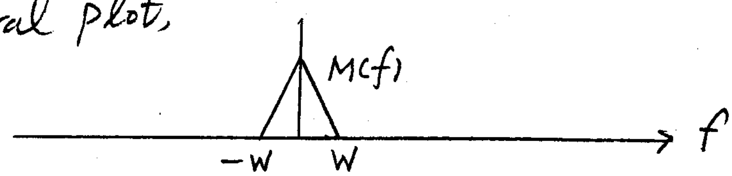
Where, $v_1(t) = m(t) + A_c \cos \omega_c t$, $\omega_c \triangleq 2\pi f_c$

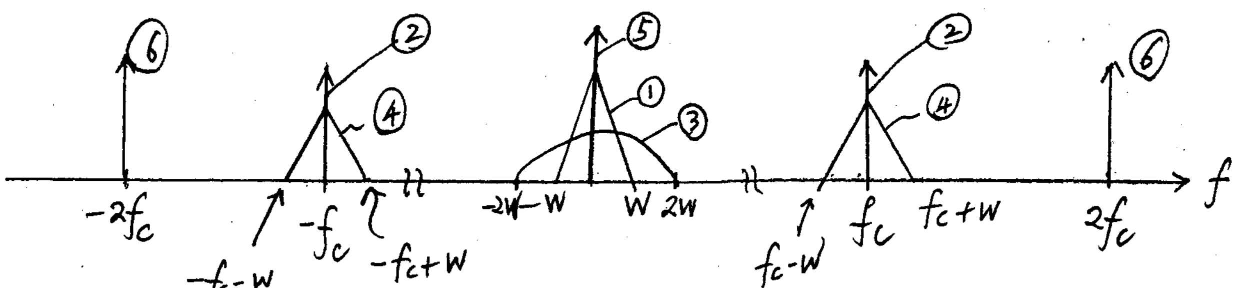
$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t) = \overbrace{a_1 m(t)}^{(1)} + \overbrace{a_1 A_c \cos \omega_c t}^{(2)}$$

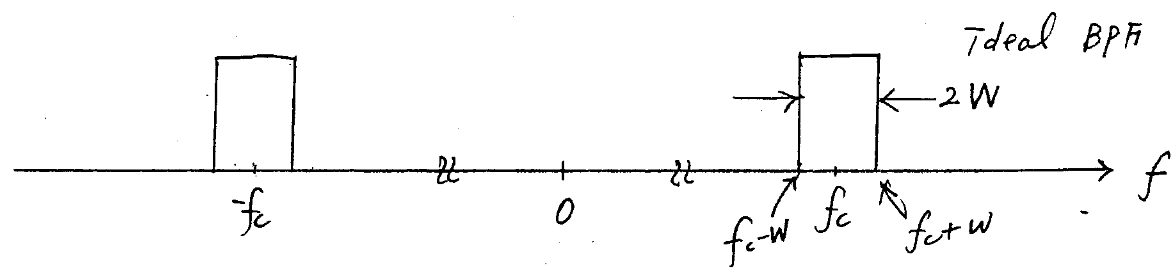
$$+ \overset{(3)}{a_2 m^2(t)} + \underbrace{2 a_2 m(t) \cos \omega_c t}_{(4)}$$

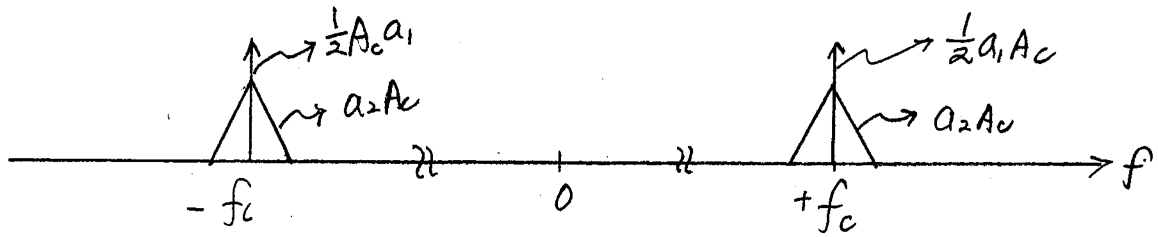
$$+ a_2 A_c^2 \cos^2 \omega_c t \left(= \underset{(5)}{\tfrac{1}{2} A_c^2 a_2} + \tfrac{1}{2} a_2 \underset{(6)}{A_c^2 \cos 2\omega_c t}\right)$$

In spectral plot,









By the result of Ideal BPF with $B_T = 2W$
We get the modulated signal (See the last plot)

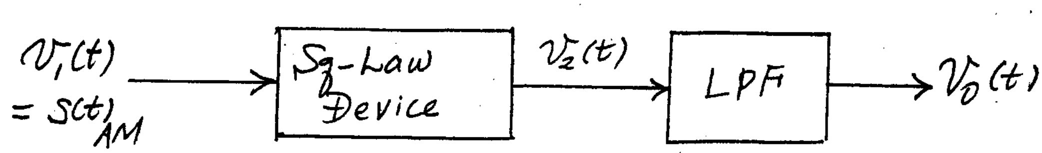
$$S(t)_{AM} = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos \omega_c t \quad (1)$$

(i) hence in this case $\underline{k_a \triangleq \frac{2a_2}{a_1}}$

(ii) In this scenario, $f_c - W > 2W$ to avoid spectral overlapping. Hence $\underline{f_c > 3W}$

AM Signal Demodulation Using Square-Law Detector

From Sq-law Modulator output (after BPF), signal $S(t)$ (Eq(1) above) is Txed. Assuming no noise in Channel, it can be detected/demodulated by using Sq-Law Device again.



Why LPF here ?

$$v_1(t) \triangleq S(t)_{AM} = A_c[1 + k_a m(t)]\cos\omega_c t$$

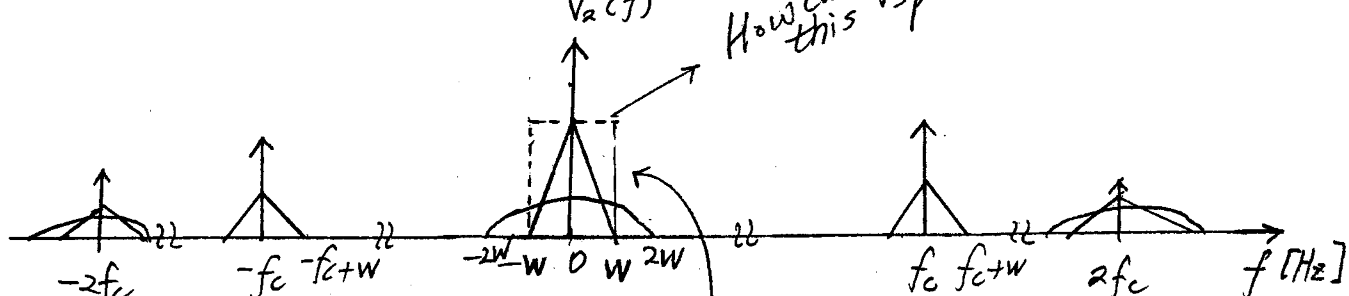
$$v_2(t) = a_1 v_1(t) + a_2 v_1^2(t)$$

$$= a_1 A_c[1+k_a m(t)]\cos\omega_c t + \frac{1}{2} a_2 A_c^2 [1+2k_a m(t) + k_a^2 m^2(t)][1+\cos 2\omega_c t]$$

$$= \frac{1}{2} a_2 A_c^2 + a_2 A_c^2 k_a m(t) + \frac{1}{2} a_2 A_c^2 k_a^2 m^2(t) + a_1 A_c[1+k_a m(t)]\cos\omega_c t + \frac{1}{2} a_2 A_c^2 [1+2k_a m(t) + k_a^2 m^2(t)]\cos 2\omega_c t$$

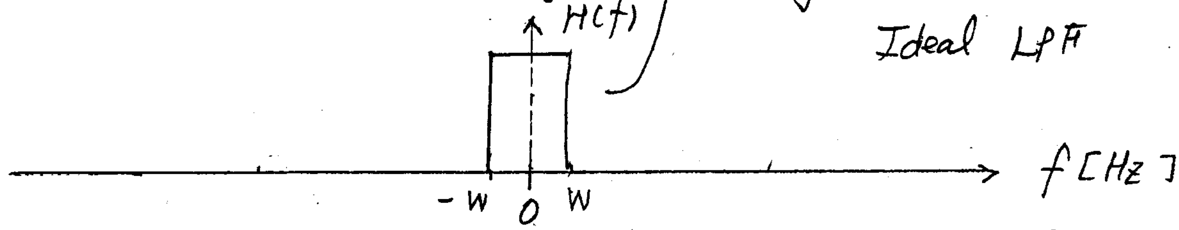
Corresponding Spectrum $V_2(f)$ be

How can you describe this spectra?

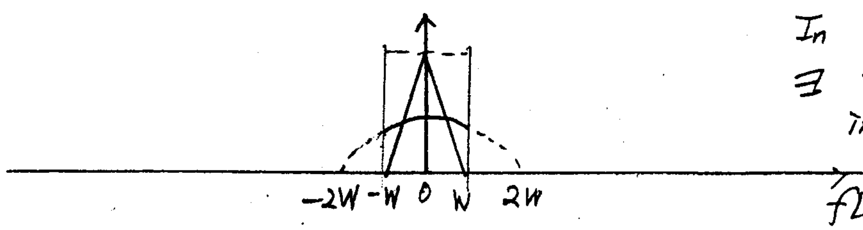


All we need is the original spectrum $M(f)$ Centered to freq 0 (why?)

Ideal LPF



In $|f| < W$ $\exists$ three components including $M(f)$



Corresponding $v_2(t)$ can be written as

$$v_2(t) = a_2 A_c^2 \left\{\frac{1}{2} + k_a m(t)\left[1 + \frac{1}{2}k_a m(t)\right]\right\} + \cdots$$

then $v_0(t) \simeq a_2 A_c^2 \left[\frac{1}{2} + k_a m(t)\right]$ if $|k_a m(t)| \ll 2$ why?

due to $a_2 v_1^2(t)$ (hence called Sq-Law Detection)

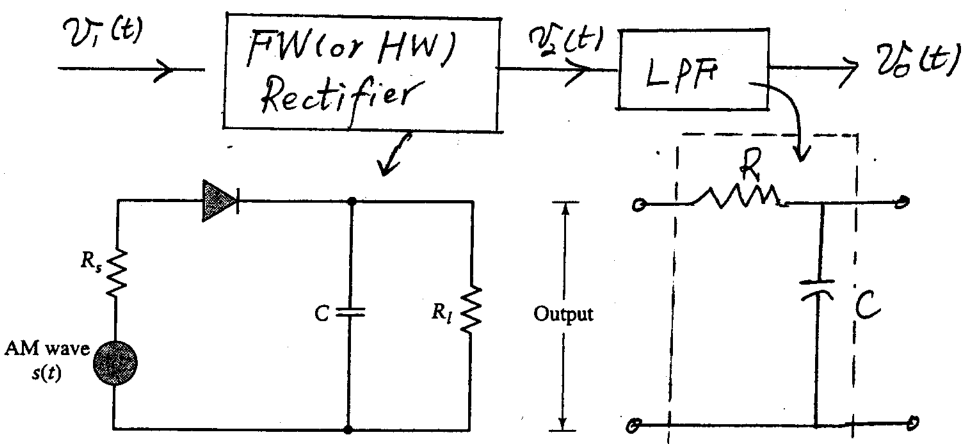
Envelope Detection of AM Signal

1) Note that, in AM, $m(t)$ is represented as the Envelope of the modulated Carrier (if no distortion)

2) To demoduate (or recover $m(t)$), Env. Detector is Widely used if,
   - (a) Message bandwidth $W \ll f_c$: Narrowband
   - (b) No Envelope distortion exist ($<100\%$ modulation)

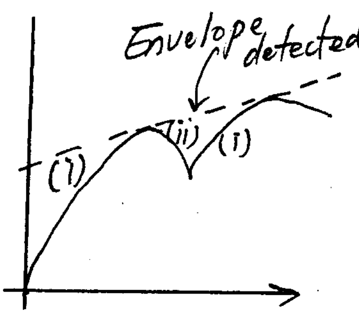
   * All commercial AM use this.

3) Env. Detection



(a)

Operation (i) For positive half cycle of input. Diode is Forward biased
$\Rightarrow$ start to charge C quickly to pick-up input peak value



(ii) If input < peak, diode is reverse-biased and C discharge slowly through R_l

(iii) process (ii) continues until next positive half cycle

(iv) If input > V_c, repeat i) and ii)

Assuming { diode forward resistance r_f / Source Resistance R_s } in Series $(r_f + R_s)$

i) The Charging time Const $< \frac{1}{f_c} = T_c$ to catch peak

$$\rightarrow \quad (r_f + R_s)\cdot C \ll \frac{1}{f_c}$$

ii) For discharge, its time constant must be slower than $\frac{1}{f_c}$, but not too slower than the change rate of $m(t)$, i.e, $\frac{1}{W}$,

$$\frac{1}{f_c} \ll R_l C \ll \frac{1}{W}$$

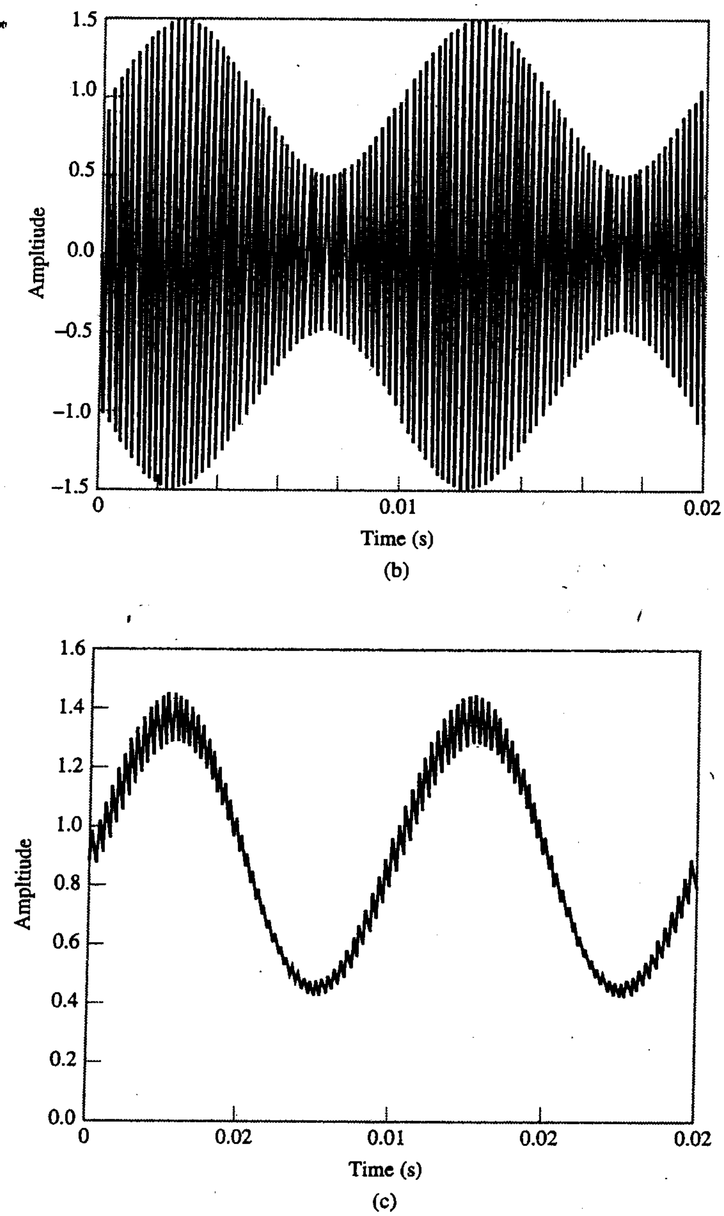
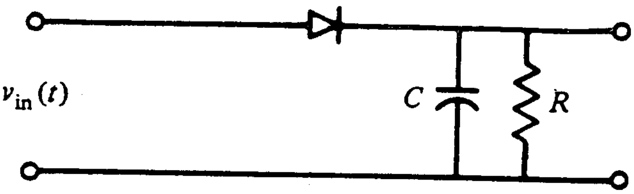
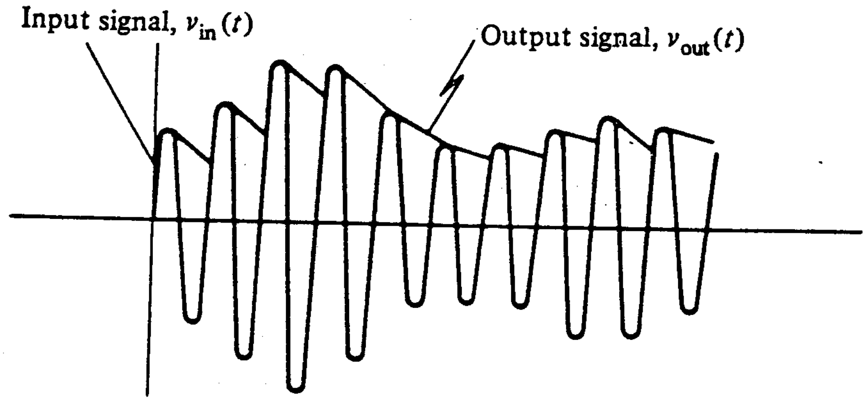


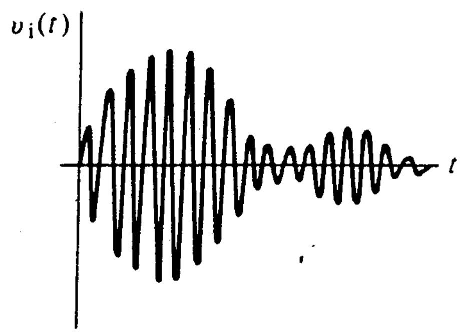
FIGURE 3.9 Envelope detector. (*a*) Circuit diagram. (*b*) AM wave input. (*c*) Envelope detector output



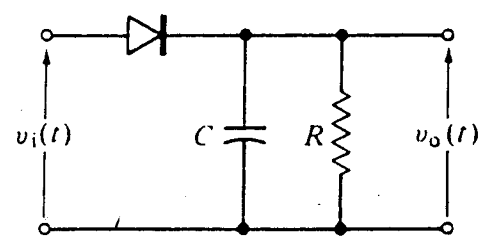
(a) A Diode Envelope Detector



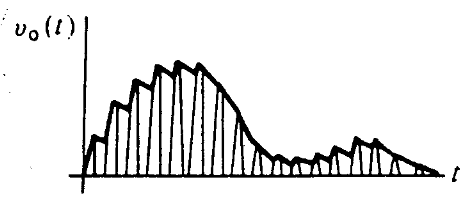
(b) Waveforms Associated with the Diode Envelope Detector



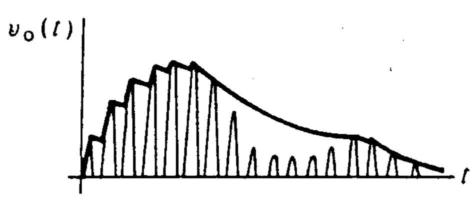
(a)



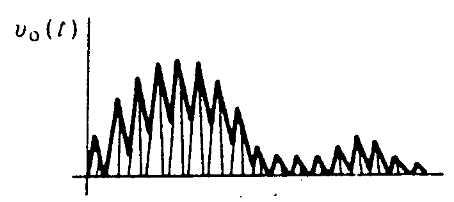
(b)



(c) Correct RC



(d) RC too large



(e) RC too small

3.2. Virtues – Read it please