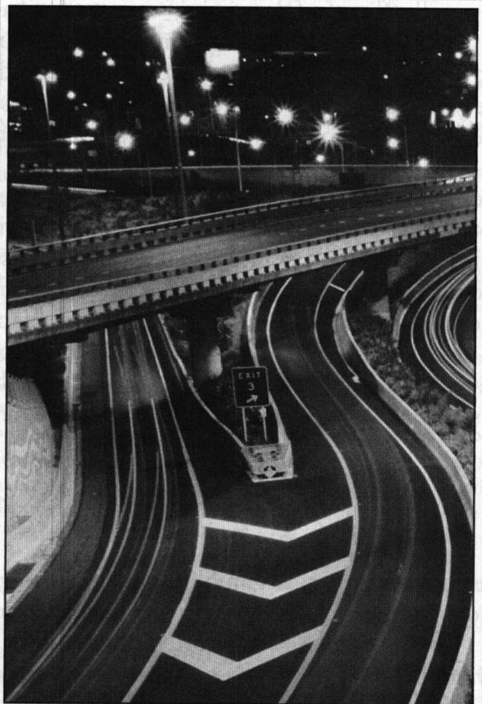
Chapter 4



UNCERTAINTY

So far, we have assumed that people's choices do not involve any degree of uncertainty; once they decide what to do, they get what they have chosen. That is not always the way things work in many real-world situations. When you buy a lottery ticket, invest in shares of common stock, or play poker, what you get back is subject to chance. In this chapter, we look at three questions raised by economic problems involving uncertainty: (1) How do people make decisions in an uncertain environment? (2) Why do people generally dislike risky situations? and (3) What can people do to avoid or reduce risks?

PROBABILITY AND EXPECTED VALUE

The study of individual behavior under uncertainty and the mathematical study of probability and statistics have a common historical origin in gambles of chance. Gamblers who try to devise ways of winning at blackjack and casinos trying to keep the gamble profitable are modern examples of this concern. Two statistical concepts that originated from studying gambles of chance, *probability* and *expected value,* are very important to our study of economic choices in uncertain situations.

Probability
The relative frequency with which an event occurs.

The probability of an event happening is, roughly speaking, the relative frequency with which it occurs. For example, to say that the probability of a head coming up on the flip of a fair coin is 1/2 means that if a coin is flipped a large number of times, we can expect a head to come up in approximately one-half of the flips. Similarly, the probability of rolling a "2" on a single die is 1/6. In approximately one out of every six rolls, a "2" should come up. Of course, before a coin is flipped or a die is rolled, we have no idea what will happen, so each flip or roll has an uncertain outcome.

Expected value
The average outcome from an uncertain gamble.

The expected value of a gamble with a number of uncertain outcomes (or prizes) is the size of the prize that the player will win on average. Suppose Jones and Smith agree to flip a coin once. If a head comes up, Jones will pay Smith \$1; if a tail comes up, Smith will pay Jones \$1. From Smith's point of view, there are two prizes or outcomes (X_1 and X_2) in this gamble: If the coin is a head, $X_1 = +\$1$; if a tail comes up, $X_2 = -\$1$ (the minus sign indicates that Smith must pay). From Jones's point of view, the gamble is exactly the same except that the signs of the outcomes are reversed. The expected value of the gamble is then

$$\frac{1}{2}X_1 + \frac{1}{2}X_2 = \frac{1}{2}(\$1) + \frac{1}{2}(-\$1) = 0. \tag{4.1}$$

The expected value of this gamble is zero. If the gamble were repeated a large number of times, it is not likely that either player would come out very far ahead.

Now suppose the gamble's prizes were changed so that, from Smith's point of view, $X_1 = \$10$, and $X_2 = -\$1$. Smith will win \$10 if a head comes up but will lose only \$1 if a tail comes up. The expected value of this gamble is \$4.50:

$$\begin{aligned}\frac{1}{2}X_1 + \frac{1}{2}X_2 &= \frac{1}{2}(\$10) + \frac{1}{2}(-\$1) \\ &= \$5 - \$0.50 = \$4.50.\end{aligned} \tag{4.2}$$

Micro Quiz 4.1

What is the actuarially fair price for each of the following gambles?

1. Winning \$1,000 with probability 0.5 and losing \$1,000 with probability 0.5
2. Winning \$1,000 with probability 0.6 and losing \$1,000 with probability 0.4
3. Winning \$1,000 with probability 0.7, winning \$2,000 with probability 0.2, and losing \$10,000 with probability 0.1

If this gamble is repeated many times, Smith will certainly end up the big winner, averaging \$4.50 each time the coin is flipped. The gamble is so attractive that Smith might be willing to pay Jones something for the privilege of playing. She might even be willing to pay as much as \$4.50, the expected value, for a chance to play. Gambles with an expected value of zero (or equivalently gambles for which the player must pay the expected value up front for the right to play, here \$4.50) are called **fair gambles**. If fair gambles are repeated many times, the monetary losses or gains are expected to be rather small. Application 4.1: Blackjack Systems looks at the importance of the expected value idea to gamblers and casinos alike.

Fair gamble
Gamble with an expected value of zero.

APPLICATION 4.1

Blackjack Systems

The game of blackjack (or twenty-one) provides an illustration of the expected-value notion and its relevance to people's behavior in uncertain situations. Blackjack is a very simple game. Each player is dealt two cards (with the dealer playing last). The dealer asks each player if he or she wishes another card. The player getting a hand that totals closest to 21, without going over 21, is the winner. If the receipt of a card puts a player over 21, that player automatically loses.

Played in this way, blackjack offers a number of advantages to the dealer. Most important, the dealer, who plays last, is in a favorable position because other players can go over 21 (and therefore lose) before the dealer plays. Under the usual rules, the dealer has the additional advantage of winning ties. These two advantages give the dealer a margin of winning of about 6 percent on average. Players can expect to win 47 percent of all hands played, whereas the dealer will win 53 percent of the time.

Card Counting

Because the rules of blackjack make the game unfair to players, casinos have gradually eased the rules in order to entice more people to play. At many Las Vegas casinos, for example, dealers must play under fixed rules that allow no discretion depending on the individual game situation; and, in the case of ties, rather than winning them, dealers must return bets to the players. These rules alter fairness of the game quite a bit. By some estimates, Las Vegas casino dealers enjoy a blackjack advantage of as little as 0.1 percent, if that. In fact, in recent years a number of systems have been developed by players that they claim can even result in a net advantage for the player. The systems involve counting face cards, systematic varying of bets, and numerous other strategies for special situations that arise in the game.[1] Computer simulations of literally billions of potential blackjack hands have shown that careful adherence to a correct strategy can result in an advantage to the player of as much as 1 or 2 percent. Actor Dustin Hoffman illustrated these potential advantages in his character's remarkable ability to count cards in the 1989 movie *Rain Man*.

Casino vs. Card Counter

It should come as no surprise that players' use of these blackjack systems is not particularly welcomed by those who operate Las Vegas casinos. The casinos made several rule changes (such as using multiple card decks to make card counting more difficult) in order to reduce system players' advantages. They have also started to refuse admission to known system players. Such care has not been foolproof, however. For example, in the late 1990s a small band of MIT students used a variety of sophisticated card counting techniques to take Las Vegas casinos for more than $2 million.[2] Their clever efforts did not amuse casino personnel, however, and the students had a number of unpleasant encounters with security personnel.

All of this turmoil illustrates the importance of small changes in expected values for a game such as blackjack that involves many repetitions. Card counters pay little attention to the outcome on a single hand in blackjack. Instead, they focus on improving the average outcome after many hours at the card table. Even small changes in the probability of winning can result in large expected payoffs.

Expected Values of Other Games

The expected value concept plays an important role in all of the games of chance offered at casinos. For example, slot machines can be set to yield a precise expected return to players. When a casino operates hundreds of slot machines in a single location it can be virtually certain of the return it can earn each day even though the payouts from any particular machine can be quite variable. Similarly, the game of roulette includes 36 numbered squares together with squares labeled "0" and "00." By paying out 36-to-1 on the numbered squares the casino can expect to earn about 5.3 cents (= 2 ÷ 38) on each dollar bet. Bets on Red or Black or on Even or Odd are equally profitable. According to some experts the game of baccarat has the lowest expected return for casinos, though in this case the game's high stakes may still make it quite profitable.

To Think About

1. If blackjack systems increase people's expected winnings, why doesn't everyone use them? Who do you expect would be most likely to learn how to use the systems?
2. How does the fact that casinos operate many blackjack tables, slot machines, and roulette tables simultaneously reduce the risk that they will lose money? Is it more risky to operate a small casino than a large one?

[1]The classic introduction to card-counting strategies is in Edward O. Thorp, *Beat the Dealer* (New York: Random House, 1962).

[2]See Ben Merzrich, *Bringing Down the House* (New York: Free Press, 2002).

RISK AVERSION

Risk aversion
The tendency of people to refuse to accept fair gambles.

Economists have found that, when people are faced with a risky situation that would be a fair gamble, they usually choose not to participate.[1] A major reason for this risk aversion was first identified by the Swiss mathematician Daniel Bernoulli in the eighteenth century.[2] In his early study of behavior under uncertainty, Bernoulli theorized that it is not the monetary payoff of a gamble that matters to people. Rather, it is the gamble's utility (what Bernoulli called the *moral value*) associated with the gamble's prizes that is important for people's decisions. If differences in a gamble's money prizes do not completely reflect utility, people may find that gambles that are fair in dollar terms are in fact unfair in terms of utility. Specifically, Bernoulli (and most later economists) assumed that the utility associated with the payoffs in a risky situation increases less rapidly than the dollar value of these payoffs. That is, the extra (or marginal) utility that winning an extra dollar in prize money provides is assumed to decline as more dollars are won.

Diminishing Marginal Utility

This assumption is illustrated in Figure 4.1, which shows the utility associated with possible prizes (or incomes) from \$0 to \$50,000. The concave shape of the curve reflects the assumed diminishing marginal utility of these prizes. Although additional income always raises utility, the increase in utility resulting from an increase in income from \$1,000 to \$2,000 is much greater than the increase in utility that results from an increase in income from \$49,000 to \$50,000. It is this assumed diminishing marginal utility of income (which is in some ways similar to the assumption of a diminishing MRS introduced in Chapter 2) that gives rise to risk aversion.

A Graphical Analysis of Risk Aversion

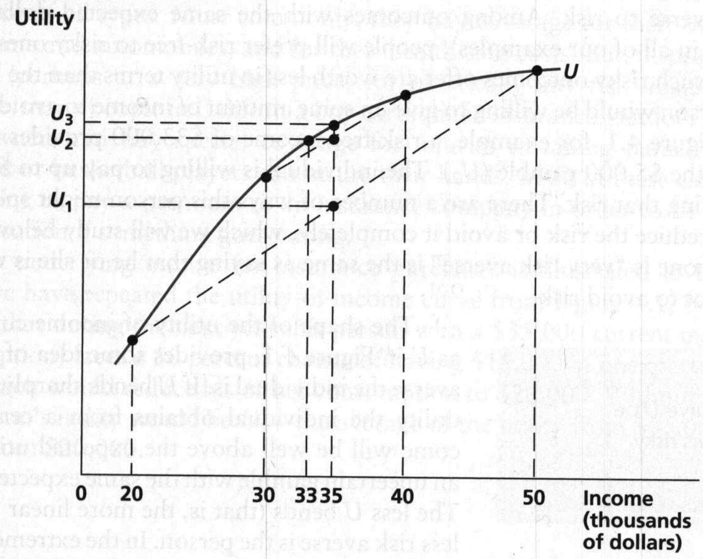
Figure 4.1 illustrates risk aversion. The figure assumes that three options are open to this person. He or she may (1) retain the current level of income (\$35,000) without taking any risk, (2) take a fair bet with a 50-50 chance of winning or losing \$5,000, or (3) take a fair bet with a 50-50 chance of winning or losing \$15,000. To examine the person's preferences among these options, we must compute the expected utility available from each.

The utility received by staying at the current \$35,000 income is given by U_3. The *U* curve shows directly how the individual feels about this current income. The utility level obtained from the \$5,000 bet is simply the average of the utility of \$40,000 (which the individual will end up with by winning the gamble) and the utility of \$30,000 (which he or she will end up with when the gamble is lost). This

[1]The gambles we discuss here are assumed to yield no utility in their play other than the prizes. Because economists wish to focus on the purely risk-related aspects of a situation, they must abstract from any pure consumption benefit that people get from gambling. Clearly, if gambling is fun to someone, he or she will be willing to pay something to play.

[2]For an English translation of the original 1738 article, see D. Bernoulli, "Exposition of a New Theory on the Measurement of Risk," *Econometrica* (January 1954): 23–36.

FIGURE 4.1 Risk Aversion



An individual characterized by the utility-of-income curve U will obtain a higher utility (U_3) from a risk-free income of \$35,000 than from a 50-50 chance of winning or losing \$5,000 (U_2). He or she will be willing to pay up to \$2,000 to avoid having to take this bet. A fair bet of \$15,000 provides even less utility (U_1) than the \$5,000 bet.

average utility is given by U_2.[3] Because it falls short of U_3, we can assume that the person will refuse to make the \$5,000 bet. Finally, the utility of the \$15,000 bet is the average of the utility from \$50,000 and the utility from \$20,000. This is given by U_1, which falls below U_2. In other words, the person likes the risky \$15,000 bet even less than the \$5,000 bet.

KEEP*in*MIND

Choosing among Gambles

To solve problems involving a consumer's choice over gambles, you should proceed in two steps. First, using the formula for expected values, compute the consumer's expected utility from each gamble. Then choose the gamble with the highest value of this number.

[3]This average utility can be found by drawing the chord joining U(\$40,000) and U(\$30,000) and finding the midpoint of that chord. Because the vertical line at \$35,000 is midway between \$40,000 and \$30,000, it will also bisect the chord.