

EXPERIMENT 9 CONSERVATION OF MOMENTUM AND KINETIC ENERGY

OBJECTIVE:

In this experiment, the conservation of kinetic energy and momentum in four different one-dimensional collisions will be tested. Specifically, we will measure the velocities of two air track gliders before and after they collide. From these velocities and the masses of the gliders, the kinetic energies and momenta of the gliders, before and after they collide, can be calculated. Momentum before collision will be compared with momentum after. Kinetic energy before collision will be compared with kinetic energy after. This experiment repeats, for linear motion, the analysis in Experiment 8 for rotational motion. Prepare a formal report using the guidance in Appendix B in this manual.

BACKGROUND:

Conservation laws play a central role in physics. Such laws are the center of the physicist's drive towards reducing the number of assumptions necessary to predict effects of specific causes. As an example, consider the history of the law of conservation of energy. Anytime an experiment provided evidence that energy was not conserved in a given physical situation, physicists simply expanded their definition of energy so that energy would always be conserved. That is, in some sense physicists have defined energy to be that which is conserved in every physical process. In this experiment, we have no way of measuring any kind of energy except kinetic. If the collisions are sufficiently inelastic, kinetic energy should not be conserved.

In this experiment, we will test **conservation of kinetic energy** and **conservation of momentum**. If we consider all types of energy (heat, sound, etc.), then energy is conserved in all processes. On the other hand, momentum is only conserved when no net external force (e.g., friction) acts on the system. This latter point can be seen by considering Newton's second law given by

$$\vec{F} = m\vec{a} \quad (9.1)$$

Momentum, $\vec{p}$, is given by $\vec{p} = m\vec{v}$. The most general form of the second law is

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{m d\vec{v}}{dt} \quad (\text{for } m = \text{constant}) \quad (9.2)$$

If $\vec{F}$ vanishes (i.e., if the net external force is zero), there is no time-rate-of-change of momentum and momentum is conserved, that is, the total momentum of the system is constant.

Thus, in testing for the conservation of energy we must be careful to account for all forms of energy and we should not expect conservation of momentum unless there is no net external force acting on the system. In this experiment, the motion is confined to one dimension, which means vector notation can be dropped.

In the absence of any net external force (e.g., friction), the conservation of linear momentum before and after collision is given by

where

$$\sum_i \vec{p}_i(\text{after}) = \sum_j \vec{p}_j(\text{before}) \quad (9.3)$$

or

$$\vec{p}_B = m_1 \vec{v}_{1B} + m_2 \vec{v}_{2B} = m_1 \vec{v}_{1A} + m_2 \vec{v}_{2A} = \vec{p}_A \quad (9.4)$$

where m is mass and $\vec{v}$ is velocity. Subscript 1 refers to the glider initially in motion, the projectile glider. Subscript 2 refers to the initially stationary glider, the target glider. Subscripts A and B refer to after-collision and before-collision respectively. (9.5)

Similarly, the kinetic energy before and after collision can be computed. The kinetic energy of an object of mass m moving in a straight line is given by

$$K = \frac{1}{2} m v^2 \quad (9.6)$$

By analogy with equation (9.3)

$$\sum_i K_i(\text{after}) = \sum_j K_j(\text{before}) \quad (9.7)$$

By analogy with equation (9.5)

$$K_B = \frac{1}{2} m_1 v_{1B}^2 + \frac{1}{2} m_2 v_{2B}^2 = \frac{1}{2} m_1 v_{1A}^2 + \frac{1}{2} m_2 v_{2A}^2 = K_A \quad (9.8)$$

We will measure the linear velocity of an in-coming glider before it strikes a stationary glider and then measure the velocities of both gliders after collision. We will then calculate and compare total momentum before with momentum after collision and try to show they are equal.

We will consider equation (9.5) confirmed when the absolute value of the percent difference between the momentum before and after is less than or equal to twice the percent uncertainty of the difference. We define the percent uncertainty of the difference between K_B and K_A to be

$$\% \sigma_{p_A - p_B} = \left(\frac{\sigma_{p_A - p_B}}{p_B} \right) \cdot 100\% \quad (9.9)$$

where

$$\sigma_{p_A - p_B} = \sqrt{\sigma_{p_A}^2 + \sigma_{p_B}^2} \quad (9.10)$$

But note that

$$\sigma_{p_A} = \sqrt{(\sigma_{p_{A1}})^2 + (\sigma_{p_{A2}})^2} \quad (9.11)$$

The percent difference of the momenta is defined as

$$\% \text{ Diff}(p) = \frac{p_A - p_B}{p_B} \cdot 100\% \quad (9.12)$$

We consider equation (9.5) confirmed when

$$|\%Diff(p)| \leq 2 \left(\frac{\sigma_{p_A - p_B}}{p_B} \right) \cdot 100\% \quad (9.13)$$

Inequality (9.13) means we are using a ± 2 -sigma uncertainty range for momentum.

The percent uncertainty in the difference between kinetic energy after and before is given by

$$\% \sigma_{K_A - K_B} = \left(\frac{\sigma_{K_A - K_B}}{K_B} \right) \cdot 100\% \quad (9.14)$$

where

$$\sigma_{K_A - K_B} = \sqrt{\sigma_{K_A}^2 + \sigma_{K_B}^2} \quad (9.15)$$

The percent difference between kinetic energy after and before is

$$\% Diff(K) = \left(\frac{K_A - K_B}{K_B} \right) \cdot 100\% \quad (9.16)$$

Equation 9.8 is considered confirmed when

$$|\%Diff(K)| \leq 2 \left(\frac{\sigma_{K_A - K_B}}{K_B} \right) \cdot 100\% \quad (9.17)$$

Inequality (9.17) means we are using a ± 2 -sigma uncertainty range.

The uncertainty in the momentum is propagated through equation $p = mv$ as

$$\sigma_p = \sqrt{\left(\frac{\sigma_m}{m} \right)^2 + \left(\frac{\sigma_v}{v} \right)^2} \cdot p \quad (9.18)$$

The uncertainty in kinetic energy is propagated through equation $K = 1/2mv^2$ as

$$\sigma_K = \sqrt{\left(\frac{\sigma_m}{m} \right)^2 + \left(2 \frac{\sigma_v}{v} \right)^2} \cdot K \quad (9.19)$$

There are two basic types of collisions, elastic and inelastic. Generally, inelastic collisions of simple objects are those in which kinetic energy is not conserved, generally because one or more of the colliding objects are deformed during the collision. In the inelastic collisions in our experiment, a nail on one glider will deform wax on the other glider. Energy is converted to heat as a result of deformation of the wax and the two gliders stick together after the collision. In an inelastic collision, kinetic energy is not conserved. That is, the deformation converts some of the initial kinetic energy into a different form. An elastic collision is one in which there is no deformation or external force such as friction during the collision. This takes place, strictly

speaking, only at the atomic level. However, there are some collisions not at the atomic level where the collisions are almost elastic, that is, kinetic energy is almost conserved. We will examine some such cases in this experiment. It should be noted that the laws of conservation of momentum and energy are independent. The total momentum of a system is conserved in the absence of any external net force regardless of whether the collision is elastic or inelastic.

NOTE: The reason momentum is conserved (when no external force acts) while kinetic energy is lost is that the kinetic energy lost in the inelastic collision stays IN THE SYSTEM of the two colliding bodies as another form of energy such as heat resulting from deformation of the wax/clay.

In this experiment, we will measure the velocities of two gliders before and after they collide. From these data, the sum of the kinetic energies and sum of the momenta of the two masses before and after the collision can be calculated. You are then equipped to make statements about the conservation (i.e., equality) of these two total quantities in the various collisions.

We can now summarize the conservation laws for two gliders, 1 and 2, undergoing a one-dimensional collision on an air track.

For an elastic collision, (A is after, B is before, 1 is projectile glider, 2 is target glider)

$$K_{A1} + K_{A2} = K_{B1} + K_{B2} \quad (9.20)$$

For all collisions, when there is no net external force, we expect that

$$p_{A1} + p_{A2} = p_{B1} + p_{B2} \quad (9.21)$$

We will perform the experiment using two gliders on an air track with the target glider always initially at rest. The speeds are measured with two photogates triggered by flags mounted on top of the gliders. Be sure to record the directions of the gliders because momentum is still a vector quantity for one dimensional motion. Define the initial direction of the projectile glider as the plus, "+", direction and the opposite direction as the minus, "-", direction.

HYPOTHESIS 1:

Inequality (9.17) holds for elastic collisions.

HYPOTHESIS 2:

Inequality (9.17) does not hold for inelastic collisions.

HYPOTHESIS 3:

Inequality (9.13) holds for both elastic and inelastic collisions when there is no net external force (e.g., friction) acting.

PROCEDURE:

Near-elastic collisions:

1. Level the air track with the target glider positioned at the center of the track where the collision will occur. The air track is level when the target glider does not move for about 10 seconds. Adjust the two-foot support on the air track to raise or lower that end of the track as necessary. For the three near-elastic collisions, a small, metal collider with a flat metal blade on one end is inserted into the lower hole in the front of the projectile glider with the blade vertical, and a rubber-band collider is inserted into the lower hole in the adjacent end of the target glider.

These accessories provide near-elastic collisions as long as the glider speeds are low enough to keep the deformation of the rubber band elastic during the collision. It is important to place the colliders in the lower hole in each glider in order to reduce upward deflection of the gliders during collision, which can result in significant energy loss during elastic collisions. It does not matter which collider goes in which glider.

2. **Place** the two gliders at the leveled position with colliders just touching each other. **Position** two photogates on the two sides of the two gliders and as close together as possible without having either glider flag in a photogate. The Capstone program should be running on the computer with the Experiment 9 file open. We estimate that the time interval readings, Δt , provided by Capstone have an uncertainty of ± 0.0010 s.

Position the target glider as close as possible to the second photogate without the flag being in the gate before collision. Because of the influence of airflow over the edges of the gliders, the target glider must be gently held in place until just before the collision. The initial speed of the projectile glider must not be so high as to cause a significant wobbling of the gliders upon collision but must be high enough so that the final velocities of the two gliders are not so slow as to be significantly influenced by the airflow from the air track. **Give** the projectile glider a brief push with your hand for a velocity in the first photogate in the range of **0.7 to 0.9 m/s**.

In **Collision 1**, the projectile glider should come to a complete stop at the instant of collision if the leveling and positioning were done right. Conduct several practice collisions before taking data.

In Collisions 2, 3, and 4, two 0.0500kg masses are added to one or the other glider.

Inelastic collision:

In **Collision 4**, **replace** the flat metal collider and rubber band collider with a collider with wax and a collider with a nail. As before, place the colliders in the lower holes on the ends of the gliders. When the nail strikes the wax, it sticks causing the two gliders to stick together. The nail or wax colliders can go on either glider Re-level the track with the two gliders temporarily stuck together before taking data for collision 4. Before inserting the wax-containing-cylinder collider into the glider, use an eraser or a banana plug from another collider to tamp the wax so it presents a smooth surface without holes. Tamp the wax again before each data-taking collision.

3. **Weigh** both gliders with any collider attached—we will assume all colliders have the same mass—and **record** the masses in Table 9.1. We estimate the mass uncertainty to be $\sigma_m = 0.0010$ kg. The flag lengths, d , are .1000m. This value for d has been input into the Capstone software. The uncertainty of the flag length is estimated to be $\sigma_d \pm 0.0010$ m.

4. Create your tables in Excel. In Excel, Tables 9.2, 9.3, 9.4, and 9.5 are identical in structure. In Tables 9.2, 9.3, 9.4, and 9.5 below, subscript **B** indicates the values **Before** collision and subscript **A** indicates the values **After** collision. **Practice** each collision before taking data. Then **perform** each collision twice (two trials) and **immediately calculate** at least the **total momentum** before and after at least one of the trials to confirm that your experimental technique was correct and your data for that collision are reasonable. If the difference in momentum is greater than 10% check your calculations and, if necessary, repeat the experiment. **Do not**

Does Part P

calculate any uncertainties until you have calculated total momentum and total kinetic energy before and after collision for all collisions.

In this experiment, you are asked to use Capstone to record the linear speeds before and after collision for the following four collisions.

Collision 1: Equal masses - near elastic. Record in Table 9.2.

Collision 2: Smaller mass at rest - near elastic. Record in Table 9.3.

Collision 3: Larger mass at rest - near elastic. Record in Table 9.4.

Collision 4: Larger mass at rest - inelastic. Record in Table 9.5.

Watch sign of v .

ANALYSIS:

5. Complete all the calculations in Tables 9.2 - 9.5. If, after collision, the projectile glider moves in the direction opposite to its initial movement, its final momentum is negative.

6. Summarize your results in Table 9.6. The percent differences in momenta for all collisions usually are less than 5%, but should not be larger than 10%. The percent differences in kinetic energy in Collisions 1, 2 and 3 are usually less than 10%, but not always. Use the before values as the accepted values in all cases.

7. Estimated uncertainties: $\sigma_m = \pm 0.0010 \text{ kg}$, $\sigma_d = \pm 0.0010 \text{ m}$, $\sigma_{\Delta t} = \pm 0.0010 \text{ s}$.

8. Construct Tables 9.1 to 9.6 using Excel. Populate them using the data from your collisions. When computing percent difference, use the before-collision value as the accepted value. In Collision 4, there will be two velocities after collision. Use the velocity of the target glider (the first through the second photogate) for calculating momentum and kinetic energy after collision.

Table 9.1. Glider masses—with attachment

Projectile, m_p (kg)	0.1998
Target, m_t (kg)	0.2006

Table 9.2. COLLISION 1: Equal masses - near elastic

	Projectile		Target		Total	
	Trial 1	Trial 2	Trial 1	Trial 2	Trial 1	Trial 2
Δt_B (s)			At rest	At rest		
Δt_A (s)						
v_B (m/s)			0	0		
v_A (m/s)						
p_B (kg-m/s)			0	0		
p_A (kg-m/s)						
K_B (J)			0	0		
K_A (J)						

Table 9.3. COLLISION 2: Smaller mass at rest - near elastic

	Projectile + 0.1000 kg		Target		Total	
	Trial 1	Trial 2	Trial 1	Trial 2	Trial 1	Trial 2
Δt_B (s)			At rest	At rest		
Δt_A (s)						
v_B (m/s)			0	0		
v_A (m/s)						
p_B (kg-m/s)			0	0		
p_A (kg-m/s)						
K_B (J)			0	0		
K_A (J)						

Table 9.4. COLLISION 3: Larger mass at rest - near elastic

	Projectile		Target + 0.1000 kg		Total	
	Trial 1	Trial 2	Trial 1	Trial 2	Trial 1	Trial 2
Δt_B (s)			At rest	At rest		
Δt_A (s)						
v_B (m/s)			0	0		
v_A (m/s)						
p_B (kg-m/s)			0	0		
p_A (kg-m/s)						
K_B (J)			0	0		
K_A (J)						

Table 9.5. COLLISION 4: Larger mass at rest - inelastic

	Projectile		Target + 0.1000 kg		Total	
	Trial 1	Trial 2	Trial 1	Trial 2	Trial 1	Trial 2
Δt_B (s)			At rest	At rest		
Δt_A (s)						
v_B (m/s)			0	0		
v_A (m/s)						
p_B (kg-m/s)			0	0		
p_A (kg-m/s)						
K_B (J)			0	0		
K_A (J)						

Table 9.6. Summary of conservation measurements

Collision No.	Trial	Total p_B (kg-m/s)	Total p_A (kg-m/s)	Percent Difference*	Total K_B (J)	Total K_A (J)	Percent Difference*
1	1						
1	2						
2	1						
2	2						
3	1						
3	2						
4	1						
4	2						

*Compute % Diff using before values as accepted values.

**Add columns to show $\% \sigma p_A$, $\% \sigma p_B$, $\% \sigma(p_A - p_B)$, $\% \sigma K_A$, $\% \sigma K_B$, and $\% \sigma(K_A - K_B)$.

REMEMBER: $p_A = p_{A1} + p_{A2}$, $K_A = K_{A1} + K_{A2}$.

To test for conservation of momentum, you will determine the percent difference between before and after momenta, with the before-collision momentum taken as the reference value.

$$\% \text{Diff}(p_A - p_B) = \left(\frac{p_A - p_B}{p_B} \right) \cdot 100\% \quad (9.22)$$

You will also determine the percent uncertainty between the before and after momenta. If we divide by the difference, the resulting percent uncertainty would be a very large value because typically the difference between before and after momenta is quite small. Since we determine the percent difference by comparison with the momentum before collision, it makes sense to calculate the percent uncertainty in the difference in momenta using the before-collision momentum.

$$\% \text{Unc}(p_A - p_B) = \left(\frac{\sigma_{(p_A - p_B)}}{p_B} \right) \cdot 100\% \quad (9.23)$$

where

$$\sigma_{(p_A - p_B)} = \sqrt{\sigma_{p_A}^2 + \sigma_{p_B}^2} \quad (9.24)$$

And don't forget

$$\sigma_{p_A} = \sqrt{\sigma_{p_{A1}}^2 + \sigma_{p_{A2}}^2} \quad (9.25)$$

A similar equation applies to σ_{K_A} .
We will assume our results support Hypothesis 3 if the following inequality is satisfied

$$|\%Diff(p_A - p_B)| \leq 2\%Unc\sigma_{(p_A - p_B)} \quad (9.26)$$

Inequality (9.26) is equivalent to assuming 2-sigma error bars on our experimental results.

We will assume our kinetic energy values support hypothesis 1 if the following inequality is satisfied

$$\underline{|\%Diff(K_A - K_B)| \leq 2\%Unc\sigma_{(K_A - K_B)}} \quad (9.27)$$

Date _____

Experiment 9
Conservation of Energy and Momentum

Name _____
Partners _____

QUESTIONS:

Provide the information asked for in the Data and Analysis and/or Results and Conclusions section(s) of your formal report. **DO NOT USE A QUESTION/ANSWER FORMAT.** Integrate the information into your report in such a way that the reader cannot tell you are answering questions.

1. From Hypothesis 3, if a collision is isolated (no external forces) the change in momentum as a result of the collision should be zero within experimental uncertainty. From Hypothesis 1, if a collision is elastic, the change in kinetic energy as a result of the collision should be zero within experimental uncertainty.

Show quantitatively whether your experimental data support Hypothesis 3 for collisions 1, 2, 3 and 4. **Show all calculations** for the Collision 2 trial with the smallest absolute value of the percent difference in momentum in Table 9.6. **Use** Equation Editor at least to **propagate** the measurement uncertainties. **Determine** the percent difference and the percent uncertainty in the difference between total momentum before and total momentum after collision for this single trial. **Set up** the equations in variable form, then insert the numbers and solve. **Use** equations (9.22) to (9.27).

In Table 9.6, after you add the additional columns directed, you should have all the values you need to answer 2 and 3 below without any additional calculations beyond multiplying by 2.

2. From Hypothesis 1, kinetic energy should be conserved in collisions 1 through 3. **Show quantitatively** whether your data did or did not support Hypothesis 1. Just show whether inequality (9.17) is satisfied or not.

3. From Hypothesis 2, kinetic energy should not be conserved in collision 4 because the gliders stick together after collision and hence the collision should be inelastic. **Show quantitatively** whether your data did or did not support Hypothesis 2. Just show whether inequality (9.17) is satisfied or not.

4. **Assume** two gliders of exactly equal mass collide and stick together with one moving and one stationary prior to collision. **Use** variables $v_B(1)$, $v_B(2)$, $v_A(1)$, $v_A(2)$ for velocities of the gliders before and after collision. **Use** m for the equal masses of the gliders. Assume conservation of momentum to **find** v_A in terms of v_B . Substitute this value for v_A in terms of v_B in equation (9.9) for collision 4—the sticking collision. If you do this right, the kinetic energy after collision for Collision 4 should be 50% of the kinetic energy before collision. Show this in your Introduction and Theory or Analysis section. A lot of kinetic energy is lost in the collision just because the two gliders stuck together after collision. The energy is not really lost; it is converted to other forms of energy and remains as internal energy of the two-glider system.

5. **Show quantitatively** (using experimental data) how you know that there was friction between the gliders and the air track. (HINT: No calculations are required. You have data from Collision 4 after collision that will confirm the existence, or non-existence of friction between gliders and air

track.) (HINT: Using differences in KE and P makes no sense because it conflates (combines) the effects of degree of elasticity of the collision with friction between glider and track.

6. **Explain** how moving the photogates farther apart would impact conservation of momentum. I.e., would you expect to see better or worse agreement between momentum before and after collision with the photogates farther apart? If so, why? If not, why not?

7. **Explain** whether friction has the same quantitative effect on conservation of momentum as on conservation of kinetic energy. **Explain** why the effect on KE should be larger or smaller than on P?

8. **Describe** a method of testing conservation of momentum if the friction between the gliders and the air track is significant. (HINT: Just saying "measure something" is not acceptable. You must describe a physical process for making any measurement you propose. This is not difficult but it will take some thought. There are techniques we used in earlier air track experiments that can be modified and used here.) (HINT: In Experiment 7, we measured the deceleration due to friction for a rotating disk. Think about how you would do this for an air track in such a way that friction is the **ONLY** force acting on the glider. (HINT: It is possible to use the data from Collision 4 in Experiment 9 to measure the acceleration due to friction. Think about item 5 above.) You will get no credit if you do not describe a complete procedure for measuring something with enough detail that another student could do it. We have used the air tracks and the photogates in more than one experiment in ways that would allow you to propose a minor modification of one of our experiments to measure the force of friction. The ability to do this type of critical thinking is vital to your professional success. (NOTE: Inclining the air track can be used as part of a method to measure the acceleration due to friction **BUT** inclining the air track to compensate for friction **WILL NOT WORK** because in Collision 3, the projectile glider goes in the reverse direction after collision—I add this because innumerable students have made this mistake.)

REPORT-WRITING GUIDANCE:

9. Your goal is to prepare a formal report that is complete enough that someone with your familiarity with science and with the equipment used in this lab, but totally unfamiliar with this experiment (e.g., the next class to take Physics 210L) could reproduce your work and verify your results using **ONLY** your report. You may assume your customers (readers) have the same expertise in physics as you do and the same understanding of the equipment you had when you came into class to do the experiment.

10. The time you spend making your written report look "professional" will enable me to critique your best work based on my extensive professional experience in writing and editing technical reports. The feedback will be valuable to you in the professional world. Few students come into my classes able to do professional-quality writing. The ability to write clear, concise, technical prose is BIG money in your pocket, and steady employment, over a technical career. It is up to you how much advantage you choose to take of my offer of professional critique of your work.

11. **Integrate** equations, tables and figures into the text. Try to insert tables and figures digitally, but you may cut and paste. When mentioned in the text, each table, equation, or figure should follow as closely as possible after the reference. Use my sample formal report for guidance on how/when to refer to figures and tables and to equations in the text. Note that I do not refer in

advance to equations as I do to tables and figures. The point here is not whether my format is the "perfect" format. The point is that in the professional world you will be required to follow a format from your employer or customer. This format was designed by my predecessor as consistent with the format used in peer-reviewed physics journals in which he had published extensively.

12. **Do not number** figures, tables, or equations 9.1, 9.2,... Use sequential whole numbers, 1, 2, 3,... This is a stand-alone report intended to be published in a technical journal. It is not one in a continuing series of weekly lab reports.

13. **Number** all equations, tables, and figures using the format used in the sample formal report for Experiment 1 on my web site.

14. Acceptable organization means you used the section headings directed in Appendix B and the material in each section is appropriate to the heading.

15. When you first introduce a new variable, e.g., m , v , d , KE , p , Δt , %Diff, %Unc, etc., **define it** for the reader.

16. Do not use present tense.

17. Do not use I, we, or us. Do not use "experimenters," the "group," etc. as stand-ins for personal pronouns.

18. Double-space **ALL** text—including the abstract. Tables are not "text."

19. **DO NOT** allow tables to spread across two pages. This is highly unprofessional.

20. DO NOT leave "widows" or "orphans." The following is quoted from Wikipedia. "...a **widow** is the final line of a paragraph if it falls at the top of the following page [or column] of text, separated from the remainder of the paragraph on the previous page [or column]. A related term, **orphan**, refers to the first line of a paragraph if it appears on its own at the bottom of a page [or column] with the remaining portion of the paragraph appearing on the following page..." In addition, do not permit headings to stand alone at the bottom of a page, and separated from their text.

21. Use the feedback I provided on your first formal report. This should be too obvious to need stating. However, I can assure you from observing formal reports over many years that it is not a common step that students take before writing the second report. Failure to learn from earlier critiques in the professional world will destroy your career.

Edited by E. Smith 06/10/15
Revised by B. Ballard 06/01/16
Revised by P. Yaney 7/11/00