

Problem 3 - Two 3-m-long and 0.4-cm-thick cast iron ($k = 52 \text{ W/mK}$, $\epsilon = 0.8$) steam pipes of outer diameter 10 cm are connected to each other through two 1-cm-thick flanges of outer diameter 20 cm, as shown in the figure. The steam flows inside the pipe at an average temperature of 200°C with a heat transfer coefficient of $180 \text{ W/m}^2\text{K}$. The outer surface of the pipe is exposed to convection with ambient air at 8°C with a heat transfer coefficient of $25 \text{ W/m}^2\text{K}$ as well as radiation with the surrounding surfaces at an average temperature of $T_{\text{surr}} = 290 \text{ K}$. Assuming steady one-dimensional heat conduction along the flanges and taking the nodal spacing to be 1 cm along the flange (a) obtain the finite difference formulation for all nodes, (b) determine the temperature at the tip of the flange by solving those equations, and (c) determine the rate of heat transfer from the exposed surfaces of the flange.

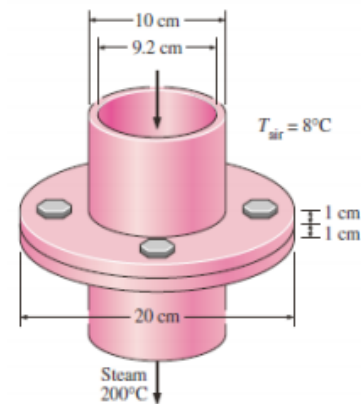
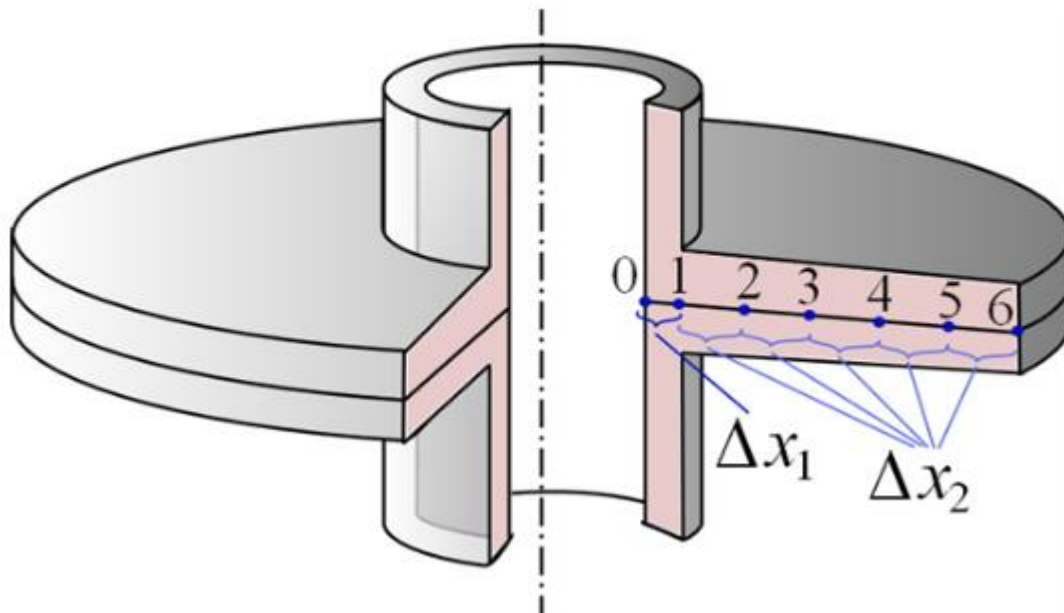


Figure showing the sectioned view of the flange and pipe combination and the location of nodes on it.



(a)

Calculate the number of nodes required for flange (M) if the nodal spacing (Δx_2) is 1 cm and the total length (or radial excess of flange over pipe, L) is 5 cm

$$\begin{aligned} M &= \frac{L}{\Delta x} + 2 \\ &= \frac{5 \text{ cm}}{1 \text{ cm}} + 2 \\ &= 7 \end{aligned}$$

Consider the energy balance at node 0.

$$h_i(2\pi r_0)(T_i - T_0) + k \frac{2\pi t(r_0 + r_1)}{2} \left(\frac{T_1 - T_0}{\Delta x_1} \right) = 0 \quad \dots (1)$$

Here the thermal conductivity is k , heat transfer coefficient of steam is h_i , thickness of flange is t , the radius and temperature of a node is represented as r and T respectively with subscripts denoting the node.

Consider the energy balance at node 1.

$$\begin{aligned} k \frac{2\pi t(r_0 + r_1)}{2} \frac{T_0 - T_1}{\Delta x_1} + k \frac{2\pi t(r_1 + r_2)}{2} \frac{T_2 - T_1}{\Delta x_2} \\ + 2 \left[\frac{2\pi t}{2} \left(r_1 + \frac{r_1 + r_2}{2} \right) \right] \left(\frac{\Delta x_2}{2} \right) \left\{ h(T_\infty - T_1) + \varepsilon \sigma \left[T_{\text{surr}}^4 - (T_1 + 273)^4 \right] \right\} = 0 \quad \dots (2) \end{aligned}$$

Here the emissivity is ε , Stephan Boltzmann constant is σ , heat transfer coefficient of the air is h , temperature of the air and surrounding surfaces is T_∞ and T_{surr} respectively.

Consider the energy balance at node 2.

$$\begin{aligned} k \frac{2\pi t(r_1 + r_2)}{2} \frac{T_1 - T_2}{\Delta x_2} + k \frac{2\pi t(r_2 + r_3)}{2} \frac{T_3 - T_2}{\Delta x_2} \\ + 2(2\pi t r_2 \Delta x_2) \left\{ h(T_\infty - T_2) + \varepsilon \sigma \left[T_{\text{surr}}^4 - (T_2 + 273)^4 \right] \right\} = 0 \quad \dots (3) \end{aligned}$$

Consider the energy balance at node 3.

$$\begin{aligned} k \frac{2\pi t(r_2 + r_3)}{2} \frac{T_2 - T_3}{\Delta x_2} + k \frac{2\pi t(r_3 + r_4)}{2} \frac{T_4 - T_3}{\Delta x_2} \\ + 2(2\pi t r_3 \Delta x_2) \left\{ h(T_\infty - T_3) + \varepsilon \sigma \left[T_{\text{surr}}^4 - (T_3 + 273)^4 \right] \right\} = 0 \quad \dots (4) \end{aligned}$$

Consider the energy balance at node 4.

$$k \frac{2\pi t(r_3 + r_4)}{2} \frac{T_3 - T_4}{\Delta x_2} + k \frac{2\pi t(r_4 + r_5)}{2} \frac{T_5 - T_4}{\Delta x_2} + 2(2\pi t r_4 \Delta x_2) \left\{ h(T_\infty - T_4) + \varepsilon \sigma \left[T_{\text{surr}}^4 - (T_4 + 273)^4 \right] \right\} = 0 \quad \text{..... (5)}$$

Consider the energy balance at node 5.

$$k \frac{2\pi t(r_4 + r_5)}{2} \frac{T_4 - T_5}{\Delta x_2} + k \frac{2\pi t(r_5 + r_6)}{2} \frac{T_6 - T_5}{\Delta x_2} + 2(2\pi t r_5 \Delta x_2) \left\{ h(T_\infty - T_5) + \varepsilon \sigma \left[T_{\text{surr}}^4 - (T_5 + 273)^4 \right] \right\} = 0 \quad \text{..... (6)}$$

Consider the energy balance at node 6.

$$k \frac{2\pi t(r_5 + r_6)}{2} \frac{T_5 - T_6}{\Delta x_2} + 2 \left[2\pi t \left(\frac{\Delta x_2}{2} \right) \frac{1}{2} \left(\frac{r_5 + r_6}{2} + r_6 \right) + 2\pi r_6 t \right] \left\{ h(T_\infty - T_6) + \varepsilon \sigma \left[T_{\text{surr}}^4 - (T_6 + 273)^4 \right] \right\} = 0 \quad \text{..... (7)}$$

Thus equations 1 through 7 represent the finite difference formulations for all the nodes.

(b)

Determine the temperatures at each node by solving the above set of equations (1 to 7). Solve the equations by entering the below code in 'Equations window' of EES software.

"Nodal Equations"

"Node-0"

$$h_i(2\pi t r_0)(T_i - T_0) + k\pi t(r_0 + r_1)((T_1 - T_0)/\Delta x_1) = 0$$

"Node-1"

$$k\pi t(r_0 + r_1)((T_0 - T_1)/\Delta x_1) + k\pi t(r_1 + r_2)((T_2 - T_1)/\Delta x_2) + \pi t(r_1 + r_2)/2 \Delta x_2 (h(T_{inf} - T_1) + \epsilon \sigma (T_{surr}^4 - (T_1 + 273)^4)) = 0$$

"Node-2"

$$k\pi t(r_1 + r_2)((T_1 - T_2)/\Delta x_2) + k\pi t(r_2 + r_3)((T_3 - T_2)/\Delta x_2) + 4\pi t r_2 \Delta x_2 (h(T_{inf} - T_2) + \epsilon \sigma (T_{surr}^4 - (T_2 + 273)^4)) = 0$$

"Node-3"

$$k\pi t(r_2 + r_3)((T_2 - T_3)/\Delta x_2) + k\pi t(r_3 + r_4)((T_4 - T_3)/\Delta x_2) + 4\pi t r_2 \Delta x_2 (h(T_{inf} - T_3) + \epsilon \sigma (T_{surr}^4 - (T_3 + 273)^4)) = 0$$

"Node-4"

$$k\pi t(r_3 + r_4)((T_3 - T_4)/\Delta x_2) + k\pi t(r_4 + r_5)((T_5 - T_4)/\Delta x_2) + 4\pi t r_2 \Delta x_2 (h(T_{inf} - T_4) + \epsilon \sigma (T_{surr}^4 - (T_4 + 273)^4)) = 0$$

"Node-5"

$$k\pi t(r_4 + r_5)((T_4 - T_5)/\Delta x_2) + k\pi t(r_5 + r_6)((T_6 - T_5)/\Delta x_2) + 4\pi t r_2 \Delta x_2 (h(T_{inf} - T_5) + \epsilon \sigma (T_{surr}^4 - (T_5 + 273)^4)) = 0$$

"Node-6"

$$k\pi t(r_5 + r_6)((T_5 - T_6)/\Delta x_2) + 2(0.5\pi t \Delta x_2 (0.5(r_5 + r_6) + r_6) + 2\pi r_6 t)(h(T_{inf} - T_6) + \epsilon \sigma (T_{surr}^4 - (T_6 + 273)^4)) = 0$$

$$h_i = 180$$

$$h = 25$$

$$t = 0.01$$

$$r_0 = 0.046$$

$$r_1 = 0.05$$

$$r_2 = 0.06$$

$$r_3 = 0.07$$

$$r_4 = 0.08$$

$$r_5 = 0.09$$

$r_6=0.1$

$T_i=250$

$T_{inf}=12$

$T_{surr}=290$

$k=52$

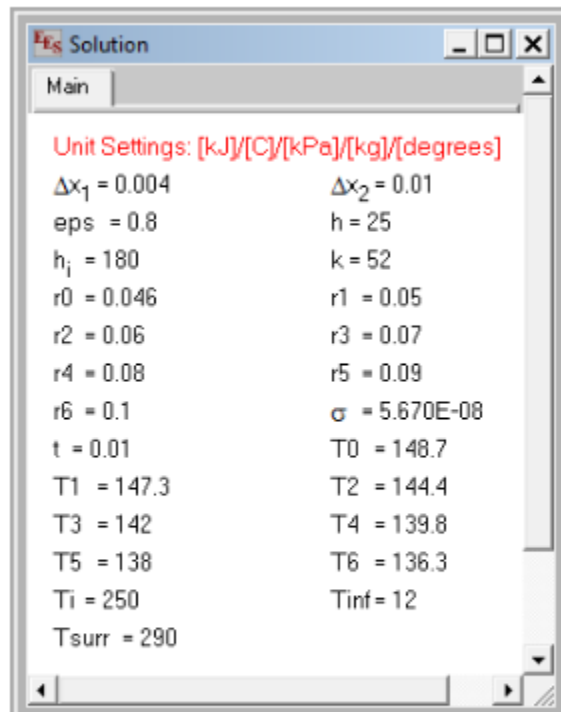
$\Delta x_1=0.004$

$\Delta x_2=0.01$

$\epsilon=0.8$

$\sigma=5.67e-8$

Click on the solve button in EES and the result gets displayed.



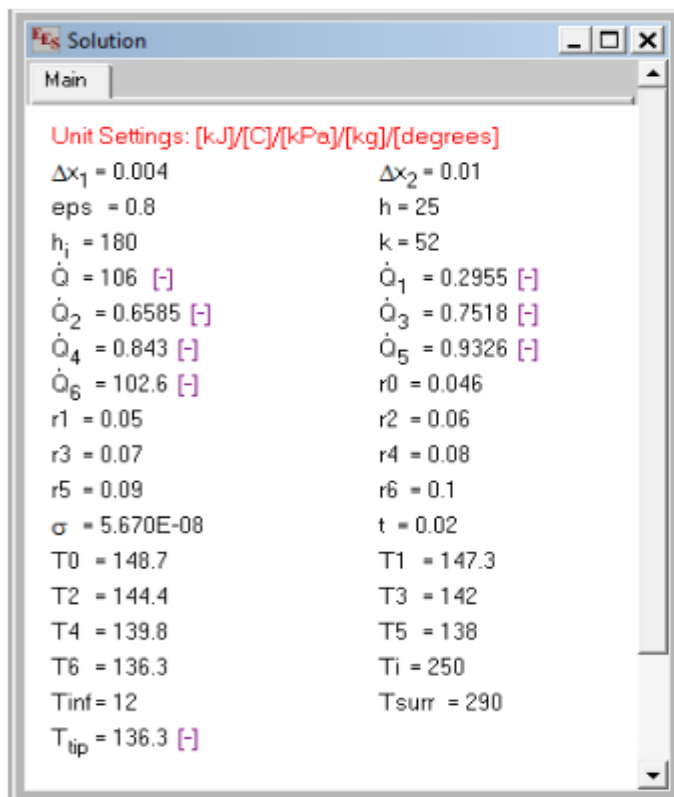
Hence the temperatures at each node are 148.7°C at node0, 147.3°C at node1, 144.4°C at node2, 142°C at node3, 139.8°C at node4, 138°C at node5, and 136.3°C at node6.

(c)

Determine the rate of heat transfer rate from single fin by adding the below code to the previous code.

```
Q_dot=Q_dot_1+Q_dot_2+Q_dot_3+Q_dot_4+Q_dot_5+Q_dot_6  
Q_dot_1=h*2*2*pi*t*(r1+0.055)/2*DELTAx_2/2*(T1-Tinf)+eps*sigma*2*2*pi*t*  
(r1+0.055)/2*DELTAx_2/2*((T1+273)^4-Tsurr^4)  
Q_dot_2=h*2*2*pi*t*r2*DELTAx_2*(T2-Tinf)+eps*sigma*2*2*pi*t*r2*DELTAx_2*((T2+273)^4-  
Tsurr^4)  
Q_dot_3=h*2*2*pi*t*r3*DELTAx_2*(T3-Tinf)+eps*sigma*2*2*pi*t*r3*DELTAx_2*((T3+273)^4-  
Tsurr^4)  
Q_dot_4=h*2*2*pi*t*r4*DELTAx_2*(T4-Tinf)+eps*sigma*2*2*pi*t*r4*DELTAx_2*((T4+273)^4-  
Tsurr^4)  
Q_dot_5=h*2*2*pi*t*r5*DELTAx_2*(T5-Tinf)+eps*sigma*2*2*pi*t*r5*DELTAx_2*((T5+273)^4-  
Tsurr^4)  
Q_dot_6=h*2*(2*pi*t*(0.095+r6)/2*(DELTAx_2/2)+2*pi*t*r6)*(T6-Tinf)+eps*sigma*2*(2*pi*t*  
(0.095+r6)/2*(DELTAx_2/2)+2*pi*t*r6)*((T6+273)^4-Tsurr^4)
```

Click on the solve button in EES and the following result gets displayed.



Hence the rate of heat transfer rate from single fin (\dot{Q}) is 106 W