Assumptions **1** Heat transfer along the fin is given to be steady, and the temperature along the fin to vary in the x direction only so that $T = T(x)$. **2** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 180$ W/m·°C. The emissivity of the fin surface is 0.9.

Analysis The fin length is given to be $L = 5$ cm, and the number of nodes is specified to be $M = 6$. Therefore, the nodal spacing Δx is

$$\Delta x = \frac{L}{M-1} = \frac{0.05 \text{ m}}{6-1} = 0.01 \text{ m}$$

The temperature at node 0 is given to be $T_0 = 200$°C, and the temperatures at the remaining 5 nodes are to be determined. Therefore, we need to have 5 equations to determine them uniquely. Nodes 1, 2, 3, and 4 are interior nodes, and the finite difference formulation for a *general interior node m* is obtained by applying an energy balance on the volume element of this node. Noting that heat transfer is steady and there is no heat generation in the fin and assuming heat transfer to be into the medium from all sides, the energy balance can be expressed as
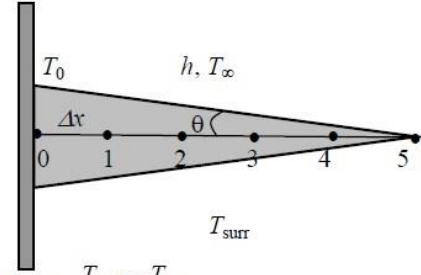
$$\sum_{\text{all sides}} \dot{Q} = 0 \quad \rightarrow \quad kA_{\text{left}} \frac{T_{m-1} - T_m}{\Delta x} + kA_{\text{right}} \frac{T_{m+1} - T_m}{\Delta x} + hA_{\text{conv}}(T_\infty - T_m) + \varepsilon\sigma A_{\text{surface}}[T_{\text{surr}}^4 - (T_m + 273)^4\} = 0$$

Note that heat transfer areas are different for each node in this case, and using geometrical relations, they can be expressed as

$A_{\text{left}} = (\text{Height} \times \text{width})_{@m-1/2} = 2w[L - (m-1/2)\Delta x]\tan\theta$

$A_{\text{right}} = (\text{Height} \times \text{width})_{@m+1/2} = 2w[L - (m+1/2)\Delta x]\tan\theta$

$A_{\text{surface}} = 2 \times \text{Length} \times \text{width} = 2w(\Delta x / \cos\theta)$

Substituting,

$$2kw[L - (m-0.5)\Delta x]\tan\theta \frac{T_{m-1} - T_m}{\Delta x} + 2kw[L - (m+0.5)\Delta x]\tan\theta \frac{T_{m+1} - T_m}{\Delta x}$$
$$+ 2w(\Delta x / \cos\theta)\{h(T_\infty - T_m) + \varepsilon\sigma[T_{\text{surr}}^4 - (T_m + 273)^4]\} = 0$$

Dividing each term by $2kwL\tan\theta / \Delta x$ gives

$$\left[1 - (m-1/2)\frac{\Delta x}{L}\right](T_{m-1} - T_m) + \left[1 - (m+1/2)\frac{\Delta x}{L}\right](T_{m+1} - T_m) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_m) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_m + 273)^4] = 0$$

Substituting,

$m = 1$: $\left[1 - 0.5\frac{\Delta x}{L}\right](T_0 - T_1) + \left[1 - 1.5\frac{\Delta x}{L}\right](T_2 - T_1) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_1) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_1 + 273)^4] = 0$

$m = 2$: $\left[1 - 1.5\frac{\Delta x}{L}\right](T_1 - T_2) + \left[1 - 2.5\frac{\Delta x}{L}\right](T_3 - T_2) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_2) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_2 + 273)^4] = 0$

$m = 3$: $\left[1 - 2.5\frac{\Delta x}{L}\right](T_2 - T_3) + \left[1 - 3.5\frac{\Delta x}{L}\right](T_4 - T_3) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_3) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_3 + 273)^4] = 0$

$m = 4$: $\left[1 - 3.5\frac{\Delta x}{L}\right](T_3 - T_4) + \left[1 - 4.5\frac{\Delta x}{L}\right](T_5 - T_4) + \frac{h(\Delta x)^2}{kL\sin\theta}(T_\infty - T_4) + \frac{\varepsilon\sigma(\Delta x)^2}{kL\sin\theta}[T_{\text{surr}}^4 - (T_4 + 273)^4] = 0$

An energy balance on the 5th node gives the 5th equation,

$m = 5$: $2k\frac{\Delta x}{2}\tan\theta\frac{T_4 - T_5}{\Delta x} + 2h\frac{\Delta x / 2}{\cos\theta}(T_\infty - T_5) + 2\varepsilon\sigma\frac{\Delta x / 2}{\cos\theta}[T_{\text{surr}}^4 - (T_5 + 273)^4] = 0$

Solving the 5 equations above simultaneously for the 5 unknown nodal temperatures gives

$T_1 = 177.0$°C, $T_2 = 174.1$°C, $T_3 = 171.2$°C, $T_4 = 168.4$°C, and $T_5 = 165.5$°C

(*b*) The total rate of heat transfer from the fin is simply the sum of the heat transfer from each volume element to the ambient, and for $w = 1$ m it is determined from

$$\dot{Q}_{fin} = \sum_{m=0}^{5} \dot{Q}_{element,m} = \sum_{m=0}^{5} hA_{surface,m}(T_m - T_\infty) + \sum_{m=0}^{5} \varepsilon\sigma A_{surface,m}[(T_m + 273)^4 - T_{surr}^4]$$

Noting that the heat transfer surface area is $w\Delta x / \cos\theta$ for the boundary nodes 0 and 5, and twice as large for the interior nodes 1, 2, 3, and 4, we have

$$\dot{Q}_{fin} = h\frac{w\Delta x}{\cos\theta}\left[(T_0 - T_\infty) + 2(T_1 - T_\infty) + 2(T_2 - T_\infty) + 2(T_3 - T_\infty) + 2(T_4 - T_\infty) + (T_5 - T_\infty)\right]$$

$$+ \varepsilon\sigma\frac{w\Delta x}{\cos\theta}\{[(T_0 + 273)^4 - T_{surr}^4] + 2[(T_1 + 273)^4 - T_{surr}^4] + 2[(T_2 + 273)^4 - T_{surr}^4] + 2[(T_3 + 273)^4 - T_{surr}^4]$$

$$+ 2[(T_4 + 273)^4 - T_{surr}^4] + [(T_5 + 273)^4 - T_{surr}^4]\}$$

$$= 533 \text{ W}$$