

14. (12 points) Consider two random variables, X and Y , and constants a and b .

- (a) Use the definition of the covariance (that is, $\text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$) to prove (mathematically) that $\text{Corr}(aX, bY) = \text{Corr}(X, Y)$.
- (b) Explain (with an example if you wish) why this is a desirable property of the correlation coefficient ρ .

$$(a) \quad \text{Cov}(X, Y) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y] \rightarrow \text{Corr}(aX, bY) = \text{Corr}(X, Y)$$

$$\mathbb{E}[XY] = \sum_{x,y} \sum_{G_S} (x,y) f(x,y)$$

$$\mathbb{E}[X] = \mu_X$$

$$\mathbb{E}[Y] = \mu_Y$$