

2 Written Answer Questions

Show all your work. Note that it is your responsibility to convince me of your solution with clear and concise arguments that are easy to follow. No credit will be given otherwise. (Partial) Credit will be given when due.

13. (14 points) The continuous random variables X and Y have the joint p.d.f.

$$f(x, y) = xy, \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 1.$$

(a) Find $\mathbb{P}(X < Y)$.

(b) Calculate $\mathbb{E}[X^2 Y]$.

(a) $0 \leq x \leq 2, \quad 0 \leq y \leq 1$
 $0 \leq x - y \leq 1$

? = $\int_0^1 \int_0^{1-x} xy \, dy \, dx$
 $= \int_0^1 \left[\frac{1}{2} xy^2 \right]_0^{1-x} dx$
 $= \frac{1}{2} x (1-x)^2 \Big|_0^1 dx$

k

$$= \frac{1}{2}$$

$$= \frac{1}{2}$$

$$=$$

(b) $\mathbb{E}[X^2 Y] = \mathbb{E}[X^2] \cdot \mathbb{E}[Y]$

$= x^2 \cdot f(x, y) \cdot y \cdot f(y)$

