

5. A professor is known for writing challenging exams. In his experience, only one in five students tends to do very well ('A' or 'B'), around half perform o.k. ('C' or 'D'), and the others do not attain a passing score ('F'). It seems reasonable to assume that students' performances on the exam are independent of one another. The professor notices that after grading the first eight students at random, four of the students got an 'A' or 'B'. Conditional on that information, what is the expected number of students among those eight who failed the exam?

A. 0.9 B. 1.2 C. 1.5 D. 2.4 E. 3.0

6. Let X and Y have the joint p.m.f.

$$f(x, y) = \frac{x + 2y}{18}, \quad x = 1, 2, 3, \quad y = 0, 1.$$

- (a) Find $f_X(2)$.

A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{2}$ E. $\frac{2}{3}$

- (b) Calculate $\text{Var}(Y|X = 1)$.

A. 0.03 B. 0.07 C. 0.13 D. 0.19 E. 0.75

7. Let the joint p.m.f. of X and Y be given by

$$f(0, 0) = 0.5; \quad f(1, 1) = f(-1, 1) = 0.25.$$

Then X and Y are:

- A. Negatively correlated, independent.
B. Negatively correlated, dependent.
C. Positively correlated, independent.
D. Positively correlated, dependent.
E. Uncorrelated, independent.
F. Uncorrelated, dependent.