2. [10 pts]

(a) Let $F: \mathbb{R}^3 \to \mathbb{R}^2$. Explain why F must have a nontrivial kernel.

(b) Must $G: \mathbb{R}^3 \to \mathbb{R}^3$ have a nontrivial kernel? Why or why not? Explain.

3. [10 pts] Recall the space

$$\mathbb{P}_3 = \{a_0 + a_1x + a_2x^2 + a_3x^3 \mid a_0, a_1, a_2, a_3 \in \mathbb{R}\}$$

of polynomials at most degree 3. Does the set

$$\{x+1, x^2, x^3 + x^2 + x + 1, x^3 + x^2\}$$

form a basis for $\mathbb{P}_3$? Why or why not? Explain.

4. [10 pts] Let $F(x) = Ax$ where A is given by

$$A = \begin{pmatrix} -3 & 2 & -3 & -2 & 2 \\ 3 & 3 & 3 & 2 & 3 \\ 2 & 2 & -4 & -3 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & 3 & -1 & 4 \end{pmatrix}.$$

Can the vector

$$c = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

be in the range of F? Explain. *Hint*: Do not try to find an explicit solution of $Ax = c$.

5. [10 pts] Let the vectors $\{a, b, c\}$ form a basis for $\mathbb{R}^3$. Explain why the vectors $\{a+b+c, a+b, a\}$ also form a basis for $\mathbb{R}^3$.