The statement of the Pythagorean theorem is an *if. . . then* statement. If the antecedent (the statement following the word "if") and the consequent (the statement following the word "then") are interchanged, the new statement is called the *converse* of the original. Although the converse of a true statement may not be true, the *converse* of the Pythagorean theorem *is* also a true statement and can be used to determine if a triangle is a right triangle, given the lengths of the three sides.

Converse of the Pythagorean Theorem

If a triangle has sides of lengths a, b, and c, where c is the length of the longest side, and if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

EXAMPLE 10 Applying the Converse of the Pythagorean Theorem

Following **Hurricane Katrina** in August 2005, the pine trees of southeastern Louisiana provided thousands of examples of **right triangles.** See the photo.

Suppose the vertical distance from the base of a broken tree to the point of the break is 55 inches. The length of the broken part is 144 inches. How far along the ground is it from the base of the tree to the point where the broken part touches the ground?

Jonathan Wooding has been contracted to complete an unfinished 8-foot-by-12-foot laundry room in an existing house. He finds that the previous contractor built the floor so that the length of its diagonal is 14 feet, 8 inches. Is the floor "squared off" properly?

SOLUTION

Because 14 feet, 8 inches $= 14\frac{2}{3}$ feet, he must check to see whether the following statement is true.

$$8^2 + 12^2 \stackrel{?}{=} \left(14\frac{2}{3}\right)^2 \qquad a^2 + b^2 = c^2$$

$$8^2 + 12^2 \stackrel{?}{=} \left(\frac{44}{3}\right)^2 \qquad 14\tfrac{2}{3} = \tfrac{44}{3}$$

$$208 \stackrel{?}{=} \frac{1936}{9} \qquad \text{Simplify.}$$

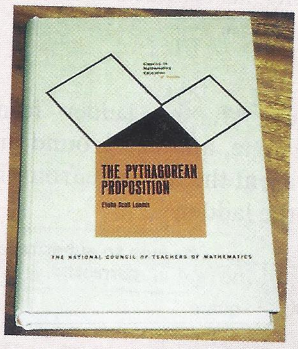
$$208 \neq 215\frac{1}{9} \qquad \text{The two values are not equal.}$$

He needs to fix the problem, since the diagonal, which measures 14 feet, 8 inches, should actually measure $\sqrt{208} \approx 14.4 \approx 14$ feet, 5 inches.

For Further Thought

Proving the Pythagorean Theorem

The Pythagorean theorem has probably been proved in more different ways than any theorem in mathematics. A book titled *The Pythagorean Proposition*, by Elisha Scott Loomis, was first published in 1927. It contained more than 250 different proofs of the theorem.



One of the most popular proofs of the theorem follows. This proof requires two formulas for area.

The area $\mathscr{A}$ of a square is given by

$$\mathscr{A} = s^2,$$

where s is the length of a side of the square. The area of a triangle is given by

$$\mathscr{A} = \frac{1}{2}bh,$$

where b is the base of the triangle and h is the height corresponding to that base. (In a right triangle, b and h are the legs.)

For Group or Individual Investigation

In the figure given on the next page, the area of the large square must always be the same, no matter how it is determined. It is made up of four right triangles and a smaller square.