

Inventory

Chapter Outline

Inventory Functionality and Definitions

Inventory Functionality

Inventory Definitions

Inventory Carrying Cost

Capital

Taxes

Insurance

Obsolescence

Storage

Planning Inventory

When to Order

How Much to Order

Managing Uncertainty

Demand Uncertainty

Performance Cycle Uncertainty

Safety Stock with Combined Uncertainty

Estimating Fill Rate

Dependent Demand Replenishment

Inventory Management Policies

Inventory Control

Reactive Methods

Planning Methods

Collaborative Inventory Replenishment

Inventory Management Practices

Product/Market Classification

Segment Strategy Definition

Policies and Parameters

Summary

Inventory decisions are both high risk and high impact throughout the supply chain. Inventory committed to support future sales drives a number of anticipatory supply chain activities. Without the proper inventory assortment, lost sales and customer dissatisfaction may occur. Likewise, inventory planning is critical to manufacturing. Material or component shortages can shut down manufacturing or force production schedule modification, added cost and potential finished goods shortages. Just as shortages can disrupt marketing and manufacturing plans, inventory overstocks also create operating problems. Overstocks increase cost and reduce profitability as a result of added

warehousing, working capital, insurance, taxes, and obsolescence. Management of inventory resources requires an understanding of functionality, principles, cost, impact, and dynamics.

Inventory Functionality and Definitions

Inventory management involves risk which varies depending upon a firm's position in the distribution channel. The typical measures of inventory exposure are time duration, depth, and width of commitment.

For a manufacturer, inventory risk is long-term. The manufacturer's inventory commitment begins with raw material and component parts purchase, includes work-in-process, and ends with finished goods. In addition, finished goods are often positioned in warehouses in anticipation of customer demand. In some situations, manufacturers are required to consign inventory to customer facilities. In effect, this practice shifts all inventory risk to the manufacturer. Although a manufacturer typically has a narrower product line than a retailer or wholesaler, the manufacturer's inventory commitment is deep and of long duration.

A wholesaler purchases large quantities from manufacturers and sells smaller quantities to retailers. The economic justification of a wholesaler is the capability to provide customers an assortment of merchandise from different manufacturers in specific quantities. When products are seasonal, the wholesaler may be required to take an inventory position far in advance of the selling season, thus increasing depth and duration of risk. One of the greatest challenges of wholesaling is product-line expansion to the point where the width of inventory risk approaches that of the retailer while depth and duration of risk remain characteristic of traditional wholesaling. In recent years, powerful retailers have driven a substantial increase in depth and duration by shifting inventory responsibility back to wholesalers.

For a retailer, inventory management is about the velocity of buying and selling. Retailers purchase a wide variety of products and assume substantial risk in the marketing process. Retail inventory risk can be viewed as wide but not deep. Because of the high cost of store location, retailers place prime emphasis on inventory turnover. Inventory turnover is a measure of inventory velocity and is calculated as the ratio of sales for a time period divided by average inventory.

Although retailers assume a position of risk on a wide variety of products, their position on any one product is not deep. Risk is spread across more than 30,000 stockkeeping units (SKUs) in a typical supermarket. A mass retailer offering general merchandise and food often exceeds 50,000 SKUs. Faced with this width of inventory, retailers attempt to reduce risk by pressing manufacturers and wholesalers to assume greater and greater inventory responsibility. Pushing inventory back up the channel has resulted in retailer demand for fast delivery of mixed-product shipments from wholesalers and manufacturers. Specialty retailers, in contrast to mass merchandisers, normally experience less width of inventory risk as a result of handling narrower assortments. However, they must assume greater risk with respect to depth and duration of inventory holding.

If a business plans to operate at more than one level of the distribution channel, it must be prepared to assume the associated inventory risk. For example, a food chain that operates a regional warehouse assumes risk related to wholesale functionality over and above normal retail operations. To the extent that an enterprise becomes vertically integrated, inventory must be managed at multiple supply chain levels.

Inventory Functionality

From an inventory perspective, the ideal situation would be a response-based supply chain. At various points in early chapters, the practicality of implementing a fully response-based supply chain was discussed in terms of the total costs and timeliness of customer support. While a zero-inventory supply chain is typically not attainable, it is important to remember that each dollar invested in inventory is a trade-off to an alternative use of assets.

Inventory is a current asset that should provide return on the capital invested. The return on inventory investments is the marginal profit on sales that would not occur without inventory. Accounting experts have long recognized that measuring the true cost and benefits of inventory on the corporate profit-and-loss is difficult.¹ Lack of measurement sophistication makes it difficult to evaluate the trade-offs between service levels, operating efficiencies, and inventory levels. While aggregate inventory levels throughout sectors of the economy have decreased, many enterprises still carry more inventory than needed to support actual business requirements. The forces driving this generalization are understood better through a review of the four prime functions of inventory. Table 7.1 summarizes inventory functionality.

These four functions, **geographical specialization, decoupling, balancing supply and demand, and buffering uncertainty**, require inventory investment to achieve operating objectives. While logistics, as discussed in Chapter 2, has made significant progress in reducing overall supply chain inventory, inventory properly deployed creates value and reduces total cost. Given a specific manufacturing/marketing strategy, inventories planned and committed to operations can only be reduced to a level consistent with performing the four inventory functions. All inventory exceeding the minimum level represents excess commitments.

At the minimum level, inventory invested to achieve geographical specialization and decoupling can be modified only by changes in network facility location and operational processes of the enterprise. The minimum level of inventory required to balance supply and demand depends on ability to estimate seasonal requirements. With increased experience over a number of seasonal periods, the inventory required to achieve marginal sales during periods of high demand can be projected fairly well. A seasonal inventory plan can be formulated on the basis of this experience.

¹ Douglas M. Lambert, *The Development of an Inventory Costing Methodology* (Chicago: National Council of Physical Distribution Management, 1976), p. 3; and *Inventory Carrying Cost, Memorandum 611* (Chicago: Drake Sheahan/Stewart Dougall, Inc., 1974).

TABLE 7.1
Inventory
Functionality

Geographical Specialization	Allows geographical positioning across multiple manufacturing and distributive units of an enterprise. Inventory maintained at different locations and stages of the value-creation process allows specialization.
Decoupling	Allows economy of scale within a single facility and permits each process to operate at maximum efficiency rather than having the speed of the entire process constrained by the slowest.
Supply/Demand Balancing	Accommodates elapsed time between inventory availability (manufacturing, growing, or extraction) and consumption.
Buffering Uncertainty	Accommodates uncertainty related to demand in excess of forecast or unexpected delays in order receipt and order processing in delivery and is typically referred to as safety stock.

Inventories committed to safety stocks represent the greatest potential for improved logistical performance. These commitments are operational in nature and can be adjusted rapidly in the event of an error or policy change. A variety of techniques are available to assist management in planning safety stock commitments. The focus of this chapter is the analysis of safety stock relationships and inventory policy development.

Inventory Definitions

In designing inventory policy, specific inventory relationships must be considered. Management must understand these relationships to determine inventory policy with respect to when and how much to order. The inventory policy drives desired inventory performance. The two key indicators of inventory performance are **service level** and **average inventory**.

Inventory Policy

Inventory policy consists of guidelines concerning what to purchase or manufacture, when to take action, and in what quantity. It also includes decisions regarding geographical inventory positioning. For example, some firms may decide to postpone inventory positioning by maintaining stock at the plant. Other firms may use a more speculative strategy of positioning product in local markets or regional warehouses. The development of sound inventory policy is the most difficult dimension of inventory management.

A second aspect of policy concerns inventory management practice. One approach is to independently manage inventory at each stocking facility. At the other extreme is central inventory management of all stocking locations. Centralized inventory management requires effective communication and coordination. The increased availability of information technology and integrated planning systems allows more firms to implement centralized inventory planning. Centralized inventory planning systems can reduce demand uncertainty between distribution locations.

Service Level

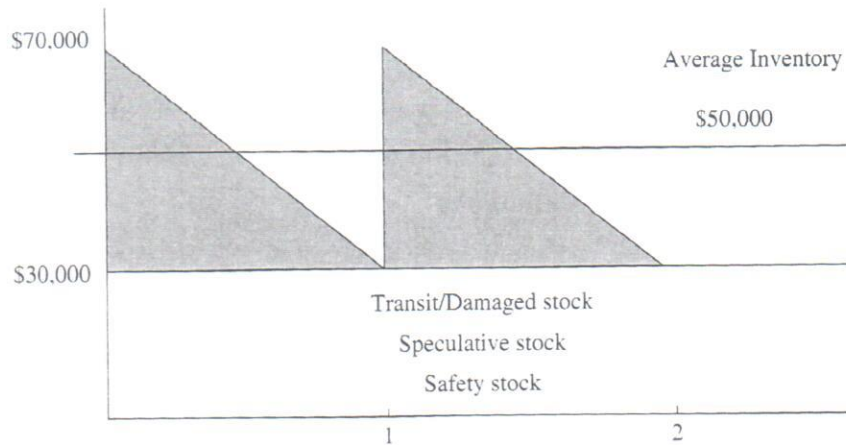
*Service level is a performance target specified by management. It defines inventory performance objectives. Service level is often measured in terms of an order cycle time, case fill rate, line fill rate, order fill rate, or any combination of these. The **performance cycle** is the elapsed time between the release of a purchase order by a buyer to the receipt of shipment. A **case fill rate** is the percent of cases or units ordered that are shipped as requested. For example, a 95 percent case fill rate indicates that, on average, 95 cases out of 100 are filled from available stock. The remaining five cases are back-ordered or deleted. The **line fill rate** is the percent of order lines filled completely. **Order fill** is the percent of customer orders filled completely.*

Inventory management is a major element of supply chain logistics strategy that must be integrated to achieve overall service objectives. While one strategy to achieve a high service level is to increase inventory, an alternative approach may be the use of faster or more reliable transportation and collaboration with customers and service providers to reduce uncertainty.

Average Inventory

The materials, components, work-in-process, and finished product typically stocked in the logistical system are referred to as **inventory** and the typical amount across time is referred to as **average inventory**. From a policy viewpoint, target inventory levels must be planned for each facility. Figure 7.1 illustrates the performance cycles for one item at one location. At the maximum, the facility has in stock during the normal performance cycle \$70,000 and a minimum of \$30,000. The difference between these two levels, \$40,000 (\$70,000 - \$30,000), is the order quantity, resulting in a cycle inventory of \$20,000. Cycle inventory or base stock

FIGURE 7.1
Inventory Cycle for
Typical Product



is the portion of average inventory that results from replenishment. Stock level is at a maximum following stock receipt from the supplier. Customers deplete inventory until the stock level reaches its minimum. Prior to the stock level reaching the minimum, a replenishment order is initiated so that inventory will arrive before an out-of-stock occurs. The replenishment order must be initiated when the available inventory is less than or equal to forecasted demand during the performance cycle time. The amount ordered for replenishment is termed the **order quantity**. Given this basic order formulation, average cycle inventory or base stock equals one-half order quantity.

The **transit inventory** represents the amount typically in transit between facilities or on order but not received. The **obsolete inventory** is the stock that is out-of-date or that has not experienced recent demand. Eventually, it is donated, destroyed, or sold at a loss. **Speculative inventory** is bought prior to need to hedge a currency exchange or to take advantage of a discount.

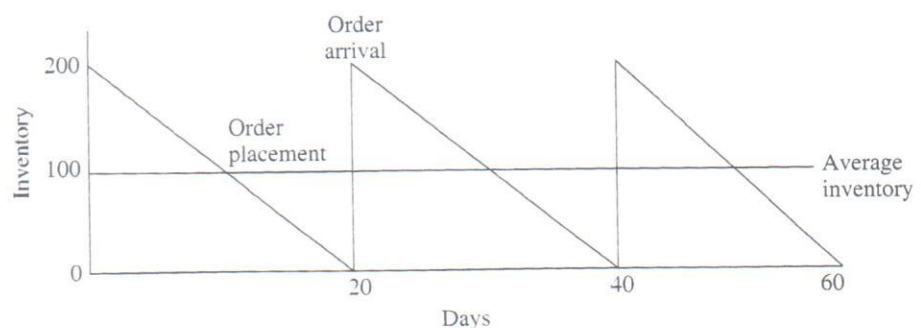
The remainder of inventory in the typical logistics system is **safety stock**. Safety stock is maintained in a logistical system to protect against demand and performance cycle uncertainty. Safety stock is used only near the end of replenishment cycles when uncertainty has caused higher-than-expected demand or longer-than-expected performance cycle times. Thus, typical average inventory is **one-half order quantity plus safety stock and in-transit stock**.

Average Inventory across Multiple Performance Cycles

In initial policy formulation, it is necessary to determine how much inventory to order at a specified time. To illustrate, assume the replenishment performance cycle is a constant 10 days and daily sales rate is 10 units per day. Also assume the order quantity is 200 units.

Figure 7.2 illustrates this relationship. This type of chart is referred to as a **sawtooth diagram** because of the series of right triangles. Since complete certainty exists with

FIGURE 7.2
Inventory
Relationship for
Constant Sales and
Performance Cycle



respect to usage and performance cycle, orders are scheduled to arrive just as the last unit is sold. Thus, no safety stock is necessary. Since the rate of sale in the example is 10 units per day and it takes 10 days to complete inventory replenishment, a sound reorder policy might be to order 200 units every 20 days. Given these conditions, common terminology related to policy formulation can be specified.

First, the **reorder point** is specified as 100 units on hand. The reorder point defines when a replenishment order is initiated. In this example, whenever the quantity on hand drops below 100, an additional order for 200 units is placed. The result of this policy is that daily inventory level ranges from a maximum of 200 to a minimum of zero over the performance cycle.

Second, average inventory is 100 units, since stock on hand exceeds 100 units one-half of the time, or for 10 days, and is less than 100 units one-half of the time. In fact, average inventory is equal to one-half the 200-unit order quantity.

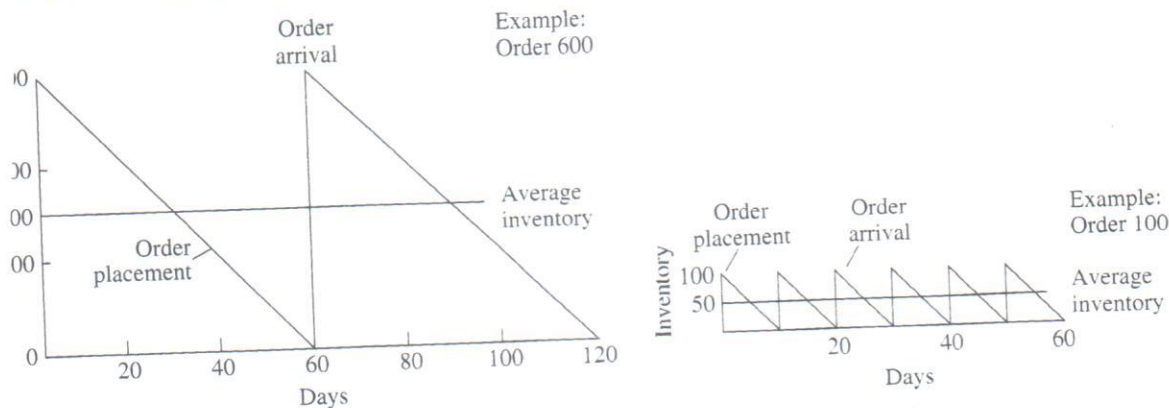
Third, assuming a work year of 240 days, 12 purchases will be required during the year. Therefore, over a period of 1 year, 200 units will be purchased 12 times for a total of 2400 units. Sales are expected to equal 10 units per/day over 240 days for a total of 2400 units. As discussed above, average inventory is 100 units. Thus, **inventory turns** will be 24 (2400 total sales/100 units of average inventory).

In time, the sheer boredom of such routine operations would lead management to ask some questions concerning the arrangement. What would happen if orders were placed more frequently than once every 20 days? Why not order 100 units every 10 days? Why order as frequently as every 20 days? Why not reorder 600 units once every 60 days? Assuming that the inventory performance cycle remains a constant 10 days, what would be the impact of each of these alternative ordering policies on reorder point, average base inventory, and inventory turnover?

The policy of ordering a smaller volume of 100 units every 10 days means that two orders would always be outstanding. Thus, the reorder point would remain 100 units on hand or on order to service average daily sales of 10 units over the 20-day inventory cycle. However, average inventory on hand would drop to 50 units, and inventory turnover would increase to 48 times per year. The policy of ordering 600 units every 60 days would result in an average base inventory of 300 units and a turnover of approximately eight times per year. These alternative ordering policies are illustrated in Figure 7.3.

The figure illustrates that average inventory is a function of the reorder quantity. Smaller replenishment order quantities do result in lower average inventory, but there are other factors such as performance cycle uncertainty, purchasing discounts, and transportation economies that are important when determining order quantity.

E 7.3 Alternative Order Quantity and Average Inventory



An exact order quantity policy can be determined by balancing the cost of ordering and the cost of maintaining average inventory. The **Economic Order Quantity (EOQ)** model provides a specific quantity balancing of these two critical cost components. By determining the EOQ and dividing it into annual demand, the frequency and size of replenishment orders minimizing the total cost of cycle inventory are identified. Prior to reviewing EOQ, it is necessary to identify costs typically associated with ordering and maintaining inventory.

Inventory Carrying Cost

Inventory carrying cost is the expense associated with maintaining inventory. Inventory expense is calculated by multiplying annual inventory carrying cost percent by average inventory value. Standard accounting practice is to value inventory at purchase or standard manufacturing cost rather than at selling price.

Assuming an annual inventory carrying cost percentage of 20 percent, the annual inventory expense for an enterprise with \$1 million in average inventory would be \$200,000 ($20\% \times \$1,000,000$). While the calculation of inventory carrying expense is basic, determining the appropriate carrying cost percent is less obvious.

Determining carrying cost percent requires assignment of inventory-related costs. Financial accounts relevant to inventory carrying cost percent are capital, insurance, obsolescence, storage, and taxes. While cost of capital is typically based on managerial policy, expense-related taxes, insurance, obsolescence, and storage varies depending on the specific attributes of individual products.

Capital

The appropriate charge to place on capital invested in inventory varies widely. Capital assessments range from the prime interest rate to a higher managerially determined percent. The logic for using the prime interest rate or a specified rate pegged to the prime rate is that cash to replace capital invested in inventory can be obtained in the money markets at that rate. Higher managerially specified capital costs are based on expected or target return on investment for capital deployed. Such target rates are typically termed **hurdle rates**.

The cost of capital may vary significantly by firm and industry. Firms that are aggressive in uses of cash will typically employ a higher cost of capital percentage. Similarly, industries with high value or short life cycle product will employ a higher cost of capital to drive lower inventories.

Confusion often results from the fact that senior management frequently does not establish a clear-cut capital cost policy. For supply chain logistics planning, the cost of capital must be clearly specified since it has significant impact on system design and performance.

Taxes

Local taxing authorities in many areas assess taxes on inventory held in warehouses. The tax rate and means of assessment vary by location. The tax expense is usually a direct levy based on inventory level on a specific day of the year or average inventory level over a period of time.

Insurance

Insurance cost is an expense based upon estimated risk or loss over time. Loss risk depends on the product and the facility storing the product. For example, high-value products that are easily stolen and hazardous products result in high insurance cost. Insurance cost is also influenced by facility characteristics such as security cameras and sprinkler systems

that might help reduce risk. Since September 11, 2001, issues related to terrorist risk have become of greater concern in supply chain design.

Obsolescence

Obsolescence cost results from deterioration of product during storage. A prime example of obsolescence is product that ages beyond recommended sell-by date, such as food and pharmaceuticals. Obsolescence also includes financial loss when a product no longer has fashion appeal or no longer has any demand. Obsolescence costs are typically estimated on the basis of past experience concerning markdowns, donations, or quantity destroyed. This expense is the percent of average inventory value declared obsolete each year.

Storage

Storage cost is facility expense related to product holding rather than product handling. Storage cost must be allocated on the requirements of specific products since it is not related directly to inventory value. In public or contract warehouses, storage charges are billed on an individual basis. The cost of total annual occupancy for a given product can then be assigned by multiplying the average daily physical space occupied by the standard cost factor for a specified time. This figure can then be divided by the total number of units of merchandise processed through the facility to determine average storage cost per merchandise unit.

Table 7.2 illustrates the components of annual inventory carrying cost and typical range of component costs. It should be clear that the final carrying cost percent used by a firm is a matter of managerial policy. Decisions regarding inventory cost are important because they trade off against other logistics cost components in system design and operating decisions.

Planning Inventory

Inventory planning consists of determining when and how much to order. When to order is determined by average and variation in demand and replenishment. How much to order is determined by the order quantity. Inventory control is the process of monitoring inventory status.

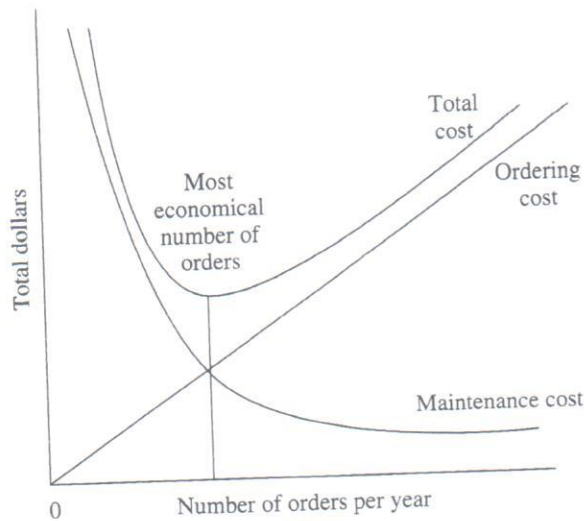
When to Order

As discussed earlier, the reorder point defines when a replenishment shipment should be initiated. A reorder point can be specified in terms of units or days' supply. This discussion focuses on determining reorder points under conditions of demand and performance cycle certainty.

TABLE 7.2
Inventory Carrying
Cost Components

Element	Average Percent	Percent Ranges
Cost of capital	10.00%	4–40%
Taxes	1.00	.5–2
Insurance	.05	0–2
Obsolescence	1.20	.5–2
Storage	2.00	0–4
Totals	14.25%	5–50%

Order



The most efficient method for calculating EOQ is mathematical. Earlier in this chapter a policy dilemma regarding whether to order 100, 200, or 600 units was presented. The answer can be found by calculating the applicable EOQ for the situation. Table 7.3 contains the necessary information.

To make the appropriate calculations, the standard formulation for EOQ is:

$$EOQ = \sqrt{\frac{2C_o D}{C_i U}}$$

where

EOQ = Economic order quantity;

C_o = Cost per order;

C_i = Annual inventory carrying cost;

D = Annual sales volume, units; and

U = Cost per unit.

Substituting from Table 7.3,

$$\begin{aligned} EOQ &= \sqrt{\frac{2 \times 19 \times 2400}{0.20 \times 5.00}} \\ &= \sqrt{91,200} \\ &= 302 \text{ (round to 300).} \end{aligned}$$

Total ordering cost would amount to \$152 ($2400/300 \times \19.00), and inventory carrying cost to \$150 [$300/2 \times (5 \times 0.20)$]. Thus, rounding to allow ordering in multiples of 100 units, annual reordering, and inventory carrying cost have been equated.

LE 7.3
ors for
etermining EOQ

Annual demand volume	2400 units
Unit value at cost	\$5.00
Inventory carrying cost percent	20% annually
Ordering cost	\$19.00 per order

To benefit from the most economical purchase arrangement, orders should be placed in the quantity of 300 units rather than 100, 200, or 600. Thus, over the year, eight orders would be placed and average base inventory would be 150 units. Referring back to Figure 7.4, the impact of ordering in quantities of 300 rather than 200 can be identified. An EOQ of 300 implies that additional inventory in the form of base stock has been introduced into the system. Average inventory has been increased from 100 to 150 units on hand.

While the EOQ model determines the optimal replenishment quantity, it does require some rather stringent assumptions. The major assumptions of the simple EOQ model are: (1) all demand is satisfied; (2) rate of demand is continuous, constant, and known; (3) replenishment performance cycle time is constant and known; (4) there is a constant price of product that is independent of order quantity or time; (5) there is an infinite planning horizon; (6) there is no interaction between multiple items of inventory; (7) no inventory is in transit; and (8) no limit is placed on capital availability. The constraints imposed by some of these assumptions can be overcome through computational extensions; however, the EOQ concept illustrates the importance of the trade-offs associated with inventory carrying and replenishment ordering cost.

Relationships involving the inventory performance cycle, inventory cost, and economic order formulations are useful for guiding inventory planning. First, the EOQ is found at the point where annualized order placement cost and inventory carrying cost are equal. Second, average base inventory equals one-half order quantity. Third, the value of the inventory unit, all other things being equal, will have a direct relationship with replenishment order frequency. In effect, the higher the product value, the more frequently it will be ordered.

While the EOQ formulation is relatively straightforward, there are other factors that must be considered in actual application. These factors refer to various adjustments necessary to take advantage of special purchase situations and unitization characteristics. Three typical adjustments are volume transportation rates, quantity discounts, and other EOQ adjustments.

Volume Transportation Rates

In the EOQ formulation, no consideration was given to the impact of transportation cost upon order quantity. Regardless of whether product is sold on a delivered basis or ownership is transferred at origin, the cost of transportation must be paid by supply chain participants. Collaborative efforts to order in quantities that minimize total cost are essential to sound logistical arrangements.

As a general rule, the greater the weight of an order, the lower the cost per pound of transportation from any origin to destination.² A freight-rate discount for larger shipments is common across all transportation modes. Thus, all other things being equal, supply chain arrangements should facilitate quantities that offer maximum transportation economies. Such quantities may be larger than the EOQ purchase quantity. Increasing order size has a twofold impact upon inventory cost. Assume for purposes of illustration that the most desirable transportation rate is obtained when a quantity of 480 is ordered, as compared to the EOQ-recommended order of 300 calculated earlier. The first impact of the larger order is to increase the average base inventory from 150 to 240 units. Thus, ordering in larger quantities increases inventory carrying cost.

The second impact is a decrease in the number of orders required to satisfy annual requirements. Decreased number of orders increases the shipment size, facilitating lower per-unit transportation cost.

To complete the analysis it is necessary to formulate the total cost with and without transportation savings. While this calculation can be directly made by modification of the EOQ formulation, direct comparison provides a more insightful answer. The only additional data

² To determine transportation rates, the unit quantity must be converted to weight.

TABLE 7.4
EOQ Data
Requirements for
Consideration of
Transportation
Economies

Annual demand volume	2400 units
Unit value at cost	\$5.00
Inventory carrying cost percentage	20% annually
Ordering cost	\$19.00 per order
Small shipment rate	\$1.00 per unit
Large shipment rate	\$0.75 per unit

required are the applicable freight rate for ordering in quantities of 300 and 480. Table 7.4 provides the data necessary to complete the analysis.

Table 7.5 illustrates total cost analysis. Reducing total annual cost by purchasing 480 units five times per year rather than the original EOQ solution of 300 units eight times per year results in approximately a \$570 savings.

The impact of volume transportation rates upon total cost of procurement cannot be neglected. In the example above, the equivalent rate per unit dropped from \$1 to \$0.75, or by 25 percent. Thus, any EOQ must be tested for transportation cost sensitivity across a range of weight breaks.

Another point illustrated in the data in Table 7.5 is the fact that rather substantial changes in the size of an order and the number of orders placed per year result in only a modest change in the total ordering and inventory carrying cost. The EOQ quantity of 300 had a total annual cost of \$302, whereas the revised order quantity had a comparative cost of \$335.

EOQ formulations are much more sensitive to significant changes in order cycle or frequency. Likewise, substantial changes in cost factors are necessary to significantly change the economic order quantity.

Finally, two factors regarding inventory cost under conditions of origin purchase are noteworthy. FOB (Free On Board) origin purchase means that the buyer is responsible for freight cost and risk while the product is in transit. It follows that any change in weight break leading to a shipment method with a different transit time should be considered, using the added cost or savings as appropriate in a total cost analysis.

Second, the transportation cost must be added to the purchase price to determine the value of goods tied up in inventory. Once the inventory has been received, the cost of the product must be increased to reflect the inbound transportation.³

Quantity Discounts

Purchase quantity discounts represent an EOQ extension analogous to volume transportation rates. Table 7.6 illustrates a sample schedule of discounts. Quantity discounts can be handled directly with the basic EOQ formulation by calculating total cost at any given volume-related purchase price, as in the process used in calculating transportation rate impact, to determine associated EOQs. If the discount at any associated quantity is sufficient to offset the added inventory carrying cost less the reduced cost of ordering, then the quantity discount is a viable choice. It should be noted that quantity discounts and volume

³ Pricing is discussed in greater detail in Chapter 9.

TABLE 7.5
Volume
Transportation Rate
Modified EOQ

	Alternative 1: EOQ ₁ = 300	Alternative 2: EOQ ₂ = 480
Inventory Carrying Cost	\$150	\$240
Ordering Cost	\$152	\$95
Transportation Cost	\$2,400	\$1,800
Total Cost	\$2,702	\$2,135

TABLE 7.6
Example of Quantity
Discounts

Cost	Quantity Purchased
\$5.00	1-99
4.50	100-200
4.00	201-300
3.50	301-400
3.00	401-500

transportation rates each drive larger purchase quantities. This does not necessarily mean that the lowest total cost purchase will always be a larger quantity than would otherwise be the case under basic EOQ.

Other EOQ Adjustments

A variety of other special situations may justify adjustments to the basic EOQ. Examples are (1) production lot size, (2) multiple-item purchase, (3) limited capital, (4) dedicated trucking, and (5) unitization. Production lot size refers to the most economical quantities from a manufacturing perspective. Multiple-item purchase refers to situations when more than one product are bought concurrently, so quantity and transportation discounts must consider the impact of product combinations. Limited capital describes situations with budget limitations for total inventory investment. Since the multiple product order must be made within the budget limitations, order quantities must recognize the need to allocate the inventory investment across the product line. Dedicated trucking can influence order quantity since the truck has a fixed cost consideration.⁴ Once it is decided to use a dedicated fleet to transport replenishment product, the enterprise should try to purchase in quantities that fully use available capacity. Back haul capacity availability may justify purchasing products earlier than otherwise determined by EOQ considerations.

Another consideration when determining replenishment order quantity is unitization. Many products are stored and moved in standard units such as cases or pallets. Since these standardized units are often designed to fit transportation vehicles, there may be significant diseconomies when the EOQ does not reflect standard units. As an example, suppose that a full pallet quantity is 200 units of a specified product. Using an EOQ of 300 units would require shipments of 1.5 pallets. From a handling or transportation utilization perspective, it is probably more effective to order either one or two pallets alternately or permanently.

Managing Uncertainty

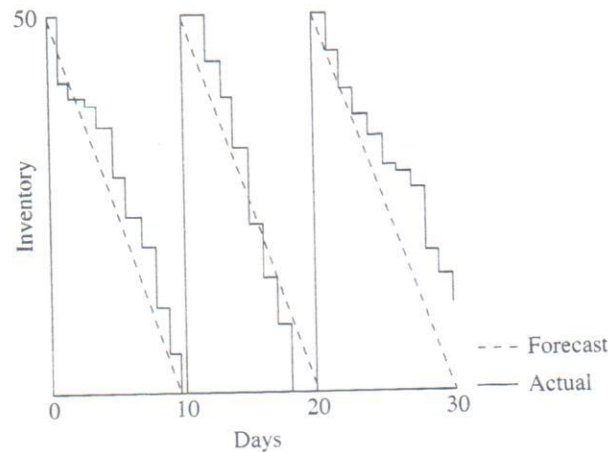
To understand basic principles, it is useful to understand inventory relationships under conditions of certainty. Formulation of inventory policy must consider uncertainty. Two types of uncertainty directly impact inventory policy. **Demand uncertainty** is rate of sale during inventory replenishment. **Performance cycle uncertainty** involves inventory replenishment time variation.

Demand Uncertainty

Sales forecasting estimates unit demand during the inventory replenishment cycle. Even with good forecasting, demand during replenishment cycle typically exceeds or falls short of what is planned. To protect against a stockout when demand exceeds forecast, safety

⁴ See Chapter 9. In such situations, the cost of money invested in inventory should be appropriately charged when the goods are paid for at the origin.

FIGURE 7.5
Inventory
Relationship,
Demand Uncertainty,
and Constant
Performance Cycle



stock is added to base inventory. Under conditions of demand uncertainty, average inventory represents one-half order quantity plus safety stock. Figure 7.5 illustrates the inventory performance cycle under conditions of demand uncertainty. The dashed line represents the forecast. The solid line illustrates inventory on hand across multiple performance cycles. The task of planning safety stock requires three steps. First, the likelihood of stockout must be gauged. Second, demand during a stockout period must be estimated. Finally, a policy decision is required concerning the desired level of stockout protection.

Assume for purposes of illustration that the inventory performance cycle is 10 days. History indicates daily sales range from 0 to 10 units with average daily sales of 5 units. The economic order is assumed to be 50, the reorder point is 50, the planned average inventory is 25, and sales during the performance cycle are forecasted to be 50 units.

During the first cycle, although daily demand experienced variation, the average of 5 units per day was maintained. Total demand during cycle 1 was 50 units, as expected. During cycle 2, demand totaled 50 units in the first 8 days, resulting in a stockout. Thus, no sales were possible on days 9 and 10. During cycle 3, demand reached a total of 39 units. The third performance cycle ended with 11 units remaining in stock. Over the 30-day period total sales were 139 units, for average daily sale of 4.6 units.

From the history recorded in Table 7.7, it is observed that stockouts occurred on 2 of 30 total days. Since sales never exceed 10 units per day, no possibility of stockout exists on the first 5 days of the replenishment cycle. Stockouts were possible on days 6 through 10 on the remote possibility that demand during the first 5 days of the cycle averaged

TABLE 7.7
Typical Demand
Experience during
Three Replenishment
Cycles

Day	Forecast Cycle 1		Stockout Cycle 2		Overstock Cycle 3	
	Demand	Accumulated	Demand	Accumulated	Demand	Accumulated
1	9	9	0	0	5	5
2	2	11	6	6	5	10
3	1	12	5	11	4	14
4	3	15	7	18	3	17
5	7	22	10	28	4	21
6	5	27	7	35	1	22
7	4	31	6	41	2	24
8	8	39	9	50	8	32
9	6	45	Stockout	50	3	35
10	5	50	Stockout	50	4	39

10 units per day and no inventory was carried over from the previous period. Since during the three performance cycles 10 units were sold on only one occasion, it is apparent that the real risk of stockout occurs only during the last few days of the performance cycle, and then only when sales exceed the average by a substantial margin.⁵ Some approximation is possible concerning sales potential for days 9 and 10 of cycle 2. A maximum of 20 units might have been sold if inventory had been available. On the other hand, it is remotely possible that even if stock had been available, no demand would have occurred on days 9 and 10. For average demand of 4 to 5 units per day, a reasonable appraisal of lost sales is 8 to 10 units.

It should be apparent that the risk of stockouts created by variations in sales is limited to a short time and includes a small percentage of total sales. Although the sales analysis presented in Table 7.7 helps achieve an understanding of the opportunity, the appropriate course of action is still not clear. Statistical probability can be used to assist management in planning safety stock.

The sales history over the 30-day period has been aggregated in Table 7.8 as a frequency distribution. The main purpose of a frequency distribution is to observe variations around the average daily demand. Given an expected average of 5 units per day, demand exceeded average on 11 days and was less than average on 12 days. An alternative way of illustrating a frequency distribution is by a bar chart, as in Figure 7.6.

Given the historical frequency of demand, it is possible to calculate the safety stock necessary to provide a specified degree of stockout protection. Probability theory is based on the random chance of a specific occurrence within a large number of occurrences. The situation illustrated uses a 28-day sample. In actual application, a larger sample size would be desirable.

The probability of occurrences assumes a pattern around a measure of central tendency, which is the average value of all occurrences. While a number of frequency distributions can be used in inventory management, the most common is the **normal distribution**.

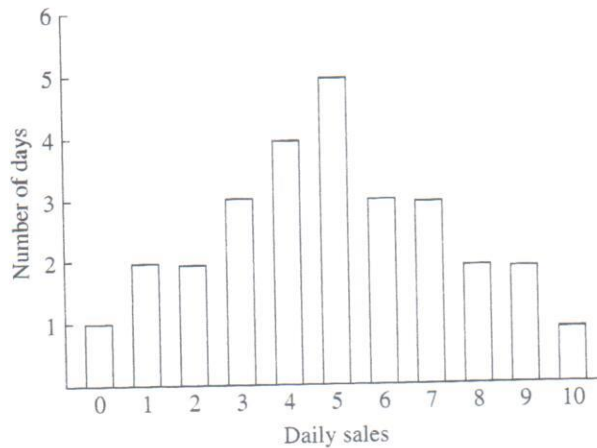
A normal distribution is characterized by a symmetrical bell-shaped curve, illustrated in Figure 7.7. The essential characteristic of a normal distribution is that the three measures of central tendency have equal value. The **mean** (average) value, the **median** (middle) observation, and the **mode** (most frequently observed) all have the same value. When these three measures are nearly identical, the frequency distribution is **normal**.

⁵ In this example, daily statistics are used. An alternative, which is technically more correct from a statistical viewpoint, is to utilize demand over multiple performance cycles. The major limitation of order cycles is the length of time and difficulty required to collect the necessary data.

TABLE 7.8
Frequency of
Demand

Daily Demand (in units)	Frequency (days)
Stockout	2
0	1
1	2
2	2
3	3
4	4
5	5
6	3
7	3
8	2
9	2
10	1

FIGURE 7.6
Historical Analysis of
Demand and History

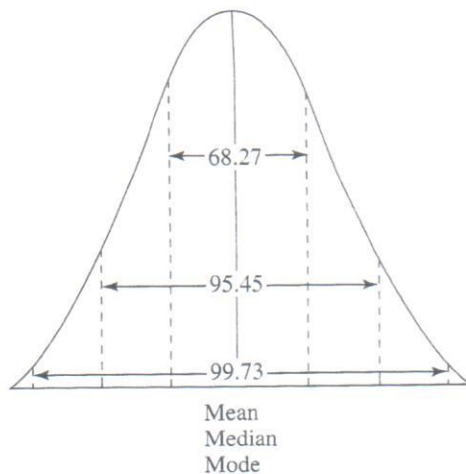


The basis for predicting demand during a performance cycle using a normal distribution is the **standard deviation** of observations around the three measures of central tendency. The standard deviation is the dispersion of observations within specified areas under the normal curve. For the inventory management application, the observation is unit sales per day and the dispersion is the variation in daily sales. Within 1 standard deviation, 68.27 percent of all events occur. This means that 68.27 percent of the days during a performance cycle will experience daily sales within ± 1 standard deviation of the average daily sales. Within ± 2 standard deviations, 95.45 percent of all observations occur. At ± 3 standard deviations, 99.73 percent of all observations are included. In terms of inventory policy, the standard deviation provides a method of estimating the safety stock required to achieve a specified degree of out-of-stock protection.

The first step in setting safety stock is to calculate the standard deviation. Most calculators and spreadsheets calculate standard deviation, but if one of these aids is not available, another method to compute the standard deviation is:

$$\sigma = \sqrt{\frac{\sum F_i D_i^2}{n}}$$

FIGURE 7.7
Normal Distribution



where

σ = Standard deviation;

F_i = Frequency of observation i ;

D_i = Deviation of observation from mean for observation i ; and

n = Total observations available.

The necessary data to determine standard deviation are contained in Table 7.9.

The standard deviation of the data in Table 7.9 is rounded to 3 units. When setting safety stocks, 2 standard deviations of protection, or 6 units, would protect against 95.45 percent of all observations included in the distribution. However, the only situations of concern in determining safety stock requirements are observations that exceed the mean value. No problem exists concerning inventory to satisfy demand equal to or below the average. Thus, on 50 percent of the days, no safety stock is required. Safety stock protection at the 95 percent level will, in fact, protect against 97.72 percent of all possible observations. The 95 percent coverage will cover all situations when daily demand is ± 2 standard deviations of the average plus the 2.72 percent of the time when demand is more than 2 standard deviations below the mean. This added benefit results from what is typically called a **one-tail** statistical application.

The above example illustrates how statistical probability can assist with the quantification of demand uncertainty, but demand conditions are not the only source of uncertainty. Performance cycles can also vary.

Performance Cycle Uncertainty

Performance cycle uncertainty means operations cannot assume consistent delivery. The planner should expect that actual performance cycle experience will cluster near the expected value and be skewed toward delayed delivery.

Table 7.10 presents a sample frequency distribution across multiple performance cycles. Although 10 days is the most frequent, replenishment experience ranges from 6 to 14 days. If the performance cycle follows a normal distribution, an individual performance cycle would be expected to fall between 8 and 12 days 68.27 percent of the time.

TABLE 7.9
Calculation of
Standard Deviation
of Daily Demand

Units	Frequency (F_i)	Deviation from Mean (D_i)	Deviation Squared (D_i^2)	$F_i D_i^2$
0	1	-5	25	25
1	2	-4	16	32
2	2	-3	9	18
3	3	-2	4	12
4	4	-1	1	4
5	5	0	0	0
6	3	+1	1	3
7	3	+2	4	12
8	2	+3	9	18
9	2	+4	16	32
10	1	+5	25	25
$n = 28$	$\bar{s} = 5$			$\sum F_i D_i^2 = 181$

$$\sigma = \sqrt{\frac{F_i D_i^2}{N}} = \sqrt{\frac{181}{28}} = 2.54$$

E 7.10 Calculation of Standard Deviation of Replenishment Cycle Duration

Performance Cycle (days)	Frequency (F_i)	Deviation from Mean (D_i)	Deviation Squared (D_i^2)	$F_i D_i^2$
6	2	-4	16	32
7	4	-3	9	36
8	6	-2	4	24
9	8	-1	1	8
10	10	0	0	0
11	8	+1	1	8
12	6	+2	4	24
13	4	+3	9	36
14	2	+4	16	32
				$\sum F_i D_i^2 = 200$

$$N = 50 \quad t = 10$$

$$\sigma = \sqrt{\frac{F_i D_i^2}{N}} = \sqrt{\frac{200}{50}} = \sqrt{4} = 2 \text{ days}$$

From a practical viewpoint, when cycle days drop below 10, there is not any need for safety stock. If the performance cycle were consistently below the planned performance cycle, then adjustment of expected duration would be in order. The situation of most immediate concern occurs when the duration of the performance cycle exceeds 10 days.

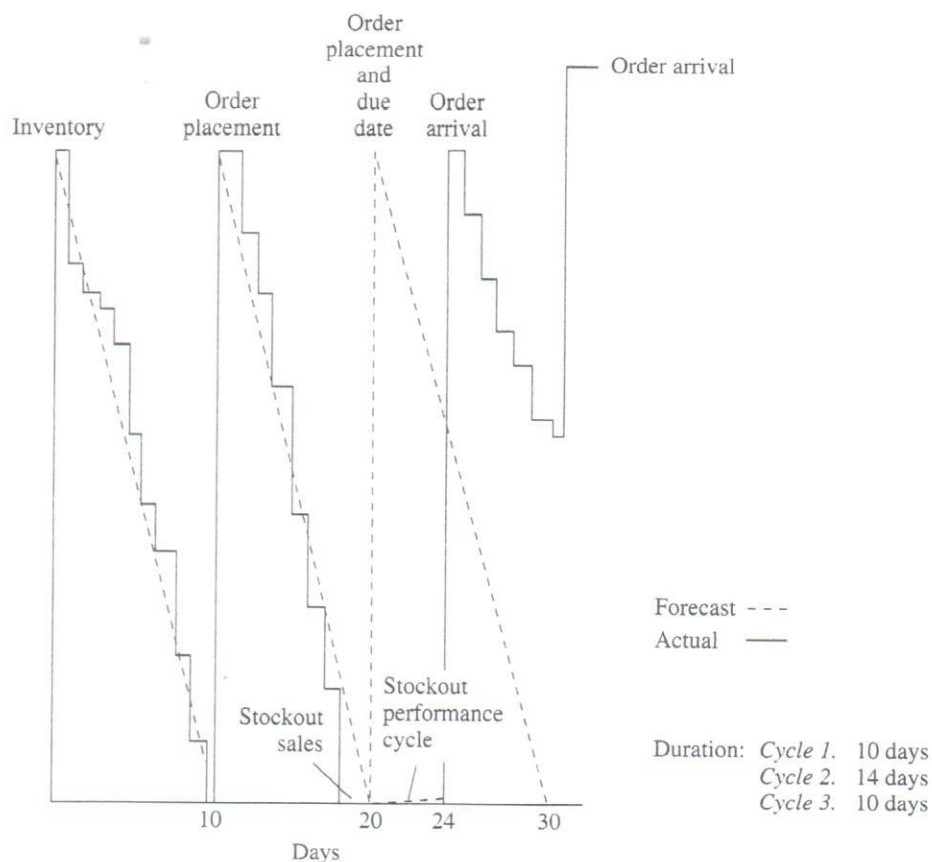
From the viewpoint of the probability of exceeding 10 days, the frequency of such occurrences, from the data in Table 7.10, can be restated in terms of performance cycles greater than 10 days and equal to or less than 10 days. In the example data, the standard deviation would not change because the distribution is normal. However, if the actual experience has been skewed in excess of the expected cycle duration, then a **Poisson distribution** may have been more appropriate. In Poisson frequency distributions, the standard deviation is equal to the square root of the mean. As a general rule, the smaller the mean, the greater the degree of skewness.

Safety Stock with Combined Uncertainty

The typical situation confronting the inventory planner is illustrated in Figure 7.8, where both demand and performance cycle uncertainties exist. Planning for both demand and performance cycle uncertainty requires combining two independent variables. The duration of the cycle is, at least in the short run, independent of the daily demand. However, in setting safety stocks, the joint impact of the probability of both demand and performance cycle variation must be determined. Table 7.11 presents a summary of sales and replenishment cycle performance. The key to understanding the potential relationships of the data in Table 7.11 is the 10-day performance cycle. Total demand during the 10 days potentially ranges from 0 to 100 units. On each day of the cycle, the demand probability is independent of the previous day for the entire 10-day duration. Assuming the full range of potential situations illustrated in Table 7.11, total sales during a performance cycle could range from 0 to 140 units. With this basic relationship between the two types of uncertainty in mind, safety stock requirements can be determined by either numerical or convolution procedures.

The **numerical compounding** of two independent variables involves multinomial expansion. This type of procedure requires extensive calculation. A direct method is to

FIGURE 7.8
Combined Demand
and Performance
Cycle Uncertainty



determine the standard deviations of demand and performance cycle uncertainty and then to approximate the combined standard deviation using the convolution formula:

$$\sigma_c = \sqrt{TS_s^2 + D^2S_t^2}$$

TABLE 7.11
Frequency
Distribution—
Demand and
Replenishment
Uncertainty

Demand Distribution		Replenishment Cycle Distribution	
Daily Sales	Frequency	Days	Frequency
0	1	6	2
1	2	7	4
2	2	8	6
3	3	9	8
4	4	10	10
5	5	11	8
6	3	12	6
7	3	13	4
8	2	14	2
9	2		
10	1		
	$n = 28$	$n = 50$	
	$D = 5$	$T = 10$	
	$S_s = 2.54$	$S_t = 2$	

where

σ_c = Standard deviation of combined probabilities:

T = Average performance cycle time:

S_T = Standard deviation of the performance cycle:

D = Average daily sales; and

S_S = Standard deviation of daily sales.

Substituting from Table 7.11,

$$\begin{aligned}\sigma_c &= \sqrt{10.00(2.54)^2 + (5.00)^2(2)^2} \\ &= \sqrt{64.52 + 100} = \sqrt{164.52} \\ &= 12.83 \text{ (round to 13).}\end{aligned}$$

This formulation estimates the convoluted or combined standard deviation of T days with an average demand of D per day when the individual standard deviations are S_T and S_S , respectively. The average for the combined distribution is the product of T and D , or 50.00 (10.00×5.00).

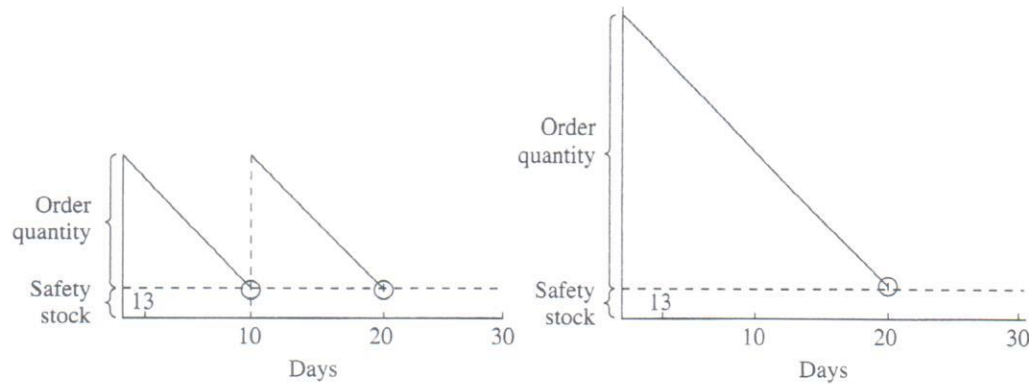
Thus, given a frequency distribution of daily sales from 0 to 10 units per day and a range in replenishment cycle duration of 6 to 14 days, 13 units (1 standard deviation multiplied by 13 units) of safety stock is required to protect 84.14 percent of all performance cycles. To protect at the 97.72 percent level, a 26-unit safety stock is necessary. These levels assume a one-tail distribution since it is not necessary to protect against leadtime demand below average.

It is important to note that the specific event being protected against is a stockout during the performance cycle. The 84.14 and 97.72 percent levels are not product availability levels. These percentages reflect the probability of a stockout during a given order cycle. For example, with a 13-unit safety stock, stockouts would be expected to occur during 15.86 ($100 - 84.14$) percent of the performance cycles. Although this percentage provides the probability of a stockout, it does not estimate magnitude. The relative stockout magnitude indicates the percentage of units stocked out relative to overall demand.

Still assuming the replenishment order quantity of 50 units, average inventory requirements would be 25 units if no safety stock were desired. The average inventory with 2 standard deviations of safety stock is 51 units [$25 + (2 \times 13)$]. This inventory level would protect against a stockout during 97.72 percent of the performance cycles. Table 7.12 summarizes the alternatives confronting the planner in terms of assumptions and corresponding impact on average inventory.

TABLE 7.12
Average Inventory
Impact Resulting
from Changes in
EOQ

	Order Quantity	Safety Stock	Average Inventory
Assume constant S sales and constant T performance cycle	50	0	25
Assume demand protection $+2\sigma$ and constant T performance cycle	50	6	31
Assume constant S demand and $+2\sigma$ performance cycle protection	50	20	45
Assume joint $+2\sigma$ for demand and performance cycle	50	26	51

FIGURE 7.9 Impact of Order Quantity on Stockout Magnitude

Estimating Fill Rate

The fill rate is the magnitude rather than the probability of a stockout. The case fill rate is the percentage of units that can be filled when requested from available inventory. Figure 7.9 graphically illustrates the difference between stockout probability and stockout magnitude. Both illustrations in Figure 7.9 have a safety stock of 1 standard deviation or 13 units. For both situations, given any performance cycle, the probability of a stockout is 31.73 percent. However, during a 20-day period, the figure illustrates two instances where the stock may be depleted. These instances are at the ends of the cycle. If the order quantity is doubled, the system has the possibility of stocking out only once during the 20-day cycle. So, while both situations face the same demand pattern, the first one has more stockout opportunities and potential. In general, for a given safety stock level, increasing the replenishment order quantity decreases the relative magnitude of potential stockouts and conversely increases customer service availability.

The mathematical formulation of the relationship is:

$$SL = 1 - \frac{f(k)\sigma_c}{Q},$$

where

SL = The stockout magnitude (the product availability level);

$f(k)$ = A function of the normal loss curve which provides the area in a right tail of a normal distribution;

σ_c = The combined standard deviation considering both demand and replenishment cycle uncertainty; and

Q = The replenishment order quantity.

To complete the example, suppose a firm desired 99 percent product availability or case fill rate. Assume the Q was calculated to be 300 units. Table 7.13 summarizes the required information.

TABLE 7.13
Information for
Determining Required
Safety Stock

Desired Service Level	99%
σ_c	13
Q	300

Since $f(k)$ is the term used to calculate safety stock requirements, the above equation must be solved for $f(k)$ using algebraic manipulation. The result is:

$$f(k) = (1 - SL) \times (Q/\sigma_c).$$

Substituting from Table 7.13,

$$\begin{aligned} f(k) &= (1 - 0.99) \times (300/13) \\ &= 0.01 \times 23.08 = .2308. \end{aligned}$$

The calculated value of $f(k)$ is then compared against the values in Table 7.14 to find the one that most closely approximates the calculated value. For this example, the value of k that fits the condition is 0.4. The required safety stock level is:

$$SS = k \times \sigma_c,$$

where

SS = Safety stock in units;

k = The k factor that corresponds with $f(k)$;

σ_c = The combined standard deviation.

So, substituting in for the example,

$$\begin{aligned} SS &= k \times \sigma_c \\ &= .04 \times 13 = 5.2 \text{ units.} \end{aligned}$$

The safety stock required to provide a 99 percent product fill rate when the order quantity is 300 units is approximately 5 units. Table 7.15 shows how the calculated safety stock and average inventory levels vary for other order quantities. An increased order size can be used to compensate for decreasing the safety stock levels, or vice versa. The existence of such a trade-off implies that there is a combination of replenishment order quantities that will result in desired customer service at the minimum cost.

E 7.14
Integral for
Standardized Normal
Distribution

k	$f(k)$	k	$f(k)$
0.0	.3989	1.6	.0232
0.1	.3509	1.7	.0182
0.2	.3068	1.8	.0143
0.3	.2667	1.9	.0111
0.4	.2304	2.0	.0085
0.5	.1977	2.1	.0065
0.6	.1686	2.2	.0049
0.7	.1428	2.3	.0037
0.8	.1202	2.4	.0027
0.9	.1004	2.5	.0020
1.0	.0833	2.6	.0015
1.1	.0686	2.7	.0011
1.2	.0561	2.8	.0008
1.3	.0455	2.9	.0005
1.4	.0366	3.0	.0004
1.5	.0293	3.1	.0003

TABLE 7.15
Impact of Order
Quantity on Safety
Stock

Order Quantity (Q)	k	Safety Stock	Average Inventory
300	0.40	5	155
200	0.70	8	108
100	1.05	14	64
50	1.40	18	43
25	1.70	22	34

Dependent Demand Replenishment

With respect to dependent demand replenishment, inventory requirements are a function of known events that are not generally random. Therefore, dependent demand does not require forecasting because there is no uncertainty. It follows that no specific safety stock should be necessary to support a time-phased procurement program such as materials requirements planning (MRP).⁶ The basic notion of time phasing is that parts and subassemblies need not be carried in inventory as long as they arrive when needed or just-in-time.

The case for carrying no safety stocks under conditions of dependent demand rests on two assumptions. First, procurement replenishment to support planning is predictable and constant. Second, vendors and suppliers maintain adequate inventories to satisfy 100 percent of purchase requirements. The second assumption may be operationally attained by use of volume-oriented purchase contracts that assure vendors and suppliers of eventual purchase. In such cases the safety stock requirement still exists for the overall supply chain, although the primary responsibility rests with the supplier.

The assumption of performance cycle certainty is more difficult to achieve. Even in situations where dedicated transportation is used, an element of uncertainty is always present. The practical result is that safety stocks do exist in most dependent demand situations.

Three basic approaches have been used to introduce safety stocks into dependent demand situations. First, a common practice is to put **safety time** into the requirements plan. For example, a component is ordered earlier than needed to assure timely arrival. A second approach is to increase the requisition by a quantity specified by some estimate of expected plan error. For example, assume that plan error will not exceed 5 percent. This procedure is referred to as **over-planning top-level demand**. The net result is to increase procurement of all components in a ratio to their expected usage plus a cushion to cover plan error. Components common to different end products or subassemblies covered by the overplanning will naturally experience greater quantity buildups than single-purpose components and parts. The third method is to utilize the previously discussed statistical techniques for setting safety stocks directly to the component rather than to the item of top-level demand.

Inventory Management Policies

Inventory management implements inventory policy. The reactive or pull inventory approach uses customer demand to pull product through the distribution channel. An alternative philosophy is a planning approach that proactively allocates inventory on the basis of forecasted demand and product availability. A third, or hybrid, logic uses a combination of push and pull.

⁶ These concepts are discussed later in this chapter under Requirements Planning.

where

I_{avg} = Average inventory in units:

Q = Order quantity in units:

P = Review period in days:

D = Average daily demand in units:

SS = Safety stock in units.

For the preceding example, the average inventory is calculated as:

$$\begin{aligned} I_{avg} &= Q/2 + (P \times D)/2 + SS \\ &= 300/2 + (7 \times 10)/2 + 0 = 150 + 35 = 185 \text{ units.} \end{aligned}$$

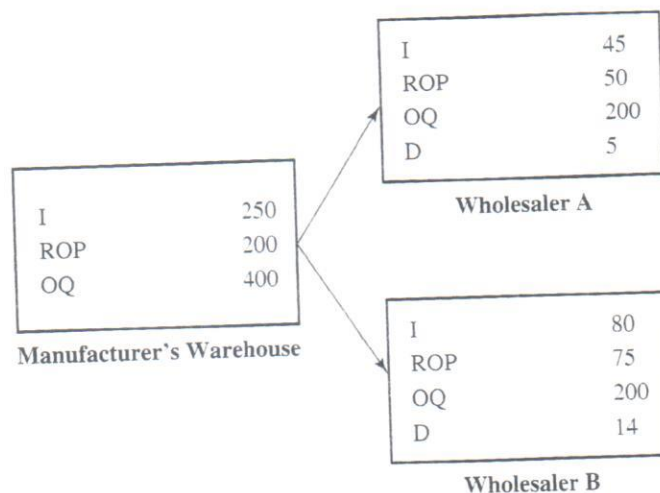
Because of the time interval introduced by periodic review, periodic control systems generally require larger average inventories than perpetual systems.

Reactive Methods

The **reactive** or **pull inventory system**, as the name implies, responds to a channel member's inventory needs by drawing the product through the distribution channel. Replenishment shipments are initiated when available warehouse stock levels fall below a predetermined minimum or order point. The amount ordered is usually based on some lot-sizing formulation, although it may be some variable quantity that is a function of current stock levels and a predetermined maximum level.

The basic perpetual or periodic review process discussed earlier exemplifies a typical reactive system. Figure 7.10 illustrates a reactive inventory environment for a warehouse serving two wholesalers. The figure shows the current inventory (I), reorder point (ROP), order quantity (OQ), and average daily demand (D) for each wholesaler. A review of the wholesaler inventory indicates that a resupply order for 200 units should be placed by wholesaler A from the warehouse. Since current inventory is above ROP for wholesaler B, no resupply action is necessary at this time. However, more thorough analysis illustrates that the independent actions by wholesaler A will likely cause a stockout at wholesaler B within a few days. Wholesaler B will likely stock out because inventory level is close to the reorder point and the supplying warehouse center will not have enough inventory to replenish wholesaler B.

FIGURE 7.10
A Reactive Inventory
Environment



For the previous example, a replenishment order of 200 is placed whenever the sum of on-hand and on-order inventory is less than or equal to 200 units. Since the reorder point equals the order quantity, the previous replenishment shipment would arrive just as the next replenishment is initiated. The average inventory level for a perpetual review system is:

$$I_{\text{avg}} = Q/2 + \text{SS},$$

where

I_{avg} = Average inventory in units;

Q = Order quantity units; and

SS = Safety stock units.

Average inventory for the previous example is calculated as:

$$\begin{aligned} I_{\text{avg}} &= Q/2 + \text{SS} \\ &= 300/2 + 0 = 150 \text{ units.} \end{aligned}$$

Most illustrations throughout this text are based on a perpetual review system with fixed reorder point. The reorder formulation assumes purchase orders will be placed when the reorder point is reached, and the method of control provides a perpetual monitoring of inventory status. If these two assumptions are not satisfied, the control parameters (**RO** and **Q**) determining the perpetual review must be refined.

Periodic Review

Periodic inventory control reviews the inventory status of an item at regular intervals such as weekly or monthly. For periodic review, the basic reorder point must be adjusted to consider the intervals between review. The formula for calculating the periodic review reorder point is:

$$\text{ROP} = D(T + P/2) + \text{SS},$$

where

ROP = Reorder point;

D = Average daily demand;

T = Average performance cycle length;

P = Review period in days; and

SS = Safety stock.

Since inventory counts occur periodically, any item could fall below the desired reorder point prior to the review period. Therefore, the assumption is made that the inventory will fall below ideal reorder status prior to the periodic count approximately one-half of the review times. Assuming a review period of 7 days and using conditions similar to those in the perpetual example, the ROP then would be as follows:

$$\begin{aligned} \text{ROP} &= D(T + P/2) + \text{SS} \\ &= 20(10 + 7/2) + 0 = 20(10 + 3.5) = 270 \text{ units.} \end{aligned}$$

The average inventory formulation for the case of periodic review is:

$$I_{\text{avg}} = Q/2 + (P \times D)/2 + \text{SS},$$

Classical reactive inventory logic is rooted in the following assumptions. First, the system is founded on the basic assumption that all customers, market areas, and products contribute equally to profits.

Second, a reactive system assumes infinite capacity at the source. This assumption implies that product can be produced as desired and stored at the production facility until required throughout the supply chain.

Third, reactive inventory logic assumes infinite inventory availability at the supply location. The combination of assumptions 2 and 3 implies relative replenishment certainty. The reactive inventory logic provides for no back orders or stockouts in processing replenishment orders.

Fourth, reactive decision rules assume that performance cycle time can be predicted and that cycle lengths are independent. This means that each performance cycle is a random event and that extended cycles don't generally occur for subsequent replenishment orders. Although reactive logic assumes no control over cycle times, many managers are, in fact, able to influence performance cycle length through expediting and alternative sourcing strategies.

Fifth, reactive inventory logic operates best when customer demand patterns are relatively stable and consistent. Ideally, demand patterns should be stable over the relevant planning cycle for statistically developed inventory parameters to operate correctly. Most reactive system decision rules assume demand patterns based on standard normal, gamma, or Poisson distributions. When the actual demand function does not resemble one of the above functions, the statistical inventory decision rules based on these assumptions will not operate correctly.

Sixth, reactive inventory systems determine each distribution warehouse's timing and quantity of replenishment orders independently of all other sites, including the supply source. Thus, there is little potential to effectively coordinate inventory requirements across multiple distribution warehouses. The ability to take advantage of inventory information is not utilized—a serious defect when information and its communication are among the few resources that are decreasing in cost in the supply chain.

The final assumption characteristic of reactive inventory systems is that performance cycle length cannot be correlated with demand. The assumption is necessary to develop an accurate approximation of the variance of the demand over the performance cycle. For many situations higher demand levels create longer replenishment performance cycle; since they also increase the demands on inventory and transportation resources. This implies that periods of high demand should not necessarily correspond to extended performance cycles caused by stockouts or limited product availability.

Operationally, most inventory managers constrain the impact of such limitations through the skillful use of manual overrides. However, these overrides often lead to ineffective inventory decisions since the resulting plan is based on inconsistent rules and managerial policy.

Planning Methods

Inventory planning methods use a shared database to coordinate inventory requirements across multiple locations or stages in the supply chain. Planning activities may occur at the plant warehouse level to coordinate inventory allocation and delivery to multiple destinations. Planning may also coordinate inventory requirements across multiple channel partners such as manufacturers and retailers. The Advanced Planning and Scheduling (APS) systems discussed in Chapter 5 illustrate the capability of planning applications. With APS systems computerize the process, it is important for logistics managers to understand

the underlying logic and assumptions. Two inventory planning methods are fair share allocation and Distribution Requirements Planning (DRP).

Fair Share Allocation

A simplified inventory management planning method that provides each distribution facility with an equitable distributor of available inventory is called **fair share allocation**. Figure 7.11 illustrates the network structure, current inventory level, and daily requirements for three warehouses served by a single plant warehouse.

Using fair share allocation, the inventory planner determines the amount of inventory that can be allocated to each warehouse from the available inventory at the plant. For this example, assume that it is desirable to retain 100 units at the plant warehouse; therefore, 500 units are available for allocation. The calculation to determine the common days supply is:

$$DS = \frac{AQ + \sum_{j=1}^n I_j}{\sum_{j=1}^n D_j}$$

where

DS = Common days supply for warehouse inventories;

AQ = Inventory units to be allocated from plant warehouse;

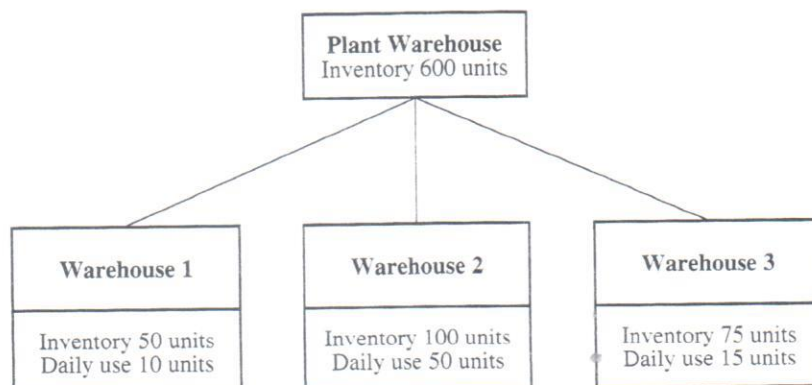
I_j = Inventory in units for warehouse j ; and

D_j = Daily demand for warehouse j .

In this example,

$$\begin{aligned} DS &= \frac{500 + (50 + 100 + 75)}{10 + 50 + 15} \\ &= \frac{500 + 225}{75} = 9.67 \text{ days.} \end{aligned}$$

FIGURE 7.11
Fair Share Allocation
Example



The fair share allocation dictates that each warehouse should be stocked to 9.67 days of inventory. The amount to be allocated to each warehouse is determined by:

$$A_j = (DS - I_j/D_j) \times D_j,$$

where

A_j = Amount allocated to warehouse j ;

DS = Days supply that each warehouse is brought up to;

I_j = Inventory in units for warehouse j ; and

D_j = Daily demand for warehouse j .

The amount allocated to warehouse 1 for this example is:

$$\begin{aligned} A_1 &= (9.67 - 50/10) \times 10 \\ &= (9.67 - 5) \times 10 \\ &= 4.67 \times 10 = 46.7 \text{ (round to 47 units).} \end{aligned}$$

The allocation for warehouses 2 and 3 can be determined similarly and is 383 and 70 units, respectively.

While fair share allocation coordinates inventory levels across multiple sites, it does not consider specific factors such as differences in performance cycle time, EOQ, or safety stock requirements. Fair share allocation methods are therefore limited in their ability to manage multistage inventories.

Requirements Planning

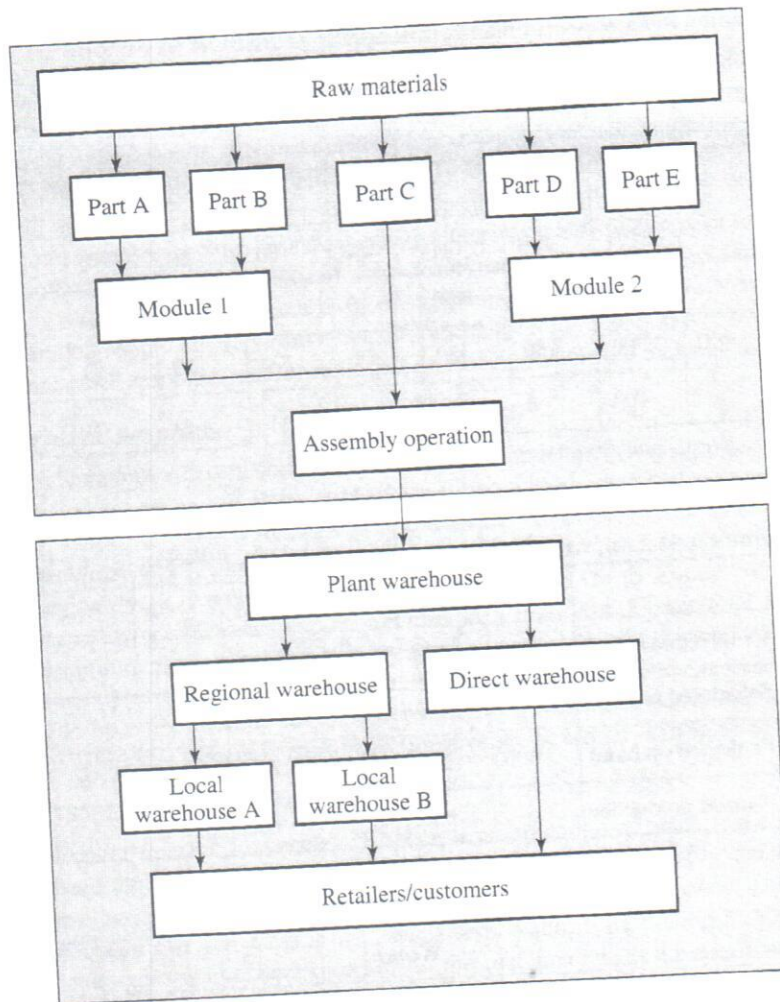
Requirements planning is an approach that integrates across the supply chain, taking into consideration unique requirements. Requirements planning is typically classified as Materials Requirements Planning (MRP) or Distribution Requirements Planning (DRP). There is one fundamental difference between the two techniques. MRP is driven by a production schedule. In contrast, DRP is driven by supply chain demand. So, while MRP generally operates in a dependent demand situation, DRP is applicable to an independent demand environment where uncertain customer requirements drive inventory requirements. MRP coordinates scheduling and integration of materials into finished goods, and so controls inventory until manufacturing or assembly is completed. DRP takes coordination responsibility once finished goods are received in the plant warehouse.

Figure 7.12 illustrates the conceptual design of a combined MRP/DRP system that integrates finished goods, work-in-process, and materials planning. The top half of the figure illustrates an MRP system that time-phases raw material arrivals to support the production schedule. The result of MRP execution is finished goods inventory at the manufacturing site. The bottom half of the figure illustrates the DRP system that allocates finished inventory from the manufacturing site to distribution warehouses and ultimately to customers. DRP time-phases movements to coordinate inventory arrivals to meet customer requirements and commitments. The MRP and DRP systems interface at the manufacturing site. Close coordination between the two systems results in minimal need for safety stock. DRP coordinates inventory level schedules, and when necessary, reschedules inventory movement between levels.

The fundamental DRP planning tool is the schedule, which coordinates requirements across the planning horizon. There is a schedule for each SKU at each warehouse. Schedules for the same SKU are integrated to determine the overall requirements for replenishment facilities such as a plant warehouse.

Figure 7.13 illustrates DRP planning schedules for two warehouses and a central supply facility. The schedules are developed using weekly time increments known as **time buckets**.

FIGURE 7.12
Conceptual Design
of Integrated MRP/
DRP System



Each bucket projects one period of activity. Although weekly increments are most common, daily or monthly increments can be used. For each site and SKU, the schedule reports current on-hand balance, safety stock, performance cycle length, and order quantity. In addition, for each planning period, the schedule reports gross requirements, scheduled receipts, and projected inventory on hand. Using the combination of requirements and projected availability, DRP determines the planned orders necessary to meet anticipated requirements. Gross requirements reflect customer demand and other distribution facilities supplied by the site under review. For Figure 7.13, the gross requirements of the central supply facility reflect the cascading demands of the Eastern and Western warehouses. Scheduled receipts are the replenishment shipments planned for arrival at the distribution warehouse. Projected on-hand inventory refers to the anticipated week-ending level. It is equal to the prior week's on-hand inventory level less the current week's gross requirements plus any scheduled receipts. While planning approaches to inventory management offer significant benefits, they have some constraints.

First, inventory planning systems require accurate and coordinated forecasts for each warehouse. The forecast is necessary to direct the flow of goods through the supply chain. Ideally, the system does not maintain excess inventory at any location, so little room for error exists in a lean inventory system. To the extent this level of forecast accuracy is possible, inventory planning systems operate well.

FIGURE 7.13 Distribution Requirements Planning Example**Eastern Warehouse**

		Weeks							
	Past due	1	2	3	4	5	6	7	8
Gross requirements		100	120	150	130	100	80	70	90
Scheduled receipts		0	0	400	0	0	0	400	0
Projected on hand	400	300	180	430	300	200	120	450	360
Planned orders		400	0	0	0	400	0	0	0

Safety stock: 100
Order quantity: 400
Leadtime: 2 weeks

Plant Warehouse

		Weeks							
	Past due	1	2	3	4	5	6	7	8
Gross requirements		400	150	0	150	550	0	0	0
Scheduled receipts		0	600	0	600	0	0	0	0
Projected on hand	600	200	650	650	1100	550	550	550	550
Planned production		600	0	600	0	0	0	0	0

Safety stock: 100
Batch size: 600
Leadtime: 1 week

Western Warehouse

		Weeks							
	Past due	1	2	3	4	5	6	7	8
Gross requirements		40	50	60	90	70	100	40	30
Scheduled receipts		0	0	150	0	150	150	0	0
Projected on hand	200	160	110	200	110	190	240	200	170
Planned orders		0	150	0	150	150	0	0	0

Safety stock: 50
Order quantity: 150
Leadtime: 1 week

Requirement
----->
Shipment
----->

Second, inventory planning requires consistent and reliable product movement between warehouse facilities. While variable performance cycles can be accommodated through safety leadtimes, such uncertainty reduces planning system effectiveness.

Third, integrated planning systems are subject to system nervousness, or frequent rescheduling, because of production breakdowns or delivery delays. System nervousness leads to fluctuation in capacity utilization, rescheduling cost, and confusion in deliveries. This is intensified by the volatile operating environment characteristic of supply chain logistics. Uncertainties such as transportation and vendor delivery reliability can cause extreme DRP nervousness.

Collaborative Inventory Replenishment

In Chapters 5 and 6 CPFR was introduced and discussed as a major collaborative effort between supply chain trading partners. Several collaborative initiatives focus only on inventory replenishment. Replenishment programs are designed to streamline the flow of goods within the supply chain. There are several specific techniques for collaborative replenishment, all of which build on supply chain relationships to rapidly replenish inventory on the basis of joint planning or actual sales experience. The intent is to reduce reliance on forecasting when and where inventory will need to be positioned to demand on a just-in-time basis. Effective collaborative replenishment programs require extensive cooperation and information sharing among supply chain partners. Specific techniques for collaborative inventory replenishment are quick response, vendor-managed inventory, and profile replenishment.

Quick Response

A technology-driven cooperative effort between retailers and suppliers to improve inventory velocity while providing merchandise supply closely matched to consumer buying patterns is quick response (**QR**). QR is implemented by sharing retail sales for specific products between supply chain participants to facilitate right product assortment availability when and where it is required. Instead of operating on a 15- to 30-day order cycle, QR arrangements can replenish retail inventories in a few days. Continuous information exchange regarding availability and delivery reduces uncertainty for the total supply chain and creates the opportunity for maximum flexibility. With fast, dependable order response, inventory can be committed as required, resulting in increased turnover and improved availability. WalMart's system is a prime example of the power of sharing sales to facilitate QR.

Vendor-Managed Inventory

Vendor-managed inventory (**VMI**) is a modification of quick response that eliminates the need for replenishment orders. The goal is to establish a supply chain arrangement so flexible and efficient that retail inventory is continuously replenished. The distinguishing factor between QR and VMI is who takes responsibility for setting target inventory levels and making restocking decisions. In QR, the customer makes the decisions. In VMI, the supplier assumes more responsibility and actually manages a category of inventory for the customer. By receiving daily transmission of retail sales or warehouse shipments, the supplier assumes responsibility for replenishing retail inventory in the required quantities, colors, sizes, and styles. The supplier commits to keeping the customer in stock and to maintaining inventory velocity. In some situations, replenishment involves cross-docking or direct store delivery (**DSD**) designed to eliminate the need for warehousing between the supplier and customer.

Profile Replenishment

Some manufacturers, wholesalers, and retailers are experimenting with an even more sophisticated collaboration known as profile replenishment (**PR**). The PR strategy extends QR and VMI by giving suppliers the right to anticipate future requirements according to their overall knowledge of a merchandise category. A category profile details the combination of sizes, colors, and associated products that usually sell in a particular type of retail outlet. Given PR responsibility, the supplier can simplify retailer involvement by eliminating the need to track unit sales and inventory level for fast-moving products.

Many firms, particularly manufacturers, are using DRP and even APS logic to coordinate inventory planning with major customers. The manufacturers are extending their planning framework to include customer warehouses and, in some cases, their retail stores. Such integrated planning capabilities facilitate manufacturer coordination and management of customer inventories.

TABLE 7.17
Comparative Service
and Inventory
Characteristics for
Anticipatory versus
Responsive Inventory
Systems

Source: Adapted from David J. Closs et al., "An Empirical Comparison of Anticipatory and Response-Based Supply Chain Strategies," *The International Journal of Logistics Management* 9, no. 2 (1998), pp. 21-34. Used with permission.

	Low Uncertainty Anticipatory	Low Uncertainty Responsive	High Uncertainty Anticipatory	High Uncertainty Responsive
Customer Service				
Fill rate percent	97.69	99.66	96.44	99.29
Inventories				
Supplier inventory	12.88	13.24	14.82	13.61
Manufacturer inventory	6.05	6.12	7.03	6.09
Distributor inventory	5.38	5.86	5.04	5.63
Retailer inventory	30.84	15.79	32.86	20.30
System inventory	55.15	41.01	59.76	45.83

Collaborative planning effectively shares inventory requirements and availability between supply chain partners, thus reducing uncertainty. Table 7.17 illustrates the service and inventory impact in a simulated environment under conditions of low and high uncertainty.⁷ Table 7.18 illustrates managerial considerations that drive adaptations of control logic.

Inventory Management Practices

An integrated inventory management strategy defines the policies and process used to determine where to place inventory and when to initiate replenishment shipments, as well as how much to allocate. The strategy development process employs three steps to classify products and markets, define segment strategies, and operationalize policies and parameters.

Product/Market Classification

The objective of product/market classification is to focus and refine inventory management efforts. Product/market classification, which is also called **fine-line** or **ABC classification**, groups products, markets, or customers with similar characteristics to facilitate inventory management. The classification process recognizes that not all products and markets have the same characteristics or degree of importance. Sound inventory management requires that classification be consistent with enterprise strategy and service objectives.

Classification can be based on a variety of measures. The most common are sales, profit contribution, inventory value, usage rate, and item category. The typical classification process sequences products or markets so that entries with similar characteristics are grouped together. Table 7.19 illustrates product classification using sales. The products are classified in descending order by sales volume so that the high-volume products are

⁷David J. Closs et al., "An Empirical Comparison of Anticipatory and Response-Based Supply Chain Strategies," *The International Journal of Logistics Management* 9, no. 2 (1998), pp. 21-34.

TABLE 7.18
Suggested Inventory
Management Logic

Use Planning Logic under Conditions of	Use Reactive Logic under Conditions of
Highly profitable segments	Cycle time uncertainty
Dependent demand	Demand uncertainty
Economies of scale	Destination capacity limitations
Supply uncertainty	
Source capacity limitations	
Seasonal supply buildup	

LE 7.19
Product Market
Classification (Sales)

Product Identification	Annual Sales (in 000s)	Percent Total Sales	Accumulated Sales (%)	Products (%)	Classification Category
1	\$45,000	30.0%	30.0%	5%	A
2	35,000	23.3	53.3	10	A
3	25,000	16.7	70.0	15	A
4	15,000	10.0	80.0	20	A
5	8,000	5.3	85.3	25	B
6	5,000	3.3	88.6	30	B
7	4,000	2.7	91.3	35	B
8	3,000	2.0	93.3	40	B
9	2,000	1.3	94.6	45	B
10	1,000	0.7	95.3	50	B
11	1,000	0.7	96.0	55	C
12	1,000	0.7	96.7	60	C
13	1,000	0.7	97.4	65	C
14	750	0.5	97.9	70	C
15	750	0.5	98.4	75	C
16	750	0.5	98.9	80	C
17	500	0.3	99.2	85	C
18	500	0.3	99.5	90	C
19	500	0.3	99.8	95	C
20	250	0.2	100.0	100	C
	\$150,000				

listed first, followed by slower movers. Classification by sales volume is one of the oldest methods used to establish selective inventory policies. For most marketing or logistics applications, a small percentage of the entities account for a large percentage of the volume. This operationalization is often called the **80/20 rule** or **Pareto's law**. The 80/20 rule, which is based on widespread observations, states that for a typical enterprise 80 percent of the sales volume is typically accounted for by 20 percent of the products. A corollary to the rule is that 80 percent of enterprise sales are accounted for by 20 percent of the customers. The reverse perspective of the rule would state that the remaining 20 percent of sales are obtained from 80 percent of the products, customers, etc. In general terms, the 80/20 rule implies that a majority of sales result from a relatively few products or customers.

Once items are classified or grouped, it is common to label each category with a character or description. High-volume, fast-moving products are often described as A items. The moderate-volume items are termed the B items, and the low-volume or slow movers are known as Cs. These character labels indicate why this process is often termed ABC analysis. While fine-line classification often uses three categories, some firms use four or five categories to further refine classifications. Grouping of similar products facilitates management efforts to establish focused inventory strategies for specific product segments. For example, high-volume or fast-moving products are typically targeted for higher service levels. This often requires that fast-moving items have relatively more safety stock. Conversely, to reduce overall inventory levels, slower-moving items may be allowed relatively less safety stock, resulting in lower service levels.

In special situations, classification systems may be based on multiple factors. For example, item gross margin and importance to customers can be weighted to develop a combined index instead of simply using sales volume. The weighted rank would then group items that have similar profitability and importance characteristics. The inventory policy, including safety stock levels, is then established using the weighted rank.

TABLE 7.20
Integrated Strategy

Fine-Line Classification	Service Objective	Forecasting Procedure	Review Period	Inventory Management	Replenishment Monitoring
A (Promotional)	99%	CPFR	Perpetual	Planning—DRP	Daily
A (Regular)	98	Sales history	Perpetual	Planning—DRP	Daily
B	95	Sales history	Weekly	Planning—DRP	Weekly
C	90	Sales history	Biweekly	Reorder point	Biweekly

The classification array defines product or market groups to be assigned similar inventory strategies. The use of item groups facilitates the identification and specification of inventory strategies without requiring tedious development of individual item strategies. It is much easier to track and manage 3 to 10 groups instead of hundreds of individual items.

Segment Strategy Definition

The second step is to define the inventory strategy for each product/market group or segment. The strategy includes specification for all aspects of the inventory management process, including service objectives, forecasting method, management technique, and review cycle.

The key to establishing selective management strategies is the realization that product segments have different degrees of importance with respect to achieving the enterprise mission. Important differences in inventory responsiveness should be designed into the policies and procedures used for inventory management.

Table 7.20 illustrates a sample integrated strategy for four item categories. In this case, the items are grouped by ABC sales volume and as a promotional or regular stock item. Promotional items are those commonly sold in special marketing efforts that result in considerable demand lumpiness. Lumpy demand patterns are characteristic of promotional periods with high volume followed by postpromotion periods with relatively low demand.

Table 7.20 illustrates a management segmentation scheme based on service objectives, forecasting process, review period, inventory management approach, and replenishment monitoring frequency. Additional or fewer characteristics of the inventory management process may be appropriate for some enterprises. Although this table is not presented as a comprehensive inventory strategy framework, it illustrates the issues that must be considered. The rationale behind each element is presented on the basis of the full-line classification.

Policies and Parameters

The final step in implementing a focused inventory management strategy is to define detailed procedures and parameters. The procedures define data requirements, software applications, performance objectives, and decision guidelines. The parameters delineate values such as review period length, service objectives, inventory carrying cost percentage, order quantities, and reorder points. The combination of parameters either determines or can be used to calculate the precise quantities necessary to make inventory management decisions.

Summary

Inventory typically represents the second largest component of logistics cost next to transportation. The risks associated with holding inventory increase as products move down the supply chain closer to the customer because the potential of having the product in the wrong place or form increases and costs have been incurred to distribute the product. In addition to the risk of lost sales due to stockouts because adequate inventory is not available, other risks include obsolescence, pilferage, and damage. Further, the cost of carrying inventory is significantly influenced by the cost of the capital tied up in the inventory. Geographic

specialization, decoupling, supply/demand balancing, and buffering uncertainty provide the basic rationale for maintaining inventory. While there is substantial interest in reducing overall supply chain inventory, inventory does add value and can result in lower overall supply chain costs with appropriate trade-offs.

From a supply chain logistics perspective, the major controllable inventory elements are replenishment cycle stock, safety stock, and in-transit stock. The appropriate replenishment cycle stock can be determined using an EOQ formula to reflect the trade-off between storage and ordering cost. Safety stock depends on the mean and variance of demand and the replenishment cycle. In-transit stock depends on the transport mode.

Inventory management uses a combination of reactive and planning logics. Reactive logic is most appropriate for items with low volume, high demand, and high performance cycle uncertainty because it postpones the risk of inventory speculation. Inventory planning logic is appropriate for high-volume items with relatively stable demand. Inventory planning methods offer the potential for effective inventory management because they take advantage of improved information and economies of scale. Adaptive logic combines the two alternatives depending on product and market conditions. Collaboration offers a way for parties in the supply chain to jointly gain inventory efficiency and effectiveness.

Questions

1. How does the cost of carrying inventory impact the traditional earnings statement of the enterprise?
2. Discuss the relationship between service level, uncertainty, safety stock, and order quantity. How can trade-offs between these elements be made?
3. Discuss the disproportionate risk of holding inventory by retailers, wholesalers, and manufacturers. Why has there been a trend to push inventory back up the channel of distribution?
4. What is the difference between the probability of a stockout and the magnitude of a stockout?
5. Data suggest that while overall average inventory levels are declining, the relative percentage being held by manufacturers is increasing. Explain why you think this observation is either true or false. Describe how such a shift could benefit the operations of the entire channel and how manufacturers could take advantage of the shift.
6. Discuss the differences between reactive and planning inventory logics. What are the advantages of each? What are the major implications of each?
7. Illustrate how fine-line inventory classification can be used with product and market segments. What are the benefits and considerations when classifying inventory by product, market, and product/market?
8. What advantage does DRP have over a fair share method of inventory deployment?
9. Discuss the importance of collaboration in the developing of supply chain inventory strategies. Provide an example.
10. Customer-based inventory management strategies allow the use of different availability levels for specific customers. Discuss the rationale for such a strategy. Are such strategies discriminatory? Justify your position.