

CMAT 541 Principles of Applied Math I study guide

September 2016

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Consider

$$-u'' = f(x), \quad 0 < x < 1; \quad u(0) = u(1) = 0.$$

with $f(x)$ piecewise continuous.

(a) By using Schwarz's inequality show that

$$\|u\|_2 \leq \left[\int_0^1 \int_0^1 g^2(x,t) dx dt \right]^{1/2} \|f\|_2$$

and evaluate the double integral.

(b) Show that

$$\|u\|_\infty \leq \frac{1}{4} \|f\|_1$$

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We introduced the L_p norm

$$\|f\|_p = \left[\int_0^1 [f(x)]^p dx \right]^{1/p}$$

Show that if $f(x)$ is continuous, then

$$\lim_{p \rightarrow \infty} \|f\|_p = \|f\|_\infty$$

(Hint: It is easy to show that $\|f\|_p \leq \|f\|_\infty$ so that $\lim_{p \rightarrow \infty} \|f\|_p \leq \|f\|_\infty$

Show the opposite inequality, fix $\epsilon > 0$ and note that

$$\|f\|_p^p = \int_0^1 |f|^p dx \geq \int_I |f|^p dx.$$

L_∞, L_1, L_2

Where I is the interval on which $|f| \geq \|f\|_\infty - \epsilon$. Show then that

$$\lim_{p \rightarrow \infty} \|f\|_p \geq \|f\|_\infty - \epsilon.$$

We take $\epsilon \rightarrow 0^+$ to obtain the desired result.

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(a) Find the exact eigenvalues and λ_m and eigenfunctions $y_n(x)$ of the eigenvalue problem

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$$-u'' = \lambda u, \quad 0 < x < 1, \quad u'(0) = u(1) = 0$$

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(b) Find the eigenvalue expansion solution for

$$-u'' = f(x), \quad 0 < x < 1, \quad u'(0) = u(1) = 0$$

$\int^s [0, 1] \rightarrow [1, 1]$

$[0, 1]$

$[-\pi, \pi]$

$[0, 2\pi]$

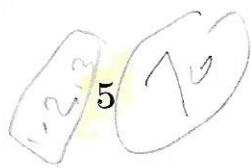
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Consider the periodic boundary value problem given by

$$\frac{d^2 u}{dx^2} + u = f(x); \quad 0 < x < 2\pi$$

$$u(0) = u(2\pi), \quad \frac{du}{dx}(0) = \frac{du}{dx}(2\pi)$$

where $f(x)$ is a given function. Determine appropriate Green's function for this problem.



Let λ be an arbitrary complex number. We shall define the principal value of $\sqrt{\lambda}$ as follows. If $\sqrt{\lambda} = 0$, then $\lambda = 0$; if $\lambda \neq 0$, then λ has a unique representation $\lambda = |\lambda|e^{i\theta}$, $0 \leq \theta < 2\pi$, and the principal value of $\sqrt{\lambda}$ is defined as $\lambda^{1/2}e^{i\theta/2}$, where $|\lambda|^{1/2}$ is the positive square root of the positive real number λ . Throughout this exercise $\sqrt{\lambda}$ will stand for the principal value just defined.

(a) The general solution of $-u'' = \lambda u$ is $u(x) = A + Bx$ if $\lambda = 0$; $u(x) = A\exp(i\sqrt{\lambda}x) + B\exp(-i\sqrt{\lambda}x)$ if $\lambda \neq 0$ (or alternatively $u(x) = C\sin\sqrt{\lambda}x + D\cos\sqrt{\lambda}x$ for this case), show that only the real values $\lambda_n = n^2\pi^2$ are eigenvalues of this boundary value problem:

$$-u'' = \lambda u, \quad 0 < x < 1, \quad u(0) = u(1) = 0.$$

(b) Find the eigenvalues and eigenfunctions of

$$-u'' = \lambda u, \quad 0 < x < 1, \quad u'(0) = u'(1) = 0.$$

(c) Obtain an eigen-function expansion solution for the problem of

$$-u'' + qu = f(x), \quad 0 < x < 1, \quad u'(0) = u'(1) = 0.$$

where q is a given positive number. What goes wrong if $q = 0$?