1. Consider the following vector fields.

e) $$\begin{aligned}\dot{x} &= -x + x^3, \\ \dot{y} &= x + y,\end{aligned} \qquad (x, y) \in \mathbb{R}^2.$$

Find all fixed points and discuss their stability.

2. Consider the following maps.

a) $$\begin{aligned} x &\mapsto x, \\ y &\mapsto x + y,\end{aligned} \qquad (x, y) \in \mathbb{R}^2.$$

Find all the fixed points and discuss their stability.

8. Suppose that the matrix A in Exercise 7 has some eigenvalues with zero real parts (and the rest have negative real parts). Does it follow that $x = 0$ is stable? Answer this question by considering the following example.

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

6. Consider the damped Duffing equation

$$\begin{aligned}\dot{x} &= y, \\ \dot{y} &= x - x^3 - \delta y, \qquad (x, y) \in \mathbb{R}^2, \quad \delta > 0.\end{aligned}$$

Use the function

$$V(x, y) = \frac{y^2}{2} - \frac{x^2}{2} + \frac{x^4}{4},$$

as a Liapunov function to show that the equilibrium points $(x, y) = (\pm 1, 0)$ are asymptotically stable.

Given

$\dot{x} = -x - y + y\sqrt{x^2+y^2}$

$\dot{y} = x - y + x\sqrt{x^2+y^2}$

Find the fix points and determine their stability and try to determine any existence of periodic sol

determine the stability of the periodic sol