WebAssign
Homework 5 (Homework)

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Math 2090, section 1, Summer 1 2016
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Current Score : – / 31 **Due :** Monday, July 11 2016 11:59 PM CDT

1. –/2 pointsGoodDiffEQLinAlg3 4.5.007.

Determine whether the given set of vectors is linearly independent or linearly dependent in $\mathbb{R}^n$.

$$\{\underset{\mathbf{v}_1}{(-1, 1, 2)}, \underset{\mathbf{v}_2}{(0, 2, -1)}, \underset{\mathbf{v}_3}{(2, 0, 3)}, \underset{\mathbf{v}_4}{(-1, -1, 1)}\}$$

- The given set of vectors is linearly independent.
- The given set of vectors is linearly dependent.

In the case of linear dependence, find a dependency relationship. (Give your answer in terms of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$. If the set of vectors is linearly independent, enter INDEPENDENT.)

= 0

2. –/2 pointsGoodDiffEQLinAlg3 4.5.008.

Determine whether the given set of vectors is linearly independent or linearly dependent in $\mathbb{R}^n$.

$$\{\underset{\mathbf{v}_1}{(1, -1, 7, 9)}, \underset{\mathbf{v}_2}{(7, -1, 1, -1)}, \underset{\mathbf{v}_3}{(-1, 1, 1, 1)}\}$$

- The given set of vectors is linearly independent.
- The given set of vectors is linearly dependent.

In the case of linear dependence, find a dependency relationship. (Give your answer in terms of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. If the set of vectors is linearly independent, enter INDEPENDENT.)

= 0

3. –/1 pointsGoodDiffEQLinAlg3 4.5.014.

Determine all values of the constant k for which the given set of vectors is linearly independent in $\mathbb{R}^4$. (Enter your answers as a comma-separated list.)

$$\{(1, 1, 0, -1), (1, k, 1, 1), (8, 1, k, 1), (-1, 1, 1, k)\}$$

$k \neq$ ______

4. –/9 pointsGoodDiffEQLinAlg3 4.5.030.

Use the Wronskian to show that the given functions are linearly independent on the given interval I.

$$f_1(x) = \sin x,\ f_2(x) = \cos x,\ f_3(x) = \tan x,\ I = (-\pi/2, \pi/2)$$

$$W[f_1, f_2, f_3](x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ \cos x & \_\_\_ & \_\_\_ \\ -\sin x & \_\_\_ & \_\_\_ \end{vmatrix}$$

$$= \begin{vmatrix} \sin x & \cos x & \tan x \\ \cos x & \_\_\_ & \_\_\_ \\ 0 & 0 & \_\_\_ \end{vmatrix}$$

$= \_\_\_$

$W[f_1, f_2, f_3](x)$ is ---Select--- zero over I, so the vectors are linerly independent.

5. –/1 pointsGoodDiffEQLinAlg3 4.5.TF.001.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

Every vector space V possesses a unique minimal spanning set.

- ○ True. Consider the vector space $V = \mathbb{R}^2$, which has only one spanning set for V.
- ○ True. Consider the vector space $V = \mathbb{R}^n$, which has only one minimal spanning set for V.
- ○ False. Consider the vector space $V = \mathbb{R}^2$, which has only one minimal spanning set for V.
- ○ False. Consider the vector space $V = \mathbb{R}^2$, which has many different minimal spanning sets for V.
- ○ False. Consider the vector space $V = \mathbb{R}^n$, which has only one minimal spanning set for V.

6. –/1 pointsGoodDiffEQLinAlg3 4.5.TF.002.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The set of column vectors of a 4 × 7 matrix A must be linearly dependent.

- ○ True. We have seven column vectors, and each of them belongs to $\mathbb{R}^4$. Therefore, the number of vectors present exceeds the number of components in those vectors.
- ○ True. We have four row vectors, and each of them belongs to $\mathbb{R}^7$. Therefore, the number of components in the vectors exceeds the number of vectors present.
- ○ False. The 4 × 7 zero matrix does not have linearly dependent columns.
- ○ False. The set of column vectors of a 4 × 7 matrix A must be linearly independent because there are more columns than rows in A.
- ○ False. The set of column vectors of a 4 × 7 matrix A must be linearly independent because A is not a square matrix.

7. –/1 pointsGoodDiffEQLinAlg3 4.5.TF.003.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The set of column vectors of a 9 × 5 matrix A must be linearly independent.

- True. We have nine row vectors, and each of them belongs to $\mathbb{R}^5$. Therefore, the number of vectors present exceeds the number of components in those vectors.
- True. We have five column vectors, and each of them belongs to $\mathbb{R}^9$. Therefore, the number of components in the vectors exceeds the number of vectors present.
- False. The set of column vectors of a 9 × 5 matrix A must be linearly independent because there are more rows than columns in A.
- False. The set of column vectors of a 9 × 5 matrix A must be linearly independent because A is not a square matrix.
- False. The 9 × 5 zero matrix does not have linearly independent columns.

8. –/1 pointsGoodDiffEQLinAlg3 4.5.TF.005.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

If the Wronskian of a set of functions is nonzero at some point x_0 in an interval I, then the set of functions is linearly independent.

- True. By the theorem that states if the Wronskian of a set of functions is zero at some point x_0 in an interval I, then the set of functions is linearly independent.
- True. By the theorem that states if the Wronskian of a set of functions is nonzero at some point x_0 in an interval I, then the set of functions is linearly independent.
- False. By the theorem that states if the Wronskian of a set of functions is zero for every x in the interval I, then the set of functions is linearly dependent.
- False. By the theorem that states if the Wronskian of a set of functions is nonzero at some point x_0 in an interval I, then the set of functions is linearly dependent.
- False. By the theorem that states if the Wronskian of a set of functions is zero for every x in the interval I, then the set of functions is linearly independent.

9. –/1 pointsGoodDiffEQLinAlg3 4.5.TF.008.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

A set of three vectors in a vector space V is linearly dependent if and only if all three vectors are proportional to one another.

- ○ True. By the theorem that states any vector space V is linearly dependent if and only if the vectors are proportional.
- ○ True. By the theorem that states any vector space V is linearly independent if and only if the vectors are proportional.
- ○ False. Consider the linearly dependent set of vectors (1, 0), (0, 1), and (1, 1) in $\mathbb{R}^2$; these are not proportional to each other.
- ○ False. Consider the linearly dependent set of vectors (0, 0), (1, 0), and (2, 0) in $\mathbb{R}^2$; these are not proportional to each other.
- ○ False. Consider the linearly independent set of vectors (1, 1), (2, 2), and (3, 3) in $\mathbb{R}^2$; these are proportional to each other.

10.–/1 pointsGoodDiffEQLinAlg3 4.6.003.

Determine whether the given set of vectors is a basis for $\mathbb{R}^n$.

$\{(1, -1, 1), (2, 6, -2), (4, 16, -6)\}$

- ○ The set of vectors form a basis for $\mathbb{R}^3$.
- ○ The set of vectors do not form a basis for $\mathbb{R}^3$.

11.–/1 pointsGoodDiffEQLinAlg3 4.6.004.

Determine whether the given set of vectors is a basis for $\mathbb{R}^n$.

$\{(1, 1, -1, 2), (1, 0, 1, -1), (2, -1, 1, -1)\}$

- ○ The set of vectors form a basis for $\mathbb{R}^4$.
- ○ The set of vectors do not form a basis for $\mathbb{R}^4$.

12.–/1 pointsGoodDiffEQLinAlg3 4.6.008.

Determine a basis S for $P_3(x)$ whose elements all have the same degree. Be sure to prove that S is a basis.

$S = \{$

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13.–/1 pointsGoodDiffEQLinAlg3 4.6.012.

Find the dimension of the null space of the given matrix A.

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \\ 2 & -1 & 3 & 4 \\ 1 & 0 & 1 & 1 \\ 3 & -1 & 4 & 5 \end{bmatrix}$$

14.–/1 pointsGoodDiffEQLinAlg3 4.6.019.

Determine a basis for the subspace of $M_2(\mathbb{R})$ spanned by the following. (Enter your answer as a list of matrices. Enter each matrix as a comma-separated list of its components in the form [[a_{11}, a_{12}], [a_{21}, a_{22}]].)

$$\begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 5 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 5 & -7 \\ -5 & 3 \end{bmatrix}$$

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15.–/1 pointsGoodDiffEQLinAlg3 4.6.023.

Determine all values of the constant α for which $\{1 + \alpha x^2, 4 + x + x^2, 5 + x\}$ is a basis for P_2. (Enter your answers as a comma-separated list.)

$\alpha \neq$

16.–/3 pointsGoodDiffEQLinAlg3 4.6.028.

Let $V = M_3(\mathbb{R})$ and let S be the subset of all vectors in V such that the sum of the entries in each row and in each column is zero.

(a) Find a basis of S.

- ○ $\left\{\begin{bmatrix}1&0&-1\\0&0&0\\-1&0&1\end{bmatrix}, \begin{bmatrix}0&1&-1\\0&0&0\\0&-1&1\end{bmatrix}, \begin{bmatrix}0&0&0\\1&0&-1\\-1&0&1\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&-1\\0&-1&1\end{bmatrix}\right\}$
- ○ $\left\{\begin{bmatrix}1&0&-1\\-1&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}1&0&-1\\0&0&0\\-1&0&1\end{bmatrix}, \begin{bmatrix}0&0&0\\1&0&-1\\-1&0&1\end{bmatrix}\right\}$
- ○ $\left\{\begin{bmatrix}1&0&-1\\-1&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&-1\\0&-1&1\\0&0&0\end{bmatrix}, \begin{bmatrix}1&0&-1\\0&0&0\\-1&0&1\end{bmatrix}, \begin{bmatrix}0&1&-1\\0&0&0\\0&-1&1\end{bmatrix}, \begin{bmatrix}0&0&0\\1&0&-1\\-1&0&1\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&-1\\0&-1&1\end{bmatrix}\right\}$
- ○ $\left\{\begin{bmatrix}0&1&-1\\0&-1&1\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&-1\\0&0&0\\0&-1&1\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&-1\\0&-1&1\end{bmatrix}\right\}$
- ○ $\left\{\begin{bmatrix}1&-1&0\\-1&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}1&-1&0\\0&0&0\\-1&1&0\end{bmatrix}, \begin{bmatrix}0&0&0\\1&-1&0\\-1&1&0\end{bmatrix}\right\}$

Find the dimension of S.

dim[S] = ______

(b) Extend the basis in (a) to a basis for V.

- ○ $\left\{\text{basis of } S, \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix}\right\}$
- ○ $\left\{\text{basis of } S, \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix}\right\}$
- ○ $\left\{\text{basis of } S, \begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}\right\}$
- ○ $\left\{\text{basis of } S, \begin{bmatrix}1&0&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\0&0&1\end{bmatrix}\right\}$
- ○ $\left\{\text{basis of } S, \begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&1&0\\0&0&0\end{bmatrix}, \begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}\right\}$

17.–/1 pointsGoodDiffEQLinAlg3 4.6.TF.004.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

$\dim[P_n] = \dim[\mathbb{R}^n]$

- True. The $\dim[P_n] = n$ and $\dim[\mathbb{R}^n] = n$.
- False. The $\dim[P_n] = n - 1$ and $\dim[\mathbb{R}^n] = n$.
- False. The $\dim[P_n] = n$ and $\dim[\mathbb{R}^n] = n - 1$.
- False. The $\dim[P_n] = n + 1$ and $\dim[\mathbb{R}^n] = n$.
- False. The $\dim[P_n] = n$ and $\dim[\mathbb{R}^n] = n + 1$.

18.–/1 pointsGoodDiffEQLinAlg3 4.6.TF.006.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

Four vectors in P_2 must be linearly dependent.

- True. The $\dim[P_2] = 3$, and so any set of more than three vectors in P_2 must be linearly dependent.
- True. The $\dim[P_2] = 1$, and so any set of more than two vectors in P_2 must be linearly dependent.
- False. The $\dim[P_2] = 4$, and so any set of four vectors in P_2 must be linearly independent.
- False. The $\dim[P_2] = 3$, and so any set of more than three vectors in P_2 must be linearly independent.
- False. The $\dim[P_2] = 1$, and so any set of more than two vectors in P_2 must be linearly independent.

19.–/1 pointsGoodDiffEQLinAlg3 4.6.TF.007.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

Two vectors in P_3 must be linearly independent.

- True. The dim[P_3] = 2, and so any set of two vectors in P_3 must be linearly independent.
- True. The dim[P_3] = 4, and so any set of less than four vectors in P_3 must be linearly independent.
- False. Consider the two vectors $1 + x$ and $1 + x^2$ in P_3.
- False. Consider the two vectors $1 + x$ and $2 + x^2$ in P_3.
- False. Consider the two vectors $1 + x$ and $2 + 2x$ in P_3.