WebAssign
Homework 4 (Homework)

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Math 2090, section 1, Summer 1 2016
Instructor: Michael Tom

Current Score : – / 36 **Due :** Monday, July 4 2016 11:59 PM CDT

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1. –/2 pointsGoodDiffEQLinAlg3 4.2.001.

Determine whether the given set of vectors is closed under addition and closed under scalar multiplication. In this case, take the set of scalars to be the set of all real numbers.

The set of all rational numbers.

The set of all rational numbers is ---Select--- under addition and the set is ---Select--- under scalar multiplication.

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2. –/2 pointsGoodDiffEQLinAlg3 4.2.002.

Determine whether the given set of vectors is closed under addition and closed under scalar multiplication. In this case, take the set of scalars to be the set of all real numbers.

The set of all upper triangular $n \times n$ matrices with real elements.

The set of all upper triangular matrices with real elements is ---Select--- under addition and the set is ---Select--- under scalar multiplication.

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3. –/2 pointsGoodDiffEQLinAlg3 4.2.003.

Determine whether the given set of vectors is closed under addition and closed under scalar multiplication. In this case, take the set of scalars to be the set of all real numbers.

The set of all solutions to the differential equation $y' + 6y = 4x^2$. (Do not solve the differential equation.)

The set of all solutions to the differential equation $y' + 6y = 4x^2$ is ---Select--- under addition and the set is ---Select--- under scalar multiplication.

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4. –/2 pointsGoodDiffEQLinAlg3 4.2.005.

Determine whether the given set of vectors is closed under addition and closed under scalar multiplication. In this case, take the set of scalars to be the set of all real numbers.

The set of all solutions to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$.

The set of all solutions to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$ is ---Select--- under addition and the set is ---Select--- under scalar multiplication.

5. –/1 pointsGoodDiffEQLinAlg3 4.2.012.

On $\mathbb{R}^+$, the set of *positive* real numbers, define the operations of addition and scalar multiplication as follows:

$$x + y = xy,$$
$$c \cdot x = x^c.$$

Note that the multiplication and exponentiation appearing on the right side of these formulas refer to the ordinary operations on real numbers. Determine whether $\mathbb{R}^+$, together with these algebraic operations, is a vector space.

- Yes, this is a vector space.
- No, this is not a vector space.

6. –/1 pointsGoodDiffEQLinAlg3 4.2.TF.002.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

If **v** is a vector in a vector space V, and r and s are scalars such that $r\mathbf{v} = s\mathbf{v}$, then $r = s$.

- True. The vector space V is closed under scalar multiplication.
- True. The vector space V is not closed under scalar multiplication.
- False. The vector space V is closed under scalar multiplication.
- False. The vector space V is not closed under scalar multiplication.
- False. The vector **v** can be the zero vector.

7. –/1 pointsGoodDiffEQLinAlg3 4.2.TF.003.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The set $\mathbb{Z}$ of integers, together with the usual operations of addition and scalar multiplication, forms a vector space.

- True. The set is closed under both addition and scalar multiplication.
- True. The set is not closed under both addition and scalar multiplication.
- False. The set is not closed under scalar multiplication.
- False. The set is not closed under addition.
- False. The set is not closed under both addition and scalar multiplication.

8. –/1 pointsGoodDiffEQLinAlg3 4.2.TF.006.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The set {0}, with the usual operations of addition and scalar multiplication, forms a vector space.

- True. The set is closed under both addition and scalar multiplication.
- True. The set is not closed under both addition and scalar multiplication.
- False. The set is not closed under scalar multiplication.
- False. The set is not closed under addition.
- False. The set is not closed under both addition and scalar multiplication.

9. –/1 pointsGoodDiffEQLinAlg3 4.2.TF.007.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The set {0, 1}, with the usual operations of addition and scalar multiplication, forms a vector space.

- True. The set is closed under both addition and scalar multiplication.
- True. The set is not closed under both addition and scalar multiplication.
- True. The set is not closed under scalar multiplication.
- False. The set is closed under both addition and scalar multiplication.
- False. The set is not closed under both addition and scalar multiplication.

10.–/2 pointsGoodDiffEQLinAlg3 4.3.003.

Express S in set notation.

$V = \mathbb{R}^2$, and S is the set of all vectors (x, y) in V satisfying $4x + 5y = 0$.

- ○ $S = \{(x, y) \in \mathbb{R}^2 : 4x + 5y = 0\}$
- ○ $S = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$
- ○ $S = \{(x, y) \in \mathbb{R}^3 : 4x + 5y = 0\}$
- ○ $S = \{(x, y) \in \mathbb{R} : x + y = 0\}$
- ○ $S = \{(x, y) \in \mathbb{R} : 4x + 5y = 0\}$

Determine whether it is a subspace of the given vector space V.

- ○ S is a subspace of $\mathbb{R}^2$.
- ○ S is not a subspace of $\mathbb{R}^2$.

11.–/2 pointsGoodDiffEQLinAlg3 4.3.005.

Express S in set notation.

$V = \mathbb{R}^3$, and S is the set of all vectors (x, y, z) in V satisfying $x + y + z = 8$.

- ○ $S = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$
- ○ $S = \{(x, y, z) \in \mathbb{R}^2 : x + y + z = 8\}$
- ○ $S = \{(x, y, z) \in \mathbb{R} : x + y + z = 0\}$
- ○ $S = \{(x, y, z) \in \mathbb{R} : x + y + z = 8\}$
- ○ $S = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 8\}$

Determine whether it is a subspace of the given vector space V.

- ○ S is a subspace of $\mathbb{R}^3$.
- ○ S is not a subspace of $\mathbb{R}^3$.

12.–/2 pointsGoodDiffEQLinAlg3 4.3.006.

Express S in set notation.

$V = \mathbb{R}^n$, and S is the set of all solutions to the nonhomogeneous linear system $A\mathbf{x} = \mathbf{b}$, where A is a fixed $m \times n$ matrix and $\mathbf{b}$ $(\neq \mathbf{0})$ is a fixed vector.

- $S = \{\mathbf{x} \in \mathbb{R}^n\text{: } A\mathbf{x} = \mathbf{b}, A \text{ is a fixed } m \times n \text{ matrix}\}$
- $S = \{\mathbf{x} \in \mathbb{R}^n\text{: } A\mathbf{x} = \mathbf{b}, A \text{ is a fixed } n \times n \text{ matrix}\}$
- $S = \{\mathbf{x} \in \mathbb{R}\text{: } A\mathbf{x} = \mathbf{b}, A \text{ is a fixed } m \times n \text{ matrix}\}$
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- $S = \{\mathbf{x} \in \mathbb{R}^n\text{: } A\mathbf{x} = \mathbf{b}, A \text{ is a fixed } m \times m \text{ matrix}\}$

Determine whether it is a subspace of the given vector space V.

- S is a subspace of $\mathbb{R}^n$.
- S is not a subspace of $\mathbb{R}^n$.

13.–/1 pointsGoodDiffEQLinAlg3 4.3.020.

Determine the null space of the given matrix A. (Express your answer in terms of parameters $r, s \in \mathbb{R}$ if necessary.)

$$A = \begin{bmatrix} 1 & -3 & 1 \\ 7 & -8 & -3 \\ -1 & 4 & 7 \end{bmatrix}$$

nullspace(A) = $\{$

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14.–/1 pointsGoodDiffEQLinAlg3 4.3.021.

Determine the null space of the given matrix A. (Express your answer in terms of parameters $r, s \in \mathbb{R}$ if necessary.)

$$A = \begin{bmatrix} 1 & 3 & -4 & -1 \\ 3 & 10 & -10 & 0 \\ 2 & 5 & -10 & -5 \end{bmatrix}$$

nullspace(A) = $\{$

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15.–/1 pointsGoodDiffEQLinAlg3 4.3.022.

Determine the null space of the given matrix A. (Express your answer in terms of parameters $r, s \in \mathbb{R}$ if necessary.)

$$A = \begin{bmatrix} 1 & i & -3 \\ 4 & 5i & -7 \\ -1 & -4i & i \end{bmatrix}$$

nullspace(A) = $\{$

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16.–/1 pointsGoodDiffEQLinAlg3 4.3.TF.001.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The null space of an $m \times n$ matrix A with real elements is a subspace of $\mathbb{R}^m$.

- True. The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^{\mathrm{m}}$.
- False. The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^n$.
- False. The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^{n \cdot m}$.
- False. The null space of an $m \times n$ matrix A is a subspace of $\mathbb{R}^{m + n}$.
- False. The null space of an $m \times n$ matrix A cannot be a subspace of $\mathbb{R}^m$ because A is not a square matrix.

17.–/1 pointsGoodDiffEQLinAlg3 4.3.TF.002.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The solution set of any linear system of m equations in n variables forms a subspace of $\mathbb{C}^n$.

- True. The linear system is always homogeneous.
- False. The solution set of a linear system may not contain $\mathbf{0}$.
- False. The linear system may not be homogeneous.
- False. The solution set of a linear system of m equations in n variables forms a subspace of $\mathbb{C}^m$.
- False. The solution set of a linear system of m equations in n variables forms a subspace of $\mathbb{R}^n$.

18.–/1 pointsGoodDiffEQLinAlg3 4.3.TF.005.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

A nonempty set S of a vector space V that is closed under scalar multiplication contains the zero vector of V.

- True. Choosing any vector $\mathbf{v}$ in S, the scalar multiple $0\mathbf{v} = \mathbf{0}$ does not belong to S.
- True. Choosing any vector $\mathbf{v}$ in S, the scalar multiple $0\mathbf{v} = \mathbf{0}$ still belongs to S.
- False. Choosing any vector $\mathbf{v}$ in S, the scalar multiple $0\mathbf{v} = \mathbf{v}$ does not belong to S.
- False. Choosing any vector $\mathbf{v}$ in S, the scalar multiple $0\mathbf{v} = \mathbf{0}$ does not belong to S.
- False. Choosing any vector $\mathbf{v}$ in S, the scalar multiple $0\mathbf{v} = \mathbf{0}$ still belongs to S.

19.–/1 pointsGoodDiffEQLinAlg3 4.3.TF.007.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

If $V = \mathbb{R}^3$ and S consists of all points on the xy-plane, the xz-plane, and the yz-plane, then S is a subspace of V.

- True. The set is closed under both addition and scalar multiplication.
- True. The set is not closed under both addition and scalar multiplication.
- False. The set is not closed under addition.
- False. The set is not scalar multiplication.
- False. The set is closed under both addition and scalar multiplication.

20.–/1 pointsGoodDiffEQLinAlg3 4.4.001.

Determine whether the given set of vectors spans $\mathbb{R}^2$.

$\{(1, -1), (2, -2), (2, 9)\}$

- The given set of vectors does span $\mathbb{R}^2$.
- The given set of vectors does not span $\mathbb{R}^2$.

21.–/1 pointsGoodDiffEQLinAlg3 4.4.002.

Determine whether the given set of vectors spans $\mathbb{R}^2$.

$\{(3, 2), (0, 0)\}$

- The given set of vectors does span $\mathbb{R}^2$.
- The given set of vectors does not span $\mathbb{R}^2$.

22.–/1 pointsGoodDiffEQLinAlg3 4.4.003.

Determine whether the given set of vectors spans $\mathbb{R}^2$.

$\{(6, -4), (-2, 4/3), (3, -2)\}$

- The given set of vectors does span $\mathbb{R}^2$.
- The given set of vectors does not span $\mathbb{R}^2$.

23.–/3 pointsGoodDiffEQLinAlg3 4.4.008.

Show that $\mathbf{v}_1 = (2, -1)$, $\mathbf{v}_2 = (3, 2)$ span $\mathbb{R}^2$ by expressing (x_1, x_2) as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$ for any given $(x_1, x_2) \in \mathbb{R}^2$.

$(x_1, x_2) = \Big(\quad \Big)\mathbf{v}_1 + \Big(\quad \Big)\mathbf{v}_2.$

Express the vector $\mathbf{v} = (5, -8)$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2$.

$(5, -8) =$

24.–/1 pointsGoodDiffEQLinAlg3 4.4.TF.001.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

The linear span of a set of vectors in a vector space V forms a subspace of V.

- True. When a linear span of a set of vectors is formed, the span becomes not closed under addition and under scalar multiplication.
- True. When a linear span of a set of vectors is formed, the span becomes closed under addition and under scalar multiplication.
- False. When a linear span of a set of vectors is formed, the span becomes closed under addition and under scalar multiplication.
- False. When a linear span of a set of vectors is formed, the span becomes closed under addition and not closed under scalar multiplication.
- False. When a linear span of a set of vectors is formed, the span becomes not closed under addition and closed under scalar multiplication.

25.–/1 pointsGoodDiffEQLinAlg3 4.4.TF.003.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

If S is a spanning set for a vector space V and W is a subspace of V, then S is a spanning set for W.

- True. Some but not all vectors in W can be expressed as a linear combination of vectors in S.
- True. Every vector in W can be expressed as a linear combination of vectors in S.
- False. Every vector in W can be expressed as a linear combination of vectors in S.
- False. No vector in W can be expressed as a linear combination of vectors in S.
- False. Every vector in V can be expressed as a linear combination of vectors in W, but not every vector in W can be.

26.–/1 pointsGoodDiffEQLinAlg3 4.4.TF.008.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

If S is a spanning set for a vector space V, then any proper subset S' of S is not a spanning set for V.

- True. Every set in S' is a subset of S so it is not a spanning set for V.
- True. Every set in S' is not a subset of S so it is not a spanning set for V.
- False. Consider $V = \mathbb{R}^2$, and consider the spanning set $S = \{(1, 0), (0, 1), (1, 1)\}$. The proper subset $S' = \{(1, 0), (0, 1)\}$ is still a spanning set for V.
- False. Consider $V = \mathbb{R}^2$, and consider the spanning set $S = \{(1, 0), (0, 1), (1, 1)\}$. The proper subset $S' = \{(1, 0), (0, 1), (1, 1)\}$ is still a spanning set for V.
- False. Consider $V = \mathbb{R}^2$, and consider the spanning set $S = \{(1, 0), (0, 1), (1, 1)\}$. The proper subset $S' = \{(1, 0), (0, 0)\}$ is still a spanning set for V.

27.–/1 pointsGoodDiffEQLinAlg3 4.4.TF.010.

Decide if the given statement is **true** or **false**, and give a brief justification for your answer. If true, you can quote a relevant definition or theorem from the text. If false, provide an example, illustration, or brief explanation of why the statement is false.

A spanning set for the vector space P_2 must contain a polynomial of each degree 0, 1, and 2.

- True. Every spanning set for P_2 is of the form $\{x^2, x, a\}$.
- False. Consider the spanning set $\{x^2, x^2 + x\}$ for P_2.
- False. Consider the spanning set $\{x^2, x, 1\}$ for P_2.
- False. Consider the spanning set $\{x^2, x^2 + x, x^2 + 1\}$ for P_2.
- False. A spanning set for the vector space P_2 must contain a polynomial of each degree 1, 2, and 3.