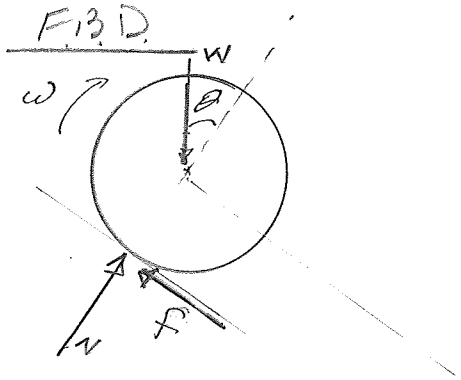
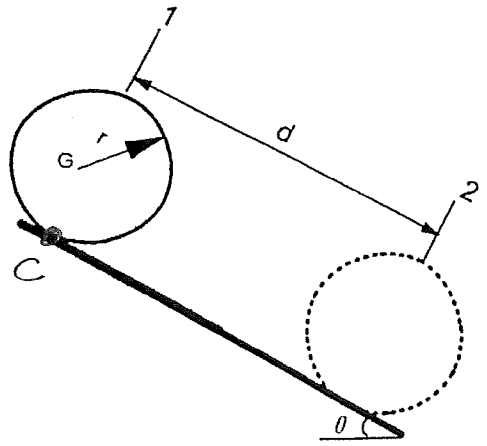
A disk of mass m starts from rest and rolls without slipping on an inclined surface. Find: a) angular velocity, b) velocity of the center of gravity





$T_1 + \sum U_{12} = T_2$ ($T_1 = 0$)

Note: ① F and N DO NO WORK since at each instant they act on a point C which has ZERO velocity [PT. of contact is the I.C.]

Therefore:

$U_{12} = mg \sin\theta\, d$

(2) ONLY force doing work is weight.

$T_2 = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$

(3) From the fact that the DISK Rolls w/o slip

$v_G = r\omega$ (NO SLIP)

Substituting

$T_2 = \frac{1}{2} m [r\omega]^2 + \frac{1}{2} I_G \omega^2$

$T_2 = \frac{1}{2}\omega^2 [I_G + mr^2] = \frac{1}{2}\omega^2 [I_{CG}]$

(4) $I_G + mr^2 = I_C$ } PARALLEL-AXIS THEOREM

$C = I.C.$ ∴ zero velocity

$\therefore T_2 = \frac{1}{2} I_C \omega^2$

(5) FOR A DISK I about point of contact is:

$I_C = \frac{3}{2} m R^2$

Thus:

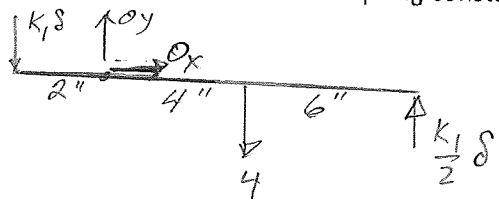
$0 + mg \sin\theta\, d = \frac{1}{2} I_C \omega^2$

$mg \sin\theta\, d = \frac{1}{2} \frac{3}{2} m R^2 \omega^2$

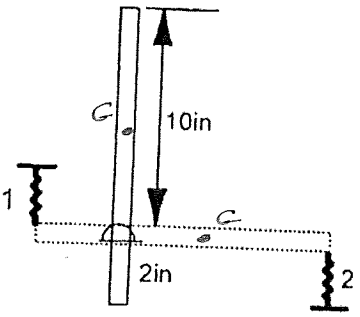
Solving:

$\therefore \omega = \sqrt{\frac{4 g \sin\theta\, d}{3R^2}}$; $v_G = r\omega = \sqrt{\frac{4}{3} g d \sin\theta}$

The angular velocity of the 4-lb homogeneous bar is 8rad/s clockwise in the vertical position. After the bar rotates 90^0 clockwise, it strikes the two springs and continues to rotate until spring 1 is compressed 0.20in. The spring constant of spring 1 is twice that of spring 2. The springs are unstressed when the bar first strikes them. Find the spring constant of spring 1.



Note:
$g = 386\ in/s^2$



O (AT rest)

$T_1 + \Sigma U_{12} = T_2$

$$4(4+0.4) - \int_0^{0.2} K_1 s\,ds - \int_0^{1} \frac{K_1}{2} s\,ds = \frac{1}{2}\left[\frac{1}{12}\left(\frac{4}{386}\right)(12)^2 + \frac{4}{386}(4)^2\right]\left[0^2 - (8)^2\right]$$

$$17.6 - K_1\frac{s^2}{2}\Big|_0^{0.2} - \frac{K_1}{2}\frac{s^2}{2}\Big|_0^{1} = -18.573$$

$$17.6 - 0.02K_1 - 0.25K_1 = -18.573$$

$$K_1 = 133.97 \approx 134\ lb/in$$