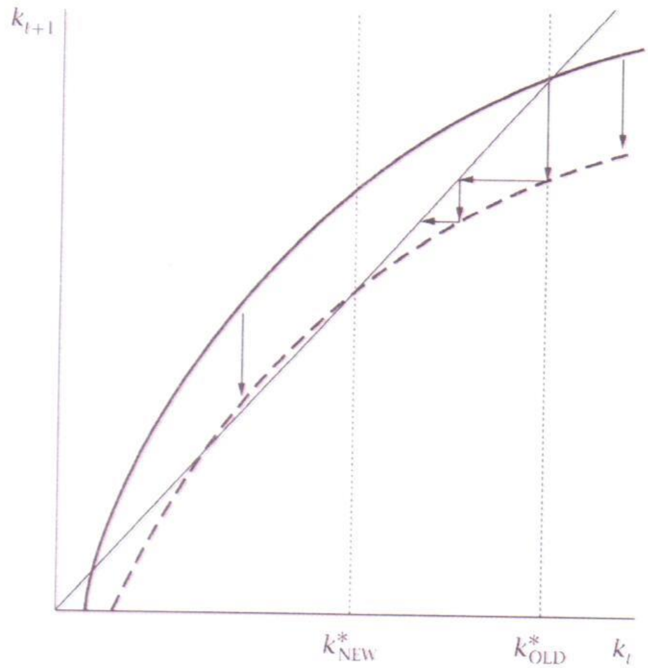

FIGURE 2.15 The effects of a permanent increase in government purchases

are high, k gradually falls and r gradually increases. Once G returns to G_L, k rises gradually back to its initial level.[29]

Problems

2.1. Consider N firms each with the constant-returns-to-scale production function $Y = F(K, AL)$, or (using the intensive form) $Y = ALf(k)$. Assume $f'(\bullet) > 0$, $f''(\bullet) < 0$. Assume that all firms can hire labor at wage wA and rent capital at cost r, and that all firms have the same value of A.

(a) Consider the problem of a firm trying to produce Y units of output at minimum cost. Show that the cost-minimizing level of k is uniquely defined and is independent of Y, and that all firms therefore choose the same value of k.

(b) Show that the total output of the N cost-minimizing firms equals the output that a single firm with the same production function has if it uses all the labor and capital used by the N firms.

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[29] The result that future values of G do not affect the current behavior of the economy does not depend on the assumption of logarithmic utility. Without logarithmic utility, the saving of the current period's young depends on the rate of return as well as on after-tax labor income. But the rate of return is determined by the next period's capital-labor ratio, which is not affected by government purchases in that period.

2.2. The elasticity of substitution with constant-relative-risk-aversion utility. Consider an individual who lives for two periods and whose utility is given by equation (2.43). Let P_1 and P_2 denote the prices of consumption in the two periods, and let W denote the value of the individual's lifetime income; thus the budget constraint is $P_1C_1 + P_2C_2 = W$.

(*a*) What are the individual's utility-maximizing choices of C_1 and C_2, given P_1, P_2, and W?

(*b*) The elasticity of substitution between consumption in the two periods is $-[(P_1/P_2)/(C_1/C_2)][\partial(C_1/C_2)/\partial(P_1/P_2)]$, or $-\partial \ln(C_1/C_2)/\partial \ln(P_1/P_2)$. Show that with the utility function (2.43), the elasticity of substitution between C_1 and C_2 is $1/\theta$.

2.3. (*a*) Suppose it is known in advance that at some time t_0 the government will confiscate half of whatever wealth each household holds at that time. Does consumption change discontinuously at time t_0? If so, why (and what is the condition relating consumption immediately before t_0 to consumption immediately after)? If not, why not?

(*b*) Suppose it is known in advance that at t_0 the government will confiscate from each household an amount of wealth equal to half of the wealth of the average household at that time. Does consumption change discontinuously at time t_0? If so, why (and what is the condition relating consumption immediately before t_0 to consumption immediately after)? If not, why not?

2.4. Assume that the instantaneous utility function $u(C)$ in equation (2.1) is $\ln C$. Consider the problem of a household maximizing (2.1) subject to (2.6). Find an expression for C at each time as a function of initial wealth plus the present value of labor income, the path of $r(t)$, and the parameters of the utility function.

2.5. Consider a household with utility given by (2.1)–(2.2). Assume that the real interest rate is constant, and let W denote the household's initial wealth plus the present value of its lifetime labor income (the right-hand side of [2.6]). Find the utility-maximizing path of C, given r, W, and the parameters of the utility function.

2.6. The productivity slowdown and saving. Consider a Ramsey–Cass–Koopmans economy that is on its balanced growth path, and suppose there is a permanent fall in g.

(*a*) How, if at all, does this affect the $\dot{k} = 0$ curve?

(*b*) How, if at all, does this affect the $\dot{c} = 0$ curve?

(*c*) What happens to c at the time of the change?

(*d*) Find an expression for the impact of a marginal change in g on the fraction of output that is saved on the balanced growth path. Can one tell whether this expression is positive or negative?

(*e*) For the case where the production function is Cobb–Douglas, $f(k) = k^{\alpha}$, rewrite your answer to part (*d*) in terms of ρ, n, g, θ, and α. (Hint: Use the fact that $f'(k^*) = \rho + \theta g$.)

2.7. Describe how each of the following affects the $\dot{c} = 0$ and $\dot{k} = 0$ curves in Figure 2.5, and thus how they affect the balanced-growth-path values of c and k:

(*a*) A rise in θ.

(*b*) A downward shift of the production function.

(*c*) A change in the rate of depreciation from the value of zero assumed in the text to some positive level.

2.8. Derive an expression analogous to (2.39) for the case of a positive depreciation rate.

2.9. A closed-form solution of the Ramsey model. (This follows Smith, 2006.) Consider the Ramsey model with Cobb–Douglas production, $y(t) = k(t)^{\alpha}$, and with the coefficient of relative risk aversion (θ) and capital's share (α) assumed to be equal.

(*a*) What is k on the balanced growth path (k^*)?

(*b*) What is c on the balanced growth path (c^*)?

(*c*) Let $z(t)$ denote the capital-output ratio, $k(t)/y(t)$, and $x(t)$ denote the consumption-capital ratio, $c(t)/k(t)$. Find expressions for $\dot{z}(t)$ and $\dot{x}(t)/x(t)$ in terms of z, x, and the parameters of the model.

(*d*) Tentatively conjecture that x is constant along the saddle path. Given this conjecture:

(*i*) Find the path of z given its initial value, $z(0)$.

(*ii*) Find the path of y given the initial value of k, $k(0)$. Is the speed of convergence to the balanced growth path, $d\ln[y(t) - y^*]/dt$, constant as the economy moves along the saddle path?

(*e*) In the conjectured solution, are the equations of motion for c and k, (2.24) and (2.25), satisfied?

2.10. Capital taxation in the Ramsey–Cass–Koopmans model. Consider a Ramsey–Cass–Koopmans economy that is on its balanced growth path. Suppose that at some time, which we will call time 0, the government switches to a policy of taxing investment income at rate τ. Thus the real interest rate that households face is now given by $r(t) = (1 - \tau)f'(k(t))$. Assume that the government returns the revenue it collects from this tax through lump-sum transfers. Finally, assume that this change in tax policy is unanticipated.

(*a*) How, if at all, does the tax affect the $\dot{c} = 0$ locus? The $\dot{k} = 0$ locus?

(*b*) How does the economy respond to the adoption of the tax at time 0? What are the dynamics after time 0?

(*c*) How do the values of c and k on the new balanced growth path compare with their values on the old balanced growth path?

(*d*) (This is based on Barro, Mankiw, and Sala-i-Martin, 1995.) Suppose there are many economies like this one. Workers' preferences are the same in each country, but the tax rates on investment income may vary across countries. Assume that each country is on its balanced growth path.

(*i*) Show that the saving rate on the balanced growth path, $(y^* - c^*)/y^*$, is decreasing in τ.

(*ii*) Do citizens in low-τ, high-k^*, high-saving countries have any incentive to invest in low-saving countries? Why or why not?

(*e*) Does your answer to part (*c*) imply that a policy of *subsidizing* investment (that is, making $\tau < 0$), and raising the revenue for this subsidy through lump-sum taxes, increases welfare? Why or why not?

(*f*) How, if at all, do the answers to parts (*a*) and (*b*) change if the government does not rebate the revenue from the tax but instead uses it to make government purchases?

2.11. Using the phase diagram to analyze the impact of an anticipated change. Consider the policy described in Problem 2.10, but suppose that instead of announcing and implementing the tax at time 0, the government announces at time 0 that at some later time, time t_1, investment income will begin to be taxed at rate τ.

(*a*) Draw the phase diagram showing the dynamics of c and k after time t_1.

(*b*) Can c change discontinuously at time t_1? Why or why not?

(*c*) Draw the phase diagram showing the dynamics of c and k before t_1.

(*d*) In light of your answers to parts (*a*), (*b*), and (*c*), what must c do at time 0?

(*e*) Summarize your results by sketching the paths of c and k as functions of time.

2.12. Using the phase diagram to analyze the impact of unanticipated and anticipated temporary changes. Analyze the following two variations on Problem 2.11:

(*a*) At time 0, the government announces that it will tax investment income at rate τ from time 0 until some later date t_1; thereafter investment income will again be untaxed.

(*b*) At time 0, the government announces that from time t_1 to some later time t_2, it will tax investment income at rate τ; before t_1 and after t_2, investment income will not be taxed.

2.13. The analysis of government policies in the Ramsey-Cass-Koopmans model in the text assumes that government purchases do not affect utility from private consumption. The opposite extreme is that government purchases and private consumption are perfect substitutes. Specifically, suppose that the utility function (2.12) is modified to be

$$U = B\int_{t=0}^{\infty} e^{-\beta t}\frac{[c(t)+G(t)]^{1-\theta}}{1-\theta}\,dt.$$

If the economy is initially on its balanced growth path and if households' preferences are given by U, what are the effects of a temporary increase in saying that all approaches have merit avoids the harder question of when different approaches are more valuable and what mix is appropriate for analyzing a particular issue. Unfortunately, as with the issue of calibration versus other approaches to evaluating models' empirical performance, we have little systematic evidence on this question. As a result, macroeconomists have little choice but to make tentative judgments, based on the currently available models and evidence, about what types of inquiry are most promising. And they must remain open to the possibility that those judgments will need to be revised.

Problems

5.1. Redo the calculations reported in Table 5.1, 5.2, or 5.3 for any country other than the United States.

5.2. Redo the calculations reported in Table 5.3 for the following:

(*a*) Employees' compensation as a share of national income.

(*b*) The labor force participation rate.

(*c*) The federal government budget deficit as a share of GDP.

(*d*) The Standard and Poor's 500 composite stock price index.

(*e*) The difference in yields between Moody's Baa and Aaa bonds.

(*f*) The difference in yields between 10-year and 3-month U.S. Treasury securities.

(*g*) The weighted average exchange rate of the U.S. dollar against major currencies.

5.3. Let A_0 denote the value of A in period 0, and let the behavior of $\ln A$ be given by equations (5.8)–(5.9).

(*a*) Express $\ln A_1$, $\ln A_2$, and $\ln A_3$ in terms of $\ln A_0$, ε_{A1}, ε_{A2}, ε_{A3}, $\overline{A}$, and g.

(*b*) In light of the fact that the expectations of the ε_A's are zero, what are the expectations of $\ln A_1$, $\ln A_2$, and $\ln A_3$ given $\ln A_0, \overline{A}$, and g?

5.4. Suppose the period-t utility function, u_t, is $u_t = \ln c_t + b(1 - \ell_t)^{1-\gamma}/(1-\gamma)$, $b > 0$, $\gamma > 0$, rather than (5.7).

(*a*) Consider the one-period problem analogous to that investigated in (5.12)–(5.15). How, if at all, does labor supply depend on the wage?

(*b*) Consider the two-period problem analogous to that investigated in (5.16)–(5.21). How does the relative demand for leisure in the two periods depend on the relative wage? How does it depend on the interest rate? Explain intuitively why γ affects the responsiveness of labor supply to wages and the interest rate.