Module 6

The coefficients on the average inflation variables are essentially the same as in the previous regression, and they remain statistically significant. The variability terms, in contrast, play little role. The null hypothesis that the coefficients on both σ_x and σ_x^2 are zero cannot be rejected at any reasonable confidence level, and the point estimates imply that reasonable changes in σ_x have quantitatively small effect on τ. For example, a change in σ_x from 0.05 to 0.10 changes τ by only 0.04. Thus the results appear to favor the menu-cost view over the Lucas model.[31]

Kiley (2000) extends the analysis to the persistence of output movements. He first notes that menu-cost models imply that departures of output from normal are less persistent when average inflation is higher. The intuition is again that higher average inflation increases the frequency of price adjustment, and therefore causes the economy to return to its flexible-price equilibrium more rapidly after a shock. He finds that the data support this implication as well.

Problems

6.1. Describe how, if at all, each of the following developments affect the curves in Figure 6.1:

(*a*) The coefficient of relative risk aversion, θ, rises.

(*b*) The curvature of $\Gamma(\bullet)$, ν falls.

(*c*) We modify the utility function, (6.2), to be $\sum \beta^t[U(C_t) + B\Gamma(M_t/P_t) - V(L_t)]$, $B > 0$, and B falls.

6.2. The Baumol-Tobin model. (Baumol, 1952; Tobin, 1956.) Consider a consumer with a steady flow of real purchases of amount αY, $0 < \alpha \leq 1$, that are made with money. The consumer chooses how often to convert bonds, which pay a constant interest rate of i, into money, which pays no interest. If the consumer chooses an interval of τ, his or her money holdings decline linearly from $\alpha YP\tau$ after each conversion to zero at the moment of the next conversion (here P is the price level, which is assumed constant). Each conversion has a fixed real cost of C. The consumer's problem is to choose τ to minimize the average cost per unit time of conversions and foregone interest.

(*a*) Find the optimal value of τ.

(*b*) What are the consumer's average real money holdings? Are they decreasing in i and increasing in Y? What is the elasticity of average money holdings with respect to i? With respect to Y?

6.3. The multiplier-accelerator. (Samuelson, 1939.) Consider the following model of income determination. (1) Consumption depends on the previous period's

[31] The lack of a discernable link between σ_x and τ, however, is a puzzle not only for the Lucas model but also for models based on barriers to price adjustment: an increase in the variability of shocks should make firms change their prices more often, and should therefore reduce the real impact of a change in aggregate demand.

income: $C_t = a + bY_{t-1}$. (2) The desired capital stock (or inventory stock) is proportional to the previous period's output: $K_t^* = cY_{t-1}$. (3) Investment equals the difference between the desired capital stock and the stock inherited from the previous period: $I_t = K_t^* - K_{t-1} = K_t^* - cY_{t-2}$. (4) Government purchases are constant: $G_t = \overline{G}$. (5) $Y_t = C_t + I_t + G_t$.

(*a*) Express Y_t in terms of Y_{t-1}, Y_{t-2}, and the parameters of the model.

(*b*) Suppose $b = 0.9$ and $c = 0.5$. Suppose there is a one-time disturbance to government purchases; specifically, suppose that G is equal to $\overline{G} + 1$ in period t and is equal to $\overline{G}$ in all other periods. How does this shock affect output over time?

6.4. The analysis of Case 1 in Section 6.2 assumes that employment is determined by labor demand. Under perfect competition, however, employment at a given real wage will equal the minimum of demand and supply; this is known as the *short-side rule*. Draw diagrams showing the situation in the labor market when employment is determined by the short-side rule if:

(*a*) P is at the level that generates the maximum possible output.

(*b*) P is above the level that generates the maximum possible output.

6.5. Productivity growth, the Phillips curve, and the natural rate. (Braun, 1984, and Ball and Moffitt, 2001.) Let g_t be growth of output per worker in period t, π_t inflation, and π_t^w wage inflation. Suppose that initially g is constant and equal to g^L and that unemployment is at the level that causes inflation to be constant. g then rises permanently to $g^H > g^L$. Describe the path of u_t that would keep price inflation constant for each of the following assumptions about the behavior of price and wage inflation. Assume $\phi > 0$ in all cases.

(*a*) (The price-price Phillips curve.) $\pi_t = \pi_{t-1} - \phi(u_t - \bar{u})$, $\pi_t^w = \pi_t + g_t$.

(*b*) (The wage-wage Phillips curve.) $\pi_t^w = \pi_{t-1}^w - \phi(u_t - \bar{u})$, $\pi_t = \pi_t^w - g_t$.

(*c*) (The pure wage-price Phillips curve.) $\pi_t^w = \pi_{t-1} - \phi(u_t - \bar{u})$, $\pi_t = \pi_t^w - g_t$.

(*d*) (The wage-price Phillips curve with an adjustment for normal productivity growth.) $\pi_t^w = \pi_{t-1} + \hat{g}_t - \phi(u_t - \bar{u})$, $\hat{g}_t = \rho\hat{g}_{t-1} + (1 - \rho)g_t$, $\pi_t = \pi_t^w - g_t$. Assume that $0 < \rho < 1$ and that initially $\hat{g} = g^L$.

6.6. The central bank's ability to control the real interest rate. Suppose the economy is described by two equations. The first is the *IS* equation, which for simplicity we assume takes the traditional form, $Y_t = -r_t/\theta$. The second is the money-market equilibrium condition, which we can write as $m - p = L(r + \pi^e, Y)$, $L_{r+\pi^e} < 0$, $L_Y > 0$, where m and p denote ln M and ln P.

(*a*) Suppose $P = \overline{P}$ and $\pi^e = 0$. Find an expression for dr/dm. Does an increase in the money supply lower the real interest rate?

(*b*) Suppose prices respond partially to increases in money. Specifically, assume that dp/dm is exogenous, with $0 < dp/dm < 1$. Continue to assume $\pi^e = 0$. Find an expression for dr/dm. Does an increase in the money supply lower the real interest rate? Does achieving a given change in r require a change in m smaller, larger, or the same size as in part (*a*)?

(*c*) Suppose increases in money also affect expected inflation. Specifically, assume that $d\pi^e/dm$ is exogenous, with $d\pi^e/dm > 0$. Continue to assume $0 < dp/dm < 1$. Find an expression for dr/dm. Does an increase in the money supply lower the real interest rate? Does achieving a given change in r require a change in m smaller, larger, or the same size as in part (*b*)?

(*d*) Suppose there is complete and instantaneous price adjustment: $dp/dm = 1$, $d\pi^e/dm = 0$. Find an expression for dr/dm. Does an increase in the money supply lower the real interest rate?

6.7. The liquidity trap. Consider the following model. The dynamics of inflation are given by the continuous-time version of (6.22)–(6.23): $\dot{\pi}(t) = \lambda[y(t) - \bar{y}(t)]$, $\lambda > 0$. The *IS* curve takes the traditional form, $y(t) = -[i(t) - \pi(t)]/\theta$, $\theta > 0$. The central bank sets the interest rate according to (6.26), but subject to the constraint that the nominal interest rate cannot be negative: $i(t) = \max[0, \pi(t) + r(y(t) - \bar{y}(t), \pi(t))]$. For simplicity, normalize $\bar{y}(t) = 0$ for all t.

(*a*) Sketch the aggregate demand curve for this model—that is, the set of points in (y, π) space that satisfy the *IS* equation and the rule above for the interest rate.

(*b*) Let $(\tilde{y}, \tilde{\pi})$ denote the point on the aggregate demand curve where $\pi + r(y, \pi) = 0$. Sketch the paths of y and π over time if:

(*i*) $\tilde{y} > 0$, $\pi(0) > \tilde{\pi}$, and $y(0) < 0$.

(*ii*) $\tilde{y} < 0$ and $\pi(0) > \tilde{\pi}$.

(*iii*) $\tilde{y} > 0$, $\pi(0) < \tilde{\pi}$, and $y(0) < 0$.[32]

6.8. Consider the model in equations (6.27)–(6.30). Suppose, however, that there are shocks to the *MP* equation but not to the *IS* equation. Thus $r_t = by_t + u_t^{MP}$, $u_t^{MP} = \rho_{MP} u_{t-1}^{MP} + e_t^{MP}$ (where $-1 < \rho_{MP} < 1$ and e^{MP} is white noise), and $y_t = E_t y_{t+1} - \frac{1}{\theta} r_t$. Find the expression analogous to (6.35).

6.9. (*a*) Consider the model in equations (6.27)–(6.30). Solve the model using the method of undetermined coefficients. That is, conjecture that the solution takes the form $y_t = Au_t^{IS}$, and find the value that A must take for the equations of the model to hold. (Hint: The fact that $y_t = Au_t^{IS}$ for all t implies $E_t y_{t+1} = AE_t u_{t+1}^{IS}$.)

(*b*) Now modify the *MP* equation to be $r_t = by_t + c\pi_t$. Conjecture that the solution takes the form $y_t = Au_t^{IS} + B\pi_{t-1}$, $\pi_t = Cu_t^{IS} + D\pi_{t-1}$. Find (but do not solve) four equations that A, B, C, and D must satisfy for the equations of the model to hold.

6.10. Multiple equilibria with menu costs. (Ball and D. Romer, 1991.) Consider an economy consisting of many imperfectly competitive firms. The profits that a firm loses relative to what it obtains with $p_i = p^*$ are $K(p_i - p^*)^2$, $K > 0$. As usual, $p^* = p + \phi y$ and $y = m - p$. Each firm faces a fixed cost Z of changing its nominal price.

[32] See Section 11.6 for more on the zero lower bound on the nominal interest rate.

price as $p_i = w_i + (1-\alpha)\ell_i - s$. The production function and the pricing equation then imply that $y_i = y - \phi(w_i - w)$, where $\phi \equiv \alpha\eta/[\alpha + (1-\alpha)\eta]$.

(*i*) What is employment at firm i, ℓ_i, as a function of $m, s, \alpha, \eta, \theta$, and θ_i?

(*ii*) What value of θ_i minimizes the variance of ℓ_i?

(*iii*) Find the Nash equilibrium value of θ. That is, find the value of θ such that if aggregate indexation is given by θ, the representative firm minimizes the variance of ℓ_i by setting $\theta_i = \theta$. Compare this value with the value found in part (*b*).

6.13. Thick-market effects and coordination failure. (This follows Diamond, 1982.)[33] Consider an island consisting of N people and many palm trees. Each person is in one of two states, not carrying a coconut and looking for palm trees (state P) or carrying a coconut and looking for other people with coconuts (state C). If a person without a coconut finds a palm tree, he or she can climb the tree and pick a coconut; this has a cost (in utility units) of c. If a person with a coconut meets another person with a coconut, they trade and eat each other's coconuts; this yields $\bar{u}$ units of utility for each of them. (People cannot eat coconuts that they have picked themselves.)

A person looking for coconuts finds palm trees at rate b per unit time. A person carrying a coconut finds trading partners at rate aL per unit time, where L is the total number of people carrying coconuts. a and b are exogenous.

Individuals' discount rate is r. Focus on steady states; that is, assume that L is constant.

(*a*) Explain why, if everyone in state P climbs a palm tree whenever he or she finds one, then $rV_P = b(V_C - V_P - c)$, where V_P and V_C are the values of being in the two states.

(*b*) Find the analogous expression for V_C.

(*c*) Solve for $V_C - V_P$, V_C, and V_P in terms of $r, b, c, \bar{u}, a$, and L.

(*d*) What is L, still assuming that anyone in state P climbs a palm tree whenever he or she finds one? Assume for simplicity that $aN = 2b$.

(*e*) For what values of c is it a steady-state equilibrium for anyone in state P to climb a palm tree whenever he or she finds one? (Continue to assume $aN = 2b$.)

(*f*) For what values of c is it a steady-state equilibrium for no one who finds a tree to climb it? Are there values of c for which there is more than one steady-state equilibrium? If there are multiple equilibria, does one involve higher welfare than the other? Explain intuitively.

6.14. Consider the problem facing an individual in the Lucas model when P_i/P is unknown. The individual chooses L_i to maximize the expectation of U_i; U_i continues to be given by equation (6.72).

[33] The solution to this problem requires dynamic programming (see Section 10.4).