

can be caused by information not about fundamentals, but about beliefs about fundamentals, or about beliefs about beliefs about fundamentals.

## Problems

- 12.1. The stability of fiscal policy.** (Blinder and Solow, 1973.) By definition, the budget deficit equals the rate of change of the amount of debt outstanding:  $\delta(t) \equiv \dot{D}(t)$ . Define  $d(t)$  to be the ratio of debt to output:  $d(t) = D(t)/Y(t)$ . Assume that  $Y(t)$  grows at a constant rate  $g > 0$ .
- (a) Suppose that the deficit-to-output ratio is constant:  $\delta(t)/Y(t) = a$ , where  $a > 0$ .
- Find an expression for  $\dot{d}(t)$  in terms of  $a$ ,  $g$ , and  $d(t)$ .
  - Sketch  $\dot{d}(t)$  as a function of  $d(t)$ . Is this system stable?
- (b) Suppose that the ratio of the primary deficit to output is constant and equal to  $a > 0$ . Thus the total deficit at  $t$ ,  $\delta(t)$ , is given by  $\delta(t) = aY(t) + r(t)D(t)$ , where  $r(t)$  is the interest rate at  $t$ . Assume that  $r$  is an increasing function of the debt-to-output ratio:  $r(t) = r(d(t))$ , where  $r'(\bullet) > 0$ ,  $r''(\bullet) > 0$ ,  $\lim_{d \rightarrow -\infty} r(d) < g$ ,  $\lim_{d \rightarrow \infty} r(d) > g$ .
- Find an expression for  $\dot{d}(t)$  in terms of  $a$ ,  $g$ , and  $d(t)$ .
  - Sketch  $\dot{d}(t)$  as a function of  $d(t)$ . In the case where  $a$  is sufficiently small that  $\dot{d}$  is negative for some values of  $d$ , what are the stability properties of the system? What about the case where  $a$  is sufficiently large that  $\dot{d}$  is positive for all values of  $d$ ?
- 12.2. Precautionary saving, non-lump-sum taxation, and Ricardian equivalence.** (Leland, 1968, and Barsky, Mankiw, and Zeldes, 1986.) Consider an individual who lives for two periods. The individual has no initial wealth and earns labor incomes of amounts  $Y_1$  and  $Y_2$  in the two periods.  $Y_1$  is known, but  $Y_2$  is random; assume for simplicity that  $E[Y_2] = Y_1$ . The government taxes income at rate  $\tau_1$  in period 1 and  $\tau_2$  in period 2. The individual can borrow and lend at a fixed interest rate, which for simplicity is assumed to be zero. Thus second-period consumption is  $C_2 = (1 - \tau_1)Y_1 - C_1 + (1 - \tau_2)Y_2$ . The individual chooses  $C_1$  to maximize expected lifetime utility,  $U(C_1) + E[U(C_2)]$ .
- Find the first-order condition for  $C_1$ .
  - Show that  $E[C_2] = C_1$  if  $Y_2$  is not random or if utility is quadratic.
  - Show that if  $U'''(\bullet) > 0$  and  $Y_2$  is random,  $E[C_2] > C_1$ .
  - Suppose that the government marginally lowers  $\tau_1$  and raises  $\tau_2$  by the same amount, so that its expected total revenue,  $\tau_1 Y_1 + \tau_2 E[Y_2]$ , is unchanged. Implicitly differentiate the first-order condition in part (a) to find an expression for how  $C_1$  responds to this change.
  - Show that  $C_1$  is unaffected by this change if  $Y_2$  is not random or if utility is quadratic.

(f) Show that  $C_1$  increases in response to this change if  $U'''(\bullet) > 0$  and  $Y_2$  is random.

- 12.3. Consider the Barro tax-smoothing model. Suppose that output,  $Y$ , and the real interest rate,  $r$ , are constant, and that the level of government debt outstanding at time 0 is zero. Suppose that there will be a temporary war from time 0 to time  $\tau$ . Thus  $G(t)$  equals  $G_H$  for  $0 \leq t \leq \tau$ , and equals  $G_L$  thereafter, where  $G_H > G_L$ . What are the paths of taxes,  $T(t)$ , and government debt outstanding,  $D(t)$ ?
- 12.4. Consider the Barro tax-smoothing model. Suppose there are two possible values of  $G(t)$ — $G_H$  and  $G_L$ —with  $G_H > G_L$ . Transitions between the two values follow Poisson processes (see Section 7.4). Specifically, if  $G$  equals  $G_H$ , the probability per unit time that purchases fall to  $G_L$  is  $a$ ; if  $G$  equals  $G_L$ , the probability per unit time that purchases rise to  $G_H$  is  $b$ . Suppose also that output,  $Y$ , and the real interest rate,  $r$ , are constant and that distortion costs are quadratic.
- (a) Derive expressions for taxes at a given time as a function of whether  $G$  equals  $G_H$  or  $G_L$ , the amount of debt outstanding, and the exogenous parameters. (Hint: Use dynamic programming, described in Section 10.4, to find an expression for the expected present value of the revenue the government must raise as a function of  $G$ , the amount of debt outstanding, and the exogenous parameters.)
- (b) Discuss your results. What is the path of taxes during an interval when  $G$  equals  $G_H$ ? Why are taxes not constant during such an interval? What happens to taxes at a moment when  $G$  falls to  $G_L$ ? What is the path of taxes during an interval when  $G$  equals  $G_L$ ?
- 12.5. If the tax rate follows a random walk (and if the variance of its innovations is bounded from below by a strictly positive number), then with probability 1 it will eventually exceed 100 percent or be negative. Does this observation suggest that the tax-smoothing model with quadratic distortion costs is not useful as either a positive or normative model of fiscal policy, since it has an implication that is both clearly incorrect as a description of the world and clearly undesirable as a prescription for policy? Explain your answer briefly.
- 12.6. **The Condorcet paradox.** Suppose there are three voters, 1, 2, and 3, and three possible policies, A, B, and C. Voter 1's preference ordering is A, B, C; voter 2's is B, C, A; and voter 3's is C, A, B. Does any policy win a majority of votes in a two-way contest against each of the alternatives? Explain.
- 12.7. Consider the Tabellini-Alesina model in the case where  $\alpha$  can only take on the values 0 and 1. Suppose that there is some initial level of debt,  $D_0$ . How, if at all, does  $D_0$  affect the deficit in period 1?
- 12.8. Consider the Tabellini-Alesina model in the case where  $\alpha$  can only take on the values 0 and 1. Suppose that the amount of debt to be issued,  $D$ , is determined before the preferences of the period-1 median voter are known. Specifically, voters vote on  $D$  at a time when the probabilities that  $\alpha_1^{\text{MED}} = 1$  and that  $\alpha_2^{\text{MED}} = 1$  are equal. Let  $\pi$  denote this common value. Assume that the draws of the two median voters are independent.

- (a) What is the expected utility of an individual with  $\alpha = 1$  as a function of  $D$ ,  $\pi$ , and  $W$ ?
- (b) What is the first-order condition for this individual's most preferred value of  $D$ ? What is the associated value of  $D$ ?
- (c) What is the most preferred value of  $D$  of an individual with  $\alpha = 0$ ?
- (d) Given these results, if voters vote on  $D$  before the period-1 median voter is known, what value of  $D$  does the median voter prefer?
- (e) Explain briefly how, if at all, the question analyzed in part (d) differs from the question of whether individuals will support a balanced-budget requirement if it is proposed before the preferences of the period-1 median voter are known.

**12.9.** Consider the Tabellini-Alesina model in the case where  $\alpha$  can only take on the values 0 and 1. Suppose, however, that there are 3 periods. The period-1 median voter sets policy in periods 1 and 2, but in period 3 a new median voter sets policy. Assume that the period-1 median voter's  $\alpha$  is 1, and that the probability that the period-3 median voter's  $\alpha$  is 1 is  $\pi$ .

- (a) Does  $M_1 = M_2$ ?
- (b) Suppose that after choosing purchases in period 1, the period-1 median voter learns that the probability that the period-3 median voter's  $\alpha$  will be 1 is not  $\pi$  but  $\pi'$ , where  $\pi' < \pi$ . How does this news affect his or her choice of purchases in period 2?

**12.10. The Persson-Svensson model.** (Persson and Svensson, 1989.) Suppose there are two periods. Government policy will be controlled by different policymakers in the two periods. The objective function of the period- $t$  policymaker is  $U + \alpha_t[V(G_1) + V(G_2)]$ , where  $U$  is citizens' utility from their private consumption;  $\alpha_t$  is the weight that the period- $t$  policymaker puts on public consumption;  $G_t$  is public consumption in period  $t$ ; and  $V(\bullet)$  satisfies  $V'(\bullet) > 0$ ,  $V''(\bullet) < 0$ . Private utility,  $U$ , is given by  $U = W - C(T_1) - C(T_2)$ , where  $W$  is the endowment;  $T_t$  is taxes in period  $t$ ; and  $C(\bullet)$ , the cost of raising revenue, satisfies  $C'(\bullet) \geq 1$ ,  $C''(\bullet) > 0$ . All government debt must be paid off at the end of period 2. This implies  $T_2 = G_2 + D$ , where  $D = G_1 - T_1$  is the amount of government debt issued in period 1 and where the interest rate is assumed to equal zero.

- (a) Find the first-order condition for the period-2 policymaker's choice of  $G_2$  given  $D$ . (Note: Throughout, assume that the solutions to the policymakers' maximization problems are interior.)
- (b) How does a change in  $D$  affect  $G_2$ ?
- (c) Think of the period-1 policymaker as choosing  $G_1$  and  $D$ . Find the first-order condition for his or her choice of  $D$ .
- (d) Show that if  $\alpha_1$  is less than  $\alpha_2$ , the equilibrium involves inefficiently low taxation in period 1 relative to tax-smoothing (that is, that it has  $T_1 < T_2$ ). Explain intuitively why this occurs.
- (e) Does the result in part (d) imply that if  $\alpha_1$  is less than  $\alpha_2$ , the period-1 policymaker necessarily runs a deficit? Explain.