In Exercises 33–38, write each expression as the sum of a polynomial and a rational function whose numerator has smaller degree than its denominator.

33 $\dfrac{2x+1}{x-3}$

34 $\dfrac{4x-5}{x+7}$

35 $\dfrac{x^2}{3x-1}$

36 $\dfrac{x^2}{4x+3}$

37 $\dfrac{x^6+3x^3+1}{x^2+2x+5}$

38 $\dfrac{x^6-4x^2+5}{x^2-3x+1}$

39 Find a number c such that $r(10^{100}) \approx 6$, where

$$r(x) = \frac{cx^3 + 20x^2 - 15x + 17}{5x^3 + 4x^2 + 18x + 7}.$$

40 Find a number c such that $r(2^{1000}) \approx 5$, where

$$r(x) = \frac{3x^4 - 2x^3 + 8x + 7}{cx^4 - 9x + 2}.$$

41 A bicycle company finds that its average cost per bicycle for producing n thousand bicycles is $a(n)$ dollars, where

$$a(n) = 700\frac{4n^2 + 3n + 50}{16n^2 + 3n + 35}.$$

What will be the approximate cost per bicycle when the company is producing many bicycles?

42 A bicycle company finds that its average cost per bicycle for producing n thousand bicycles is $a(n)$ dollars, where

$$a(n) = 800\frac{3n^2 + n + 40}{16n^2 + 2n + 45}.$$

What will be the approximate cost per bicycle when the company is producing many bicycles?

43 Suppose you start driving a car on a chilly fall day. As you drive, the heater in the car makes the temperature inside the car $F(t)$ degrees Fahrenheit at time t minutes after you started driving, where

$$F(t) = 40 + \frac{30t^3}{t^3 + 100}.$$

(a) What was the temperature in the car when you started driving?

(b) What was the approximate temperature in the car ten minutes after you started driving?

(c) What will be the approximate temperature in the car after you have been driving for a long time?

44 Suppose you start driving a car on a hot summer day. As you drive, the air conditioner in the car makes the temperature inside the car $F(t)$ degrees Fahrenheit at time t minutes after you started driving, where

$$F(t) = 90 - \frac{18t^2}{t^2 + 65}.$$

(a) What was the temperature in the car when you started driving?

(b) What was the approximate temperature in the car 15 minutes after you started driving?

(c) What will be the approximate temperature in the car after you have been driving for a long time?

PROBLEMS

45 Suppose $s(x) = \dfrac{x^2+2}{2x-1}$.

(a) Show that the point $(1, 3)$ is on the graph of s.

(b) Show that the slope of a line containing $(1, 3)$ and a point on the graph of s very close to $(1, 3)$ is approximately -4.

[*Hint:* Use the result of Exercise 25.]

46 Suppose $t(x) = \dfrac{5}{4x^3+3}$.

(a) Show that the point $(-1, -5)$ is on the graph of t.

(b) Give an estimate for the slope of a line containing $(-1, -5)$ and a point on the graph of t very close to $(-1, -5)$.

[*Hint:* Use the result of Exercise 26.]

47 Explain why the composition of a polynomial and a rational function (in either order) is a rational function.

48 Explain why the composition of two rational functions is a rational function.

49 Suppose p is a polynomial and t is a number. Explain why there is a polynomial G such that

$$\frac{p(x) - p(t)}{x - t} = G(x)$$

for every number $x \neq t$.

50 Suppose r is the function with domain $(0, \infty)$ defined by

$$r(x) = \frac{1}{x^4 + 2x^3 + 3x^2}$$

for each positive number x.

(a) Find two distinct points on the graph of r.

(b) Explain why r is a decreasing function on $(0, \infty)$.

(c) Find two distinct points on the graph of r^{-1}.

22 Find all real numbers x such that
$$x^6 - 3x^3 - 10 = 0.$$

23 Find all real numbers x such that
$$x^4 - 2x^2 - 15 = 0.$$

24 Find all real numbers x such that
$$x^4 + 5x^2 - 14 = 0.$$

25 Find a number b such that 3 is a zero of the polynomial p defined by
$$p(x) = 1 - 4x + bx^2 + 2x^3.$$

26 Find a number c such that -2 is a zero of the polynomial p defined by
$$p(x) = 5 - 3x + 4x^2 + cx^3.$$

27 Find a polynomial p of degree 3 such that -1, 2, and 3 are zeros of p and $p(0) = 1$.

28 Find a polynomial p of degree 3 such that -2, -1, and 4 are zeros of p and $p(1) = 2$.

29 Find all choices of b, c, and d such that 1 and 4 are the only zeros of the polynomial p defined by
$$p(x) = x^3 + bx^2 + cx + d.$$

30 Find all choices of b, c, and d such that -3 and 2 are the only zeros of the polynomial p defined by
$$p(x) = x^3 + bx^2 + cx + d.$$

PROBLEMS

31 Give an example of two polynomials of degree 4 whose sum has degree 3.

32 Find a polynomial p of degree 2 with integer coefficients such that 2.1 and 4.1 are zeros of p.

33 Find a polynomial p with integer coefficients such that $2^{3/5}$ is a zero of p.

34 Show that if p and q are nonzero polynomials with $\deg p < \deg q$, then $\deg(p + q) = \deg q$.

35 Give an example of polynomials p and q such that $\deg(pq) = 8$ and $\deg(p + q) = 5$.

36 Give an example of polynomials p and q such that $\deg(pq) = 8$ and $\deg(p + q) = 2$.

37 Suppose $q(x) = 2x^3 - 3x + 1$.

(a) Show that the point $(2, 11)$ is on the graph of q.

(b) Show that the slope of a line containing $(2, 11)$ and a point on the graph of q very close to $(2, 11)$ is approximately 21.

[*Hint:* Use the result of Exercise 17.]

38 Suppose $s(x) = 4x^3 - 2$.

(a) Show that the point $(1, 2)$ is on the graph of s.

(b) Give an estimate for the slope of a line containing $(1, 2)$ and a point on the graph of s very close to $(1, 2)$.

[*Hint:* Use the result of Exercise 18.]

39 Give an example of polynomials p and q of degree 3 such that $p(1) = q(1)$, $p(2) = q(2)$, and $p(3) = q(3)$, but $p(4) \neq q(4)$.

40 Suppose p and q are polynomials of degree 3 such that $p(1) = q(1)$, $p(2) = q(2)$, $p(3) = q(3)$, and $p(4) = q(4)$. Explain why $p = q$.

41 Explain why the polynomial p defined by
$$p(x) = x^6 + 7x^5 - 2x - 3$$
has a zero in the interval $(0, 1)$.

For Problems 42–43, let p be the polynomial defined by
$$p(x) = x^6 - 87x^4 - 92x + 2.$$

42 (a) Use a computer or calculator to sketch a graph of p on the interval $[-5, 5]$.

(b) Is $p(x)$ positive or negative for x near ∞?

(c) Is $p(x)$ positive or negative for x near $-\infty$?

(d) Explain why the graph from part (a) does not accurately show the behavior of $p(x)$ for large values of x.

43 (a) Evaluate $p(-2)$, $p(-1)$, $p(0)$, and $p(1)$.

(b) Explain why the results from part (a) imply that p has a zero in the interval $(-2, -1)$ and p has a zero in the interval $(0, 1)$.

(c) Show that p has at least four zeros in the interval $[-10, 10]$.
[*Hint:* We already know from part (b) that p has at least two zeros is the interval $[-10, 10]$. You can show the existence of other zeros by finding integers n such that one of the numbers $p(n)$, $p(n + 1)$ is positive and the other is negative.]

2.4

Graphs of Polynomials

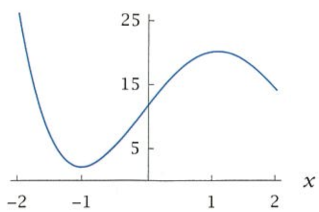
Computers can draw graphs of polynomials better than humans. However, some human thought is usually needed to select an appropriate interval on which to graph a polynomial.

EXAMPLE 8

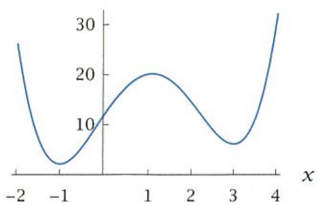
Let p be the polynomial defined by

$$p(x) = x^4 - 4x^3 - 2x^2 + 13x + 12.$$

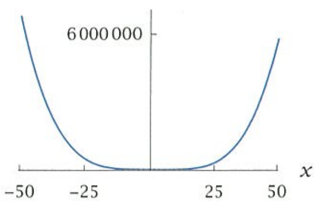
Find an interval that does a good job of illustrating the key features of the graph of p.



The graph of p on the interval $[-2, 2]$.



The graph of p on the interval $[-2, 4]$.



The graph of p on the interval $[-50, 50]$.

SOLUTION If we ask a computer to graph this polynomial on the interval $[-2, 2]$, we obtain the the first graph shown here. Because $p(x)$ behaves like x^4 for very large values of x, this first graph does not depict enough features of p.

Often a bit of experimentation is needed to find an appropriate interval to illustrate the key features of the graph. For this polynomial p, the interval $[-2, 4]$ works well, as shown in the second graph here.

The second graph here shows the graph of p beginning to look like the graph of x^4 when $|x|$ is large. Thus the interval $[-2, 4]$ provides a more complete representation of the behavior of p than does $[-2, 2]$, which was the first interval we used.

We also now see that the graph of p above contains three points that might be thought of as either the top of a peak (at $x \approx 1$) or the bottom of a valley (at $x \approx -1$ and $x \approx 3$).

To search for additional behavior of p, we might try graphing p on a much larger interval, as shown in the third graph here.

The third graph shows no peaks or valleys, even though we know that it contains a total of at least three peaks and valleys. The scale needed to display the graph on the interval $[-50, 50]$ made the peaks and valleys so small that we cannot see them. Thus using this large interval hid some key features that were visible when we used the interval $[-2, 4]$.

Conclusion: A good choice for graphing this function is the interval $[-2, 4]$.

EXERCISES

Suppose $p(x) = x^2 + 5x + 2,$

$q(x) = 2x^3 - 3x + 1, \quad s(x) = 4x^3 - 2.$

In Exercises 1-18, write the indicated expression as a polynomial.

1 $(p + q)(x)$

2 $(p - q)(x)$

3 $(3p - 2q)(x)$

4 $(4p + 5q)(x)$

5 $(pq)(x)$

6 $(ps)(x)$

7 $(p(x))^2$

8 $(q(x))^2$

9 $(p(x))^2 s(x)$

10 $(q(x))^2 s(x)$

11 $(p \circ q)(x)$

12 $(q \circ p)(x)$

13 $(p \circ s)(x)$

14 $(s \circ p)(x)$

15 $(q \circ (p + s))(x)$

16 $((q + p) \circ s)(x)$

17 $\dfrac{q(2 + x) - q(2)}{x}$

18 $\dfrac{s(1 + x) - s(1)}{x}$

19 Factor $x^8 - y^8$ as nicely as possible.

20 Factor $x^{16} - y^8$ as nicely as possible.

21 Find all real numbers x such that

$$x^6 - 8x^3 + 15 = 0.$$

2.3

EXERCISES

For Exercises 1–6, evaluate the given expression. Do not use a calculator.

1 $2^5 - 5^2$

2 $4^3 - 3^4$

3 $\frac{3^{-2}}{2^{-3}}$

4 $\frac{2^{-6}}{6^{-2}}$

5 $(\frac{2}{3})^{-4}$

6 $(\frac{5}{4})^{-3}$

The numbers in Exercises 7–14 are too large to be handled by a calculator. These exercises require an understanding of the concepts.

7 Write 9^{3000} as a power of 3.

8 Write 27^{4000} as a power of 3.

9 Write 5^{4000} as a power of 25.

10 Write 2^{3000} as a power of 8.

11 Write $2^5 \cdot 8^{1000}$ as a power of 2.

12 Write $5^3 \cdot 25^{2000}$ as a power of 5.

13 Write $2^{100} \cdot 4^{200} \cdot 8^{300}$ as a power of 2.

14 Write $3^{500} \cdot 9^{200} \cdot 27^{100}$ as a power of 3.

For Exercises 15–20, simplify the given expression by writing it as a power of a single variable.

15 $x^5(x^2)^3$

16 $y^4(y^3)^5$

17 $y^4(y^2(y^5)^2)^{3/5}$

18 $x(x^4(x^3)^2)^{5/3}$

19 $t^4(t^3(t^{-2})^5)^4$

20 $w^3(w^4(w^{-3})^6)^2$

21 Write $\frac{8^{1000}}{2^5}$ as a power of 2.

22 Write $\frac{25^{2000}}{5^3}$ as a power of 5.

23 Find integers m and n such that $2^m \cdot 5^n = 16000$.

24 Find integers m and n such that $2^m \cdot 5^n = 0.0032$.

For Exercises 25–32, simplify the given expression.

25 $\frac{(x^2)^3 y^8}{x^5(y^4)^3}$

26 $\frac{x^{11}(y^3)^2}{(x^3)^5(y^2)^4}$

27 $\frac{(x^{-2})^3 y^8}{x^{-5}(y^4)^{-3}}$

28 $\frac{x^{-11}(y^3)^{-2}}{(x^{-3})^5(y^2)^4}$

29 $\frac{(x^2 y^{4/5})^3}{(x^5 y^2)^{-4}}$

30 $\frac{(x^4 y^{3/4})^{-3}}{(x^5 y^{-2})^4}$

31 $\left(\frac{(x^2 y^{-5})^{-4}}{(x^5 y^{-2})^{-3}}\right)^2$

32 $\left(\frac{(x^{-3} y^5)^{-4}}{(x^{-5} y^{-2})^{-3}}\right)^{-2}$

For Exercises 33–44, find a formula for $f \circ g$ given the indicated functions f and g.

33 $f(x) = x^2$, $g(x) = x^3$

34 $f(x) = x^5$, $g(x) = x^4$

35 $f(x) = 4x^2$, $g(x) = 5x^3$

36 $f(x) = 3x^5$, $g(x) = 2x^4$

37 $f(x) = 4x^{-2}$, $g(x) = 5x^3$

38 $f(x) = 3x^{-5}$, $g(x) = 2x^4$

39 $f(x) = 4x^{-2}$, $g(x) = -5x^{-3}$

40 $f(x) = 3x^{-5}$, $g(x) = -2x^{-4}$

41 $f(x) = x^{1/2}$, $g(x) = x^{3/7}$

42 $f(x) = x^{5/3}$, $g(x) = x^{4/9}$

43 $f(x) = 3 + x^{5/4}$, $g(x) = x^{2/7}$

44 $f(x) = x^{2/3} - 7$, $g(x) = x^{9/16}$

For Exercises 45–56, expand the expression.

45 $(2 + \sqrt{3})^2$

46 $(3 + \sqrt{2})^2$

47 $(2 - 3\sqrt{5})^2$

48 $(3 - 5\sqrt{2})^2$

49 $(2 + \sqrt{3})^4$

50 $(3 + \sqrt{2})^4$

51 $(3 + \sqrt{x})^2$

52 $(5 + \sqrt{x})^2$

53 $(3 - \sqrt{2x})^2$

54 $(5 - \sqrt{3x})^2$

55 $(1 + 2\sqrt{3x})^2$

56 $(3 + 2\sqrt{5x})^2$

For Exercises 57–64, find all real numbers x that satisfy the indicated equation.

57 $x - 5\sqrt{x} + 6 = 0$

58 $x - 7\sqrt{x} + 12 = 0$

59 $x - \sqrt{x} = 6$

60 $x - \sqrt{x} = 12$

61 $x^{2/3} - 6x^{1/3} = -8$

62 $x^{2/3} + 3x^{1/3} = 10$

63 $x^4 - 3x^2 = 10$

64 $x^4 - 8x^2 = -15$

65 Evaluate 3^{-2x} if x is a number such that $3^x = 4$.

66 Evaluate 2^{-4x} if x is a number such that $2^x = \frac{1}{3}$.

67 Evaluate 8^x if x is a number such that $2^x = 5$.

68 Evaluate $(\frac{1}{9})^x$ if x is a number such that $3^x = 5$.

For Exercises 69–78, sketch the graph of the given function f on the interval $[-1.3, 1.3]$.

69 $f(x) = x^3 + 1$

70 $f(x) = x^4 + 2$

71 $f(x) = x^4 - 1.5$

72 $f(x) = x^3 - 0.5$

73 $f(x) = 2x^3$

74 $f(x) = 3x^4$

75 $f(x) = -2x^4$

76 $f(x) = -3x^3$

77 $f(x) = -2x^4 + 3$

78 $f(x) = -3x^3 + 4$

2.1

EXERCISES

1 What are the coordinates of the unlabeled vertex of the smaller of the two right triangles in the figure at the beginning of this section?

2 What are the coordinates of the unlabeled vertex of the larger of the two right triangles in the figure at the beginning of this section?

3 Find the slope of the line that contains the points $(3, 4)$ and $(7, 13)$.

4 Find the slope of the line that contains the points $(2, 11)$ and $(6, -5)$.

5 Find a number w such that the line containing the points $(1, w)$ and $(3, 7)$ has slope 5.

6 Find a number d such that the line containing the points $(d, 4)$ and $(-2, 9)$ has slope -3.

7 Suppose the tuition per semester at Euphoria State University is \$525 plus \$200 for each unit taken.

(a) What is the tuition for a semester in which a student is taking 10 units ?

(b) Find a linear function t such that $t(u)$ is the tuition in dollars for a semester in which a student is taking u units.

(c) Find the total tuition for a student who takes 12 semesters to accumulate the 120 units needed to graduate.

(d) Find a linear function g such that $g(s)$ is the total tuition for a student who takes s semesters to accumulate the 120 units needed to graduate.

8 Suppose the tuition per semester at Luxim University is \$900 plus \$850 for each unit taken.

(a) What is the tuition for a semester in which a student is taking 15 units ?

(b) Find a linear function t such that $t(u)$ is the tuition in dollars for a semester in which a student is taking u units.

(c) Find the total tuition for a student who takes 8 semesters to accumulate the 120 units needed to graduate.

(d) Find a linear function g such that $g(s)$ is the total tuition for a student who takes s semesters to accumulate the 120 units needed to graduate.

9 Suppose your cell phone company offers two calling plans. The pay-per-call plan charges \$14 per month plus 3 cents for each minute. The unlimited-calling plan charges a flat rate of \$29 per month for unlimited calls.

(a) What is your monthly cost in dollars for making 400 minutes per month of calls on the pay-per-call plan?

(b) Find a linear function c such that $c(m)$ is your monthly cost in dollars for making m minutes of phone calls per month on the pay-per-call plan.

(c) How many minutes per month must you use for the unlimited-calling plan to become cheaper?

10 Suppose your cell phone company offers two calling plans. The pay-per-call plan charges \$11 per month plus 4 cents for each minute. The unlimited-calling plan charges a flat rate of \$25 per month for unlimited calls.

(a) What is your monthly cost in dollars for making 600 minutes per month of calls on the pay-per-call plan?

(b) Find a linear function c such that $c(m)$ is your monthly cost in dollars for making m minutes of phone calls per month on the pay-per-call plan.

(c) How many minutes per month must you use for the unlimited-calling plan to become cheaper?

11 Find the equation of the line in the xy-plane with slope 2 that contains the point $(7, 3)$.

12 Find the equation of the line in the xy-plane with slope -4 that contains the point $(-5, -2)$.

13 Find the equation of the line that contains the points $(2, -1)$ and $(4, 9)$.

14 Find the equation of the line that contains the points $(-3, 2)$ and $(-5, 7)$.

15 Find a number t such that the point $(3, t)$ is on the line containing the points $(7, 6)$ and $(14, 10)$.

16 Find a number t such that the point $(-2, t)$ is on the line containing the points $(5, -2)$ and $(10, -8)$.

17 Find a function s such that $s(d)$ is the number of seconds in d days.

52 Suppose g is an even function whose domain is $[-5,-1] \cup [1,5]$ and whose graph on the interval $[1,5]$ is the graph used in the instructions for Exercises 15-50. Sketch the graph of g on $[-5,-1] \cup [1,5]$.

53 Suppose h is an odd function whose domain is $[-2,-1] \cup [1,2]$ and whose graph on the interval $[1,2]$ is the graph used in the instructions for Exercises 1-14. Sketch the graph of h on $[-2,-1] \cup [1,2]$.

54 Suppose h is an odd function whose domain is $[-5,-1] \cup [1,5]$ and whose graph on the interval $[1,5]$ is the graph used in the instructions for Exercises 15-50. Sketch the graph of h on $[-5,-1] \cup [1,5]$.

For Exercises 55-58, suppose f is a function whose domain is the interval $[-5,5]$ and

$$f(x) = \frac{x}{x+3}$$

for every x in the interval $[0,5]$.

55 Suppose f is an even function. Evaluate $f(-2)$.

56 Suppose f is an even function. Evaluate $f(-3)$.

57 Suppose f is an odd function. Evaluate $f(-2)$.

58 Suppose f is an odd function. Evaluate $f(-3)$.

PROBLEMS

For Problems 59-62, suppose that to provide additional funds for higher education, the federal government adopts a new income tax plan that consists of the 2011 income tax plus an additional \$100 per taxpayer. Let g be the function such that $g(x)$ is the 2011 federal income tax for a single person with taxable income x dollars, and let h be the corresponding function for the new income tax plan.

59 Is h obtained from g by a vertical function transformation or by a horizontal function transformation?

60 Write a formula for $h(x)$ in terms of $g(x)$.

61 Using the explicit formula for $g(x)$ given in Example 2 in Section 1.1, give an explicit formula for $h(x)$.

62 Under the new income tax plan, what will be the income tax for a single person whose annual taxable income is \$50,000?

For Problems 63-66, suppose that to pump more money into the economy during a recession, the federal government adopts a new income tax plan that makes income taxes 90% of the 2011 income tax. Let g be the function such that $g(x)$ is the 2011 federal income tax for a single person with taxable income x dollars, and let h be the corresponding function for the new income tax plan.

63 Is h obtained from g by a vertical function transformation or by a horizontal function transformation?

64 Write a formula for $h(x)$ in terms of $g(x)$.

65 Using the explicit formula for $g(x)$ given in Example 2 in Section 1.1, give an explicit formula for $h(x)$.

66 Under the new income tax plan, what will be the income tax for a single person whose annual taxable income is \$60,000?

67 Find the only function whose domain is the set of real numbers and that is both even and odd.

68 Show that if f is an odd function such that 0 is in the domain of f, then $f(0) = 0$.

69 The result box following Example 2 could have been made more complete by including explicit information about the domain and range of the functions g and h. For example, the more complete result box might have looked like the one shown here:

Shifting a graph up or down

Suppose f is a function and $a > 0$. Define functions g and h by

$$g(x) = f(x) + a \quad \text{and} \quad h(x) = f(x) - a.$$

Then

- g and h have the same domain as f;
- the range of g is obtained by adding a to every number in the range of f;
- the range of h is obtained by subtracting a from every number in the range of f;
- the graph of g is obtained by shifting the graph of f up a units;
- the graph of h is obtained by shifting the graph of f down a units.

Construct similar complete result boxes, including explicit information about the domain and range of the functions g and h, for each of the other five result boxes in this section that deal with function transformations.

For Exercises 49–54, suppose f is a function and a function g is defined by the given expression.

(a) ***Write g as the composition of f and one or two linear functions.***

(b) ***Describe how the graph of g is obtained from the graph of f.***

49 $g(x) = 3f(x) - 2$

50 $g(x) = -4f(x) - 7$

51 $g(x) = f(5x)$

52 $g(x) = f(-\frac{2}{3}x)$

53 $g(x) = 2f(3x) + 4$

54 $g(x) = -5f(-\frac{4}{3}x) - 8$

PROBLEMS

For Problems 55–59, suppose you are exchanging currency in the London airport. The currency exchange service there only makes transactions in which one of the two currencies is British pounds, but you want to exchange dollars for Euros. Thus you first need to exchange dollars for British pounds, then exchange British pounds for Euros. At the time you want to make the exchange, the function f for exchanging dollars for British pounds is given by the formula

$$f(d) = 0.66d - 1$$

and the function g for exchanging British pounds for Euros is given by the formula

$$g(p) = 1.23p - 2.$$

The subtraction of 1 or 2 in the number of British pounds or Euros that you receive is the fee charged by the currency exchange service for each transaction.

55 Is the function describing the exchange of dollars for Euros $f \circ g$ or $g \circ f$? Explain your answer in terms of which function is evaluated first when computing a value for a composition (the function on the left or the function on the right?).

56 Find a formula for the function given by your answer to Problem 55.

57 How many Euros would you receive for exchanging \$100 after going through this two-step exchange process?

58 How many Euros would you receive for exchanging \$200 after going through this two-step exchange process?

59 Which process gives you more Euros: exchanging \$100 for Euros twice or exchanging \$200 for Euros once?

60 Suppose $f(x) = ax + b$ and $g(x) = cx + d$, where a, b, c, and d are numbers. Show that $f \circ g = g \circ f$ if and only if $d(a-1) = b(c-1)$.

61 Suppose f and g are functions. Show that the composition $f \circ g$ has the same domain as g if and only if the range of g is contained in the domain of f.

62 Show that the sum of two even functions (with the same domain) is an even function.

63 Show that the product of two even functions (with the same domain) is an even function.

64 True or false: The product of an even function and an odd function (with the same domain) is an odd function. Explain your answer.

65 True or false: The sum of an even function and an odd function (with the same domain) is an odd function. Explain your answer.

66 Suppose g is an even function and f is any function. Show that $f \circ g$ is an even function.

67 Suppose f is an even function and g is an odd function. Show that $f \circ g$ is an even function.

68 Suppose f and g are both odd functions. Is the composition $f \circ g$ even, odd, or neither? Explain.

69 Show that if f, g, and h are functions, then

$$(f + g) \circ h = f \circ h + g \circ h.$$

70 Find functions f, g, and h such that

$$f \circ (g + h) \neq f \circ g + f \circ h.$$

WORKED-OUT SOLUTIONS *to Odd-Numbered Exercises*

For Exercises 1–10, evaluate the indicated expression assuming that f, g, and h are the functions completely defined by these tables:

x	$f(x)$
1	4
2	1
3	2
4	2

x	$g(x)$
1	2
2	4
3	1
4	3

x	$h(x)$
1	3
2	3
3	4
4	1