CHAPTER FOUR.

BEAMS IN BENDING.

The student is strongly advised to review shear and bending moment diagrams. This topic is examined in details in Statics (see Statics notes chapter five). As a result, it will not be discussed in this chapter.

This chapter examines the deformation of a beam subjected to a moment, not about its axis as was the case in torsion, but perpendicular to this axis. This load makes the beam either 'smile' at you or frown, depending on the moment direction.

The analysis is similar to the one used in the torsion analysis, i.e., first an analysis on the strain distribution across the area is made to figure out how do strain/stress vary (see equations (3.1.9) and (3.2.3) for torsion). Second, a relationship between loads and stress/strain is derived (again compare to the case of torsion, see equations (3.2.12) and (3.4.7)

4.1 Bending Deformation.

Consider the beam shown in figure 4.1.1. It represents a beam subjected to a moment M. The deformed shape of the beam is also shown, but exagerated. Recall that in the torsion case the stress strain distribution across the area is derived first, then a relationship between the load and stress/strain is obtained. It is the same thing in this case, first the stress or strain distribution, then the relationship between loads and stress or strain.

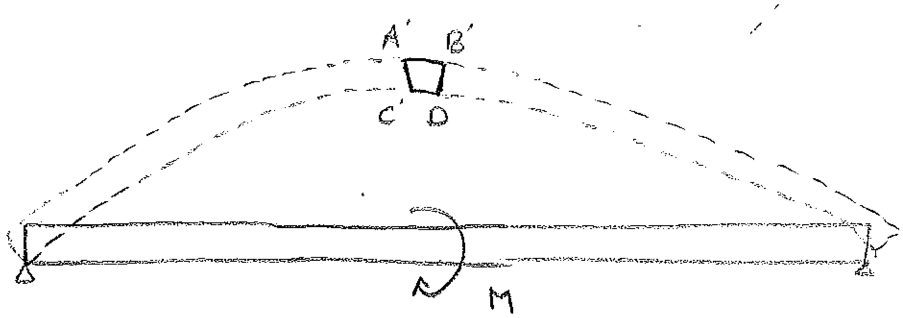


Figure 4.1.1. Beam in Bending.

When looking at the deformed shape of the beam, one realizes that the top of the beam experiences a tensile load that tends to stretch the top, while the bottom experiences compressive loads that tend to compress it. One can argue that if the stresses go from being tensile at the top to compressive at the bottom, then somewhere, the loads will be neither compressive nor tensile, i.e. zero. In other words, somewhere along the length of the beam there exist a line where the material feels no loads at all. This is called the neutral axis. In order to analyze this problem, consider the element shown in figure 4.1.2.

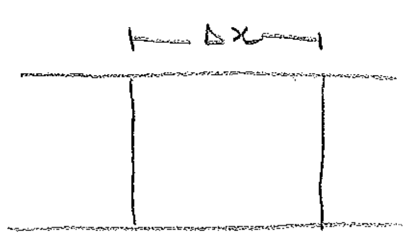
When the beam is loaded, it stops being straight and curves a bit. The radius of curvature ρ is simply the radius of the circle described by the deformed neutral axis. The angle $\Delta\theta$ is the angle spaning $\Delta S'$. Taking ΔS to be very small, then the strain associated a random segment ΔS is simply

$$\varepsilon_{\Delta S \to 0} = \frac{\Delta S' - \Delta S}{\Delta S} \qquad (4.1.1)$$

In order to understand the rest of the analysis, consider the length Δx. This is the length of the element before loading and is equal to ΔS, so

$$\Delta x = \Delta S \qquad (4.1.2)$$

After loading, the segment that does not change is the one on the neutral axis where no loads are present, but ΔS changes length to $\Delta S'$. From



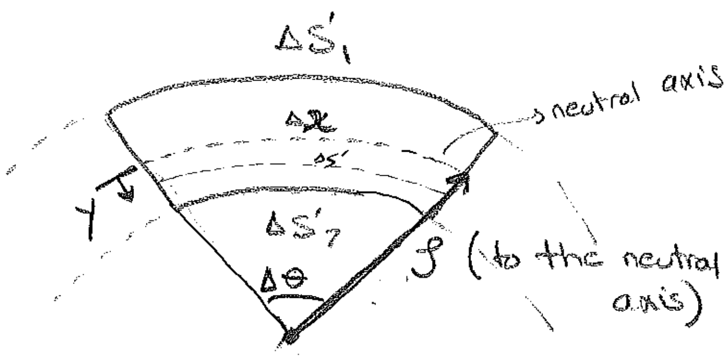


Figure 4.1.2. Deformed and Undeformed Elements

the figure one can also write

$$\Delta S' = (\rho - y)\ \Delta\theta \tag{4.1.3}$$

where y is the distance from the neutral axis.

Furthermore, at the neutral axis $(y=0)$ Δx did not change or

$$\Delta x = \Delta S = \rho\,\Delta\theta \quad \text{(at neutral axis)} \tag{4.1.4}$$

Equations (4.1.2), (4.1.3) and (4.1.4) yield

$$\varepsilon = -y/\rho \tag{4.1.5}$$

this equation states that at the neutral axis $\epsilon = 0$ and varies linearly above and below the neutral axis. This equation also states that the maximum strain occurs when y is maximum, i.e as far away from the neutral axis as possible. By denoting this maximum distance as c, the maximum strain ϵ_{max} becomes

$$\varepsilon_{max} = \frac{-c}{\rho} \tag{4.1.6}$$

It follows that

$$\varepsilon = \frac{-y}{c}\,\varepsilon_{max} \qquad (4.1.7)$$

Once again, the strain seems to vary linearly with y, in the same way it did for the case of torsion, i.e. linear with a zero value in the middle.

Equation (4.1.7) implies (assuming elastic behavior)

$$\sigma = \frac{-y}{c}\,\sigma_{max} \qquad (4.1.8)$$

In other words, the axial stresses on the cross sectional area vary linearly from a negative value to a positive one. The zero value occurs when $y=0$ or at the neutral axis. Keep in mind that a positive σ (or ε) corresponds to tension, negative values to compressive ones (check equation 4.1.8) this section determined how the stresses vary on the cross sectional area, but did not relate to the load.

Keep in mind that the previous results are valid when the following is assumed:

i) The cross sectional areas across the deformed beam remain plane and perpendicular to the beam axis.

ii) the neutral axis does not change length but becomes a curve when deformed

iii) Any deformation of the cross sectional area about its plane is negligible (no warping)

These are illustrated in figure 4.1.2

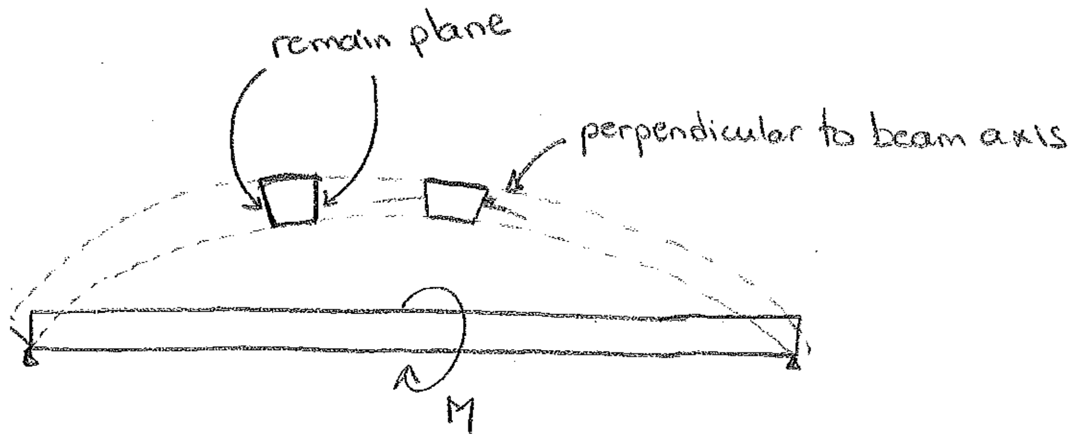


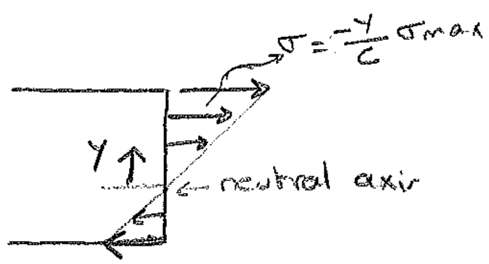
Figure 4.1.2 Assumed Deformed Shape in Bending

4.2 The Flexure Formula.

When a beam is subjected to bending, the normal stress distribution is linear, and varies from a positive to a negative value (i.e. from tension to compression), the zero point being on the neutral axis. Figure 4.2.1 shows an example. The stress at any location y with respect to the neutral axis is given by

$$\sigma = \frac{-y}{c}\,\sigma_{max} \qquad (4.1.8)$$

where c is the point farthest away from the neutral axis. The question now is how does the stress relate to the moment applied (i.e loads). Furthermore, where is the



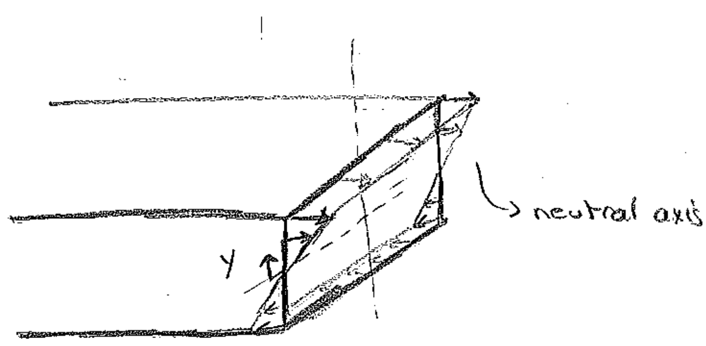


Figure 4.2.1 Stress Distribution

location of the neutral axis? In order to answer the second question, recall that no forces are applied in the axial direction. As a result, the sum of all the forces in the x direction on any area must be zero. So the compressive forces plus the tensile ones on the area must be equal.

By definition of stress, and for a differential area dA

$$dF = \sigma \, dA \qquad (4.2.1)$$

the sum of all these fores must be zero, so

$$F = \int dF = \int \sigma \, dA = 0 \qquad (4.2.2)$$

or

$$\int \frac{-y}{c} \sigma_{max} \, dA = 0 \qquad (4.2.3)$$

Since c and σ_{max} are constants, they can be written outside of the integral. ~~It~~ follows

$$\frac{-\sigma_{max}}{c} \int y \, dA = 0 \qquad (4.2.4)$$

The only meaningful solution to the previous equation is to set $\int y\,dA = 0$. Now, recall that the location of the y-component of the centroid y_c of the area is given by

$$y_c = \frac{\int y\,dA}{\int dA} \qquad (4.2.5)$$

So if $\int y\,dA$ must be zero so the sum of the forces adds up to zero, then the location of the centroid is zero. But since the y-axis chosen starts at the neutral axis, then $y_c = 0$. This means that the centroidal axis (the line along the beam made up of all the centroids) is the same as the neutral axis. Find the centroid and you'll find the neutral axis.

the next step is to try to figure out a relationship between the loads applied (moments) & the stresses on the cross sectional area.

Here again, no new concepts, simply the fact that the sum of the moments must be equal to zero. In other words, the moment M applied must be balanced by the moments that the tensile and compressive loads on the area exert. Referring to the differential area dA shown in figure 4.2.2, one can write that the moment dM due to the force dF acting on dA is

$$dM = y\,dF \qquad (4.2.6)$$

Keep in mind that

$$dF = \sigma\,dA \qquad (4.2.7)$$

so

$$dM = y\sigma\,dA \qquad (4.2.8)$$

Using equation (4.1.9), the previous equation becomes

$$dM = \frac{-\sigma_{max}}{c}\,y^2\,dA \qquad (4.2.9)$$

or

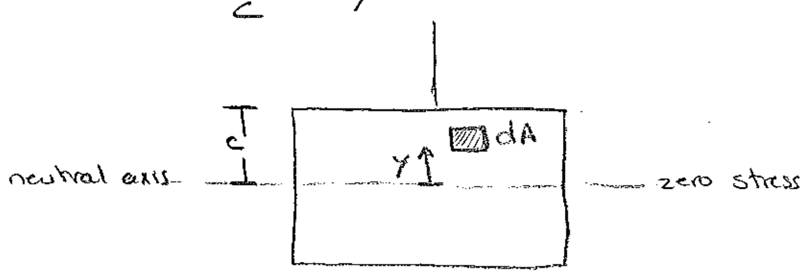


Figure 4.2.2 Differential dA

Keep in mind that this is the moment due to the small area dA. to get the total value simply integrate.

$$M = \int dM = \frac{-\sigma_{max}}{c} \int y^2 dA \qquad (4.2.10)$$

the quantity $\int y^2 dA$ is dependent on the geometry of the area but not on the load. this quantity is referred to as I, the moment of inertia of the area about the neutral axis. It is usually found in the litterature

about neutral axis.

$$M = \frac{-\sigma_{max}}{c} I \qquad (4.2.11)$$

Solving for σ_{max} yields

$$\sigma_{max} = \frac{-Mc}{I} \qquad (4.2.12)$$

It follows that when a cross sectional area of a beam is subjected to a moment M, the maximum stress is $\frac{-Mc}{I}$. The sign is simply to make the positive/negative convention valid. the stress anywhere on that area, σ is

$$\sigma = \frac{-y}{c}\sigma_{max} \qquad (4.2.13a)$$

$$\sigma = \frac{-My}{I} \qquad (4.2.13\text{-}b)$$

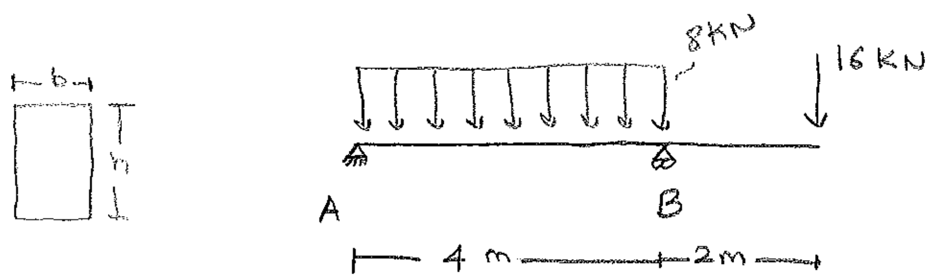
these stresses are referred to as flexural stresses, i.e. stresses that result from 'flexing' or bending the beam, as opposed to other types of loading.

It is obvious from equation (4.2.13.b) that the moment is needed. The value of the moment is obtained either from a bending moment diagram or by computing the moment at a particular position.

The following examples illustrate these concepts.

Example 4.2.1: [ENGR 241, TECH 341 PLEASE SEE NOTE AT END]

For the beam shown, find the maximum flexural stress. Take $I = \frac{1}{12} bh^3$ where b and h are the width and height of the beam's rectangular cross sectional area.



Solution

this problem involves two parts. First, one must determine the magnitude of the maximum moment (the location is not important, but can easily be obtained) Second, given this maximum value, what are the corresponding values of the flexural stresses?

Once M_{max} is known, σ_{max} is simply (see eq. (4.2.12))

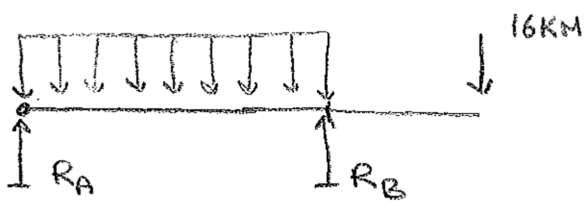
$$\sigma_{max} = \frac{-M_{max}\, c}{I}$$

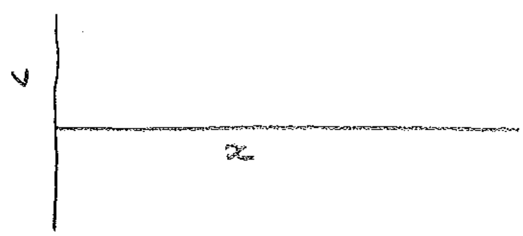
i) Shear and bending moment diagrams.

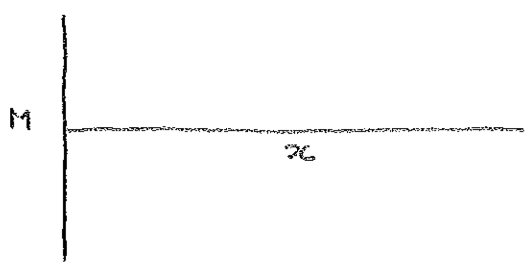
The first step is to compute the reaction forces at A and B. The quickest way to get to R_A is

so $R_A =$

to get R_B, simply







From the moment diagram, it seems that $M_{max} =$

at $x =$

ii) Maximum flexural stress

Since I is constant, the only factor that determines the value of σ_{max} is $M \cdot c$ in this case is .

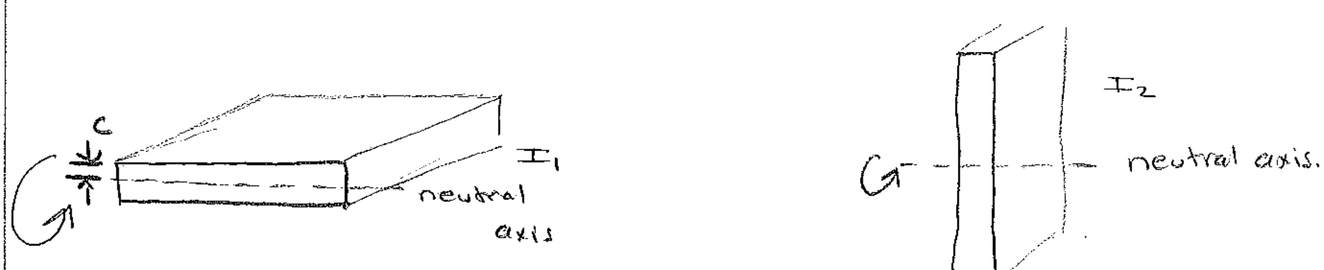
$\sigma_{max} =$.

TECH 341: IN STEAD OF A DISTRIBUTED LOAD, USE THE EQUIVALENT FORCE.

Consider the beam of rectangular cross sectional area shown in figure 4.2.3. It is obvious that in case a) the beam will flex easier, i.e. easier to bend. If the beam is rotated as shown in case b), it becomes much harder to flex. So for a given moment M; more strain will be present in case a) (and hence higher stresses) than in case b. the stress is

$$\sigma = \frac{Mc}{I} \qquad (4.2.14)$$

In case b, c and I are different. c is different for obvious reasons, but I is different because the neutral axes are different in each case (I is about neutral axis). So each beam will see different stresses for the same M.



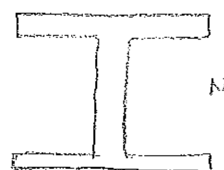
a - Easier to bend   b- Harder to bend

Figure 4.2.3. Beam Bending

Example 4.2.2:

Compute the maximum stresses when a W610x155 wide flange beam is loaded in the I and H configurations. The cross sectional area's characteristics are found in the litterature Take the moment to be simply M.

Solution



NOT TO SCALE.

4.3 Moment Curvature Equation.

In this section, the radius of curvature ρ of the deformed shape is related to the loads (i.e. moments) applied to the beam. Recall equation (4.1.5).

$$\varepsilon = \frac{-y}{\rho} \tag{4.1.5}$$

so

$$\sigma = \frac{-Ey}{\rho} \tag{4.3.1}$$

Keeping in mind that (from equation (4.2.13-b))

$$M = \frac{-\sigma I}{y} \tag{4.2.13-b}$$

so

$$M = \frac{EI}{\rho} \tag{4.3.2}$$

In other words, if a moment M is applied at a given section, the beam will have a radius of curvature ρ such that

$$\rho = \frac{EI}{M} \tag{4.3.3}$$

the term EI is referred to as the flexural rigidity of the beam. It represents the beam's resistance to bending. For high values, both M and ρ become larger. It simply means the beam is harder to bend. Keep in mind, high ρ means larger radii, i.e., less deformation.

Example 4.3.1.

Find the radii of curvature of both beams considered in example 4.2.2.

4.4 Moment of Inertia and the Parallel Axis theorem.

In order to better understand the rest of this chapter, it is important to note that the moment of inertia I considered so far was always with respect to the neutral axis, as shown in figure (4.4.1). The I values of different shapes found in the litterature are about the neutral axis. In other words when I was defined as

$$I = \int y^2 \, dA \qquad (4.4.1)$$

y was taken as the position with respect to the neutral axis, the result was the moment of inertia about the neutral axis. If another axis Ox parallel to

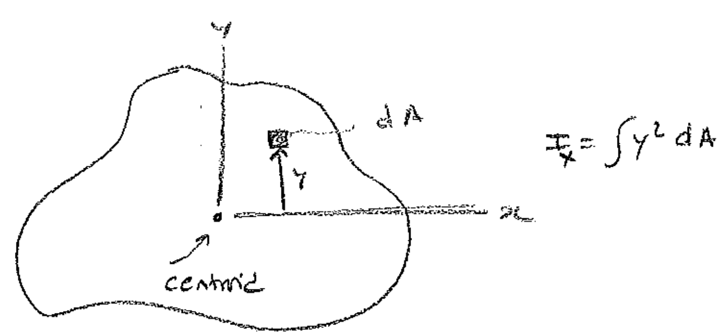


Figure 4.4.1 Moment of Inertia

the neutral axis is taken as a reference as shown in figure 4.4.2, the moment of inertia becomes

$$I_{0x} = \int (d + y')^2 \, dA \qquad (4.4.2)$$

or

$$I_{0x} = \int d^2 dA + \int y'^2 dA + 2 \int d \, y' dA \qquad (4.4.3)$$

The term $\int y'^2 dA$ is nothing more than the moment of inertia about the centroid, I. The last term is zero. Recall from the analysis of centroids (see Statics notes) that

$$\int y \, dA = \bar{y} A \qquad (4.4.4)$$

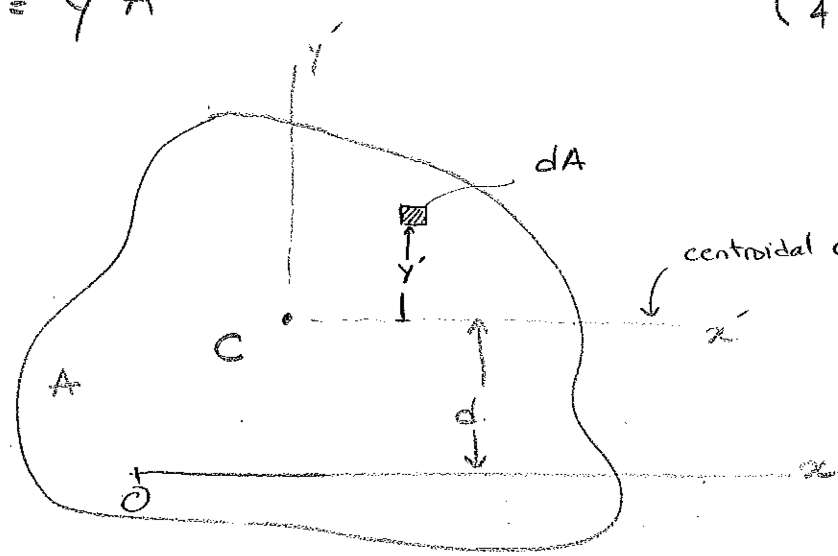


Figure 4.4.2 Parallel Axis Theorem

where $\bar{y}$ is the coordinate of the centroid. Now, since we know that the coordinate of the centroid is at zero in the $Cx'y'$ frame, then

$$\int y' \, dA = \bar{y}' A = 0 \qquad (4.4.5)$$

In other words, suppose your reference frame is the $Cx'y'$ one. If you wish to find the location of the centroid of this area simply use equation (4.4.5). The $\bar{y}'$ term is the location of the centroid with respect to the Cx' axis. But the centroid is at C, i.e. it has a $\bar{y}'$ value of zero.

With this in mind, equation (4.4.3) becomes

$$I_{Ox} = I_{centroid} + d^2 \int dA \qquad (4.4.6)$$

or

$$I_{Ox} = I_{centroid} + d^2 A \qquad (4.4.7)$$

Equation (4.4.7) is known as the parallel axis theorem It simply states that the moment of inertia about an axis a distance d from the centroid is equal to the moment of inertia with respect to the centroid plus the square of the distance times the area.

This theorem is usefull when computing the moment of inertia I of a composite area. Take as an example the T shaped area shown in figure 4.4.2. The location of the centroid is obtained from the equation (see Statics notes CH 4)

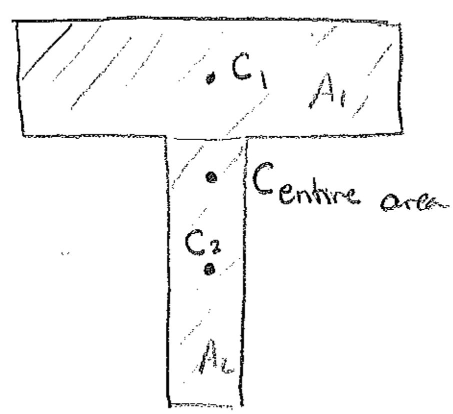


Figure 4.4.2 Example of a Composite Area

$$Y_c = \frac{\Sigma Y_i A_i}{\Sigma A_i} \qquad (4.4.8)$$

where Y_c is the centroid's location, Y_i and A_i are the centroid location and area of the different areas, respectively.

the value of the moment of inertia of the whole section with respect to its centroid (the total area's centroid) is the sum of the individual areas' moment of inertias with respect to the centroidal axis of <u>the entire area</u>. or

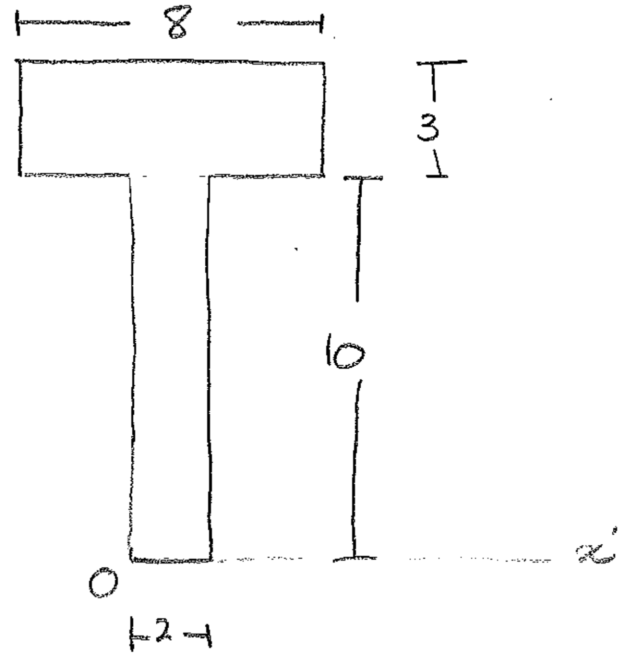
$$I_{area} = \Sigma \left(I_i\right)_{\text{about centroid } \underline{\text{ENTIRE}} \text{ area}} \qquad (4.4.9)$$

So in order to obtain the right hand side of this equation, one must find the I for each area (values are usually tabulated), then add d^2A to it (where d is the distance from the centroid of the entire area and the individual one.

In summary, when the moment of inertia of a composite area is needed, the first step is to find the locations of each individual area, then that of the entire area. For the second step, values of I of each area (about their respective centroids) are obtained (values for common shapes are tabulated, others can be obtained by integration). The third step is to compute the moment of inertia of each area with respect to the centroidal axis of the entire area, i.e., add d^2A to each of values obtained in the second step. The fourth and last step consists of summing all the values obtained to get I_{area} (equation (44.9)) this is obviously with respect to the centroidal axis of the entire area

Example 4.4.1:

Find the moment of inertia of the entire section about the axis $O'x'$.



units are in inches

Solution: the problem can be divided into the following steps:

1. Divide the area into well defined geometric areas.

2. Compute the location of the centroid of each area.

3. Compute the location of the centroid of the composite area.

4. Compute the moment of inertia of each area with respect to its centroidal axis

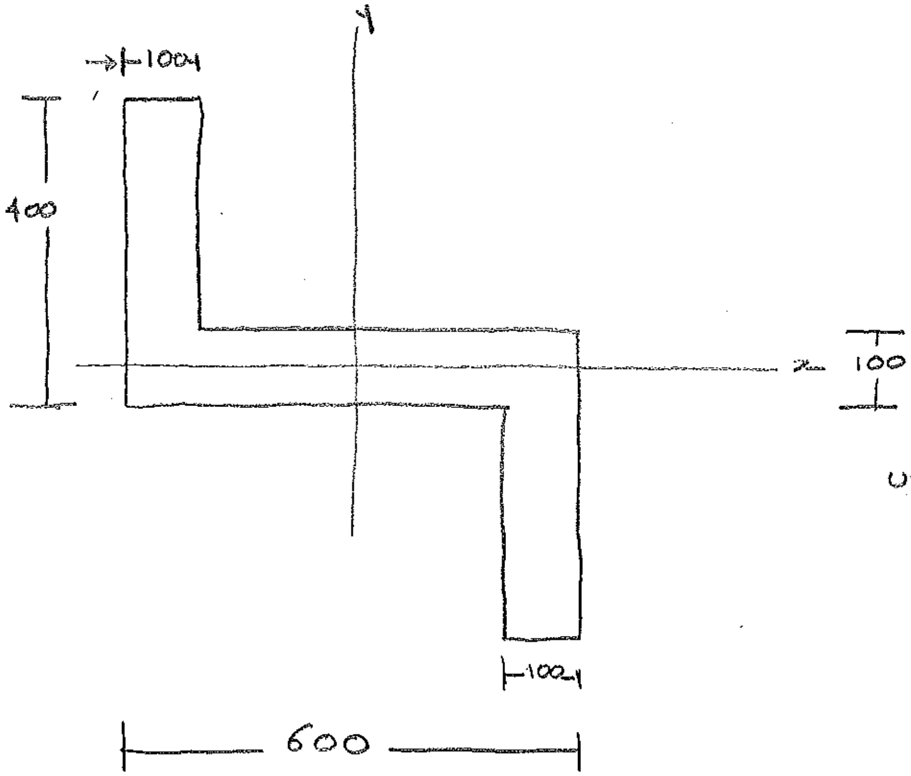
5. Compute the moment of inertia of each area with respect to the centroidal axis of the entire area

6. Sum the values from 5. This is the moment of inertia of the entire area about its centroidal axis.

7. Compute the moment of inertia of the entire area but with respect to the $O'x'$ axis.

Example 4.4.2.

Find the moment of inertia of the cross sectional area shown about the x and y axes.

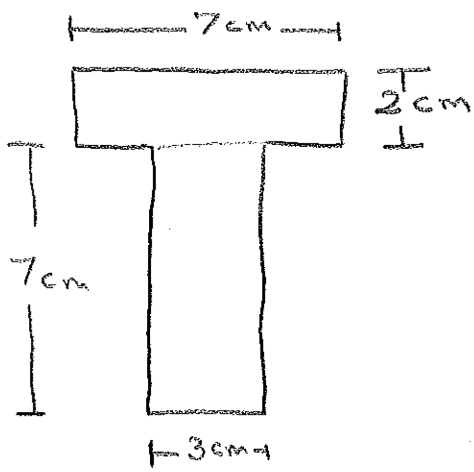


units are in mm

Solution:

Example 4.4.3.

the cross sectional area of the beam is shown below. If a torque $T = 4$ KN.m is applied, find the strain ε at P. (You must compute the location of the neutral axis by using equation (4.4.9))



Solution

4.5 Moments about a Random Direction.

This section deals with the case when the moment applied is still perpendicular to the axis of the beam but is at a random angle θ as shown in figure 4.5.1. The moment $\vec{M}$ can be expressed in terms of its y and z axes, or

$$\vec{M} = \vec{M}_y + \vec{M}_z \qquad (4.5.1)$$

where

$$M_y = M \sin\theta \qquad (4.5.2)$$

and

$$M_z = M \cos\theta \qquad (4.5.3)$$

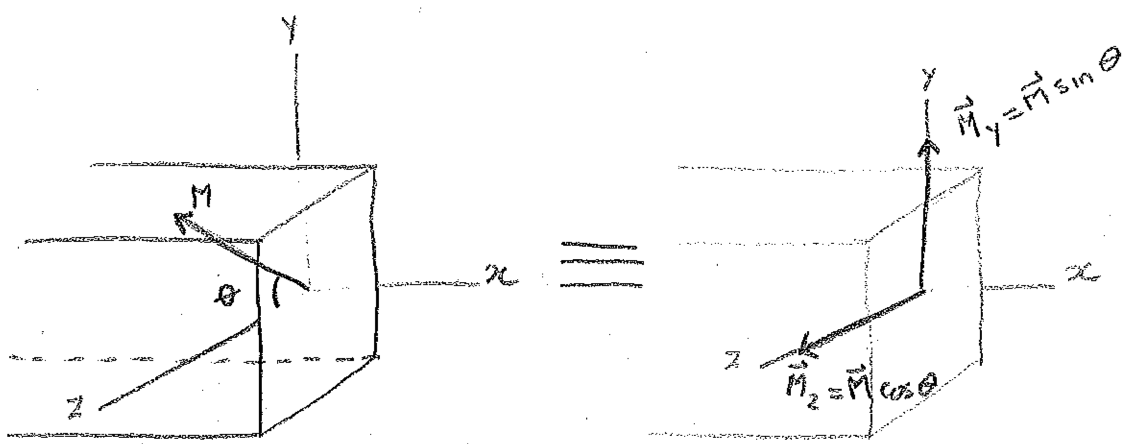
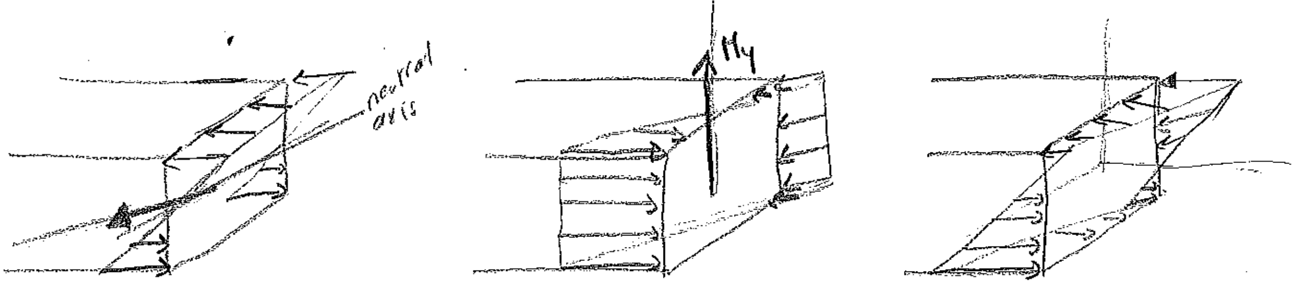


Figure 4.5.1 Moment at a Random Direction

If the material behaves elastically and if the deformations are not extreme, the superposition principle applies. So the y and z moment components are taken seperatly, the corresponding respective stresses are obtained and summed to yield the total resulting stress. Figure 4.5.2 shows the stress distribution for each moment seperatly as well as the sum of both. The stresses due to the M_z component are compressive and tensile on the top and bottom, respectively, they are given by

$$\sigma_y = \frac{-M_z Y}{I_z} \qquad (4\text{-}5\text{-}4)$$



a. σ_y due to M_z b. σ_z due to M_y c. Total $\sigma = \sigma_z + \sigma_y$ (depends on values of σ_z & σ_y)

Figure 4.5.2 σ_y and σ_z

The same can be said for the y component of M, so

$$\sigma_z = \frac{M_y\, y}{I_y} \qquad (4.5.4)$$

The student might wonder where did the negative sign go? Recall two things. first, the direction of the xyz axes must be taken following the right hand rule. Second, by convention, compressive stresses are taken as negative, so in order to satisfy the positive convetion for moment and stresses a negative sign is placed so positive moment yield the appropriate values, i.e., compression on top. The top direction is not difficult to see as it has been considered so far, i.e, a moment about the z axis. when one looks at the y component of the moment, there are no top and bottom but right and left. A careful look at the figures shows that the top direction is really in the positive y. So the corresponding top is to the left, also along the positive direction, but in this case along the z axis. Given the moment M_y's direction, the left (or top fibers) are in tension (the top are in compression in the case of M_z) and the right in compression. The M_y is positive (along the positive y) but yield the opposite situation as for M_z, hence, the negative sign is removed, otherwise sign conversions do not work. So

$$\sigma = \sigma_y + \sigma_z \qquad (4.5.5)$$

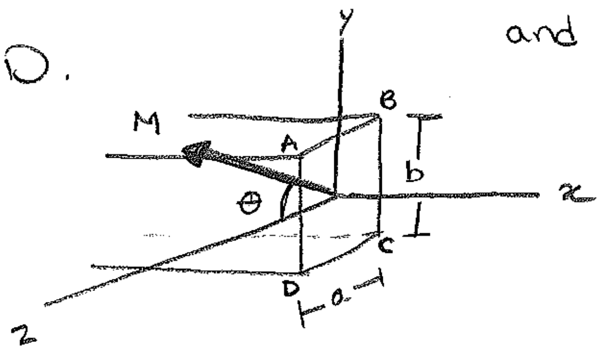
or

$$\sigma = \frac{-M_z y}{I_z} + \frac{M_y z}{I_y} \qquad (4.5.6)$$

It is important to note that, unlike the previous section where the moment was always taken about the z axis, the values for I in equation (4.5.6) must be clearly labeled as I_z or I_y and properly computed

Example 4.5.1:

Find an expression for the stresses at points A, B, C & D. and substitute for the following



$M = 1000$ N.m

$\theta = 30$

$a = 2$ cm

$b = 3$ cm

Solution:

One important question remains, where is the neutral axis? The answer lies in the definition of the neutral axis, i.e. $\sigma = 0$ along that line. Or, from equation (4.5.6)

$$\sigma = 0 \quad \text{at neutral axis} \tag{4.5.7}$$

or
$$\frac{M_z y}{I_z} = \frac{M_y z}{I_y} \tag{4.5.8}$$

or
$$y = \frac{M_y I_z}{M_z I_y} z \tag{4.5.9}$$

But $M_y = M \sin\theta$ and $M_z = M\cos\theta$ so

$$y = \left(\frac{I_z}{I_y} \tan\theta\right) z \tag{4.5.10}$$

This is the equation of the neutral axis. It obviously goes through the centroid (i.e. when $z=0$ $y=0$) with a slope of $m = \frac{I_z}{I_y}\tan\theta$.

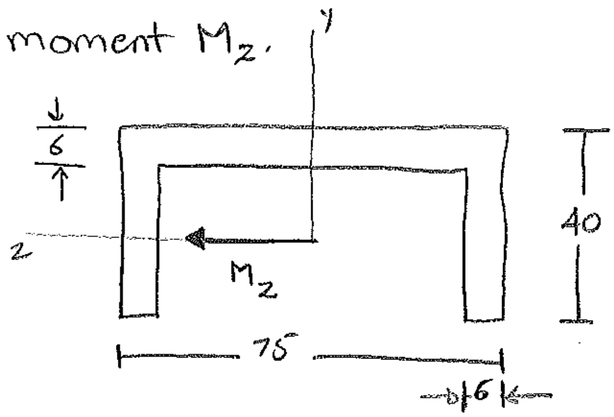
Example 4.5.2

Find the neutral axis in the previous problem.

Example 4.5.3 :

If the maximum allowable stress is 230 MPa, find the maximum allowable moment M_z.

Solution:



units are in mm

Example 4.5.4.

Re do the previous example but solve for M_y.

Example 4.5.5.

Find the maximum moment M that results from the two moments obtained in examples 4.5.3 and 4.5.4.

Solution:

4.6 Beams Made of Several Materials.

If a beam is made of two materials, the equations derived so far cannot be used as is. Consider the beam shown in figure 4.6.1. The strain will still be varying linearly except it is not zero at the centroid. In other words, the neutral axis does no go through the centroid. After all why would it? If material A is foam and B is steel, the steel part will support most of the load. In this case, the steel must feel both compressive and tensile loads so the sum of the forces in the x direction must be zero, since the foam has negligible stresses.

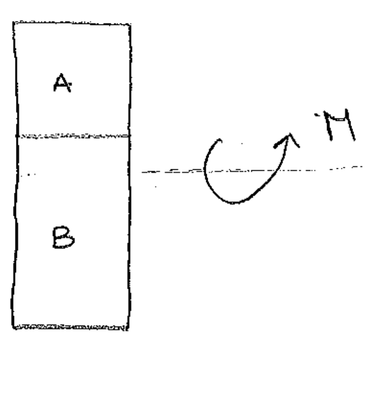


Figure 4.6.1 Beam Made of Two Materials

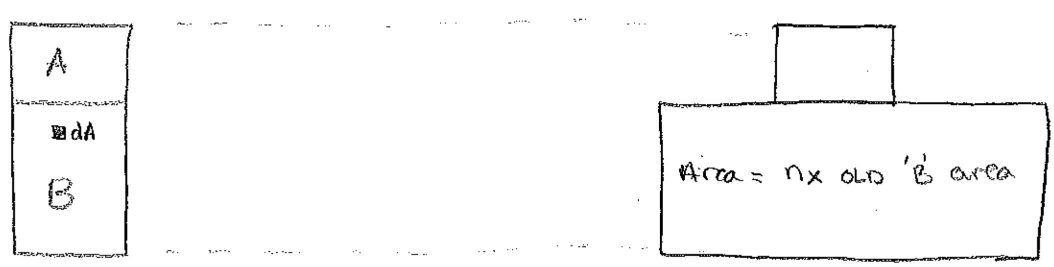
If this is the case, the location of the neutral axis must be found. To do so consider figure 4.6.2-a. It shows a beam made of two materials. Both materials experience a linear change in the strain. The stresses on the other hand are different since the Young's Moduli are different.

Consider the area dA shown in figure 4.6.2-a. It sees a force dF_1 equal to

$$dF_1 = \sigma_1 \, dA \tag{4.6.1}$$

or

$$dF_1 = \varepsilon E_1 \, dA \tag{4.6.2}$$



a) Original Area b) Transformed Area

Figure 4.6.2 Equivalent Areas ($n > 1$ in this case)

But since

$$\varepsilon = \frac{-y}{\rho} \qquad (4.1.5)$$

so

$$dF_1 = \frac{-E_1 y}{\rho}\, dA \qquad (4.6.2)$$

this is the force an area dA feels. If one wishes to replace material A with material B, so the entire area is made of material B without changing the strain distribution or changing the force applied i.e. dF_1, the area will have to change. If material A is steel and B is wood, an equivalent beam made of wood will have the steel area replaced by a larger one made of wood. If n is defined as

$$n = \frac{E_1}{E_2} \qquad (4.6.3)$$

then dF_1 can be written as

$$dF_1 = \frac{-E_2\, y}{\rho}\,(n\,dA) \qquad (4.6.4)$$

The previous equation states that if material A is to be replaced with B, in order to keep dF_1 the same, the 'new area' must be n times the 'old area'. If $n = 3$, it means the new beam's equivalent area is the one made of wood plus a new one made of wood but 3 times the steel one. Keep in mind that the vertical distance must not change (otherwise the shear distribution becomes different) so the area must become wider or thinner but not longer or shorter. This gives you a new area made up of one material. The centroid of this new area is the location of the neutral axis of this area. The stresses can be obtained using the flexure formula. However in the transformed area, the stresses must be multiplied by n. So that,

$$\sigma = \frac{-MY}{I_{transf.}} \quad \text{where no sections of area have changed} \qquad (4.6.5)$$

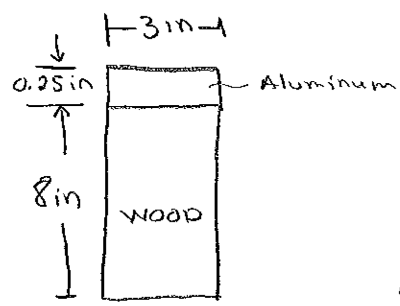
and $\sigma = \dfrac{-nMY}{I_{tranf}}$ where area sections have changed (4.6.)

where I_{trans} is the moment of <u>inertia of the transformed area</u>

<u>Example 4.6.1:</u>

A 10ft long beam has the cross-sectional area shown below. It is subjected to a distributed load of 100 lb/ft. Take $E_{wood} = 1700$ ksi and $E_{al} = 10\,200$ ksi

Sketch the stress distribution across the area.



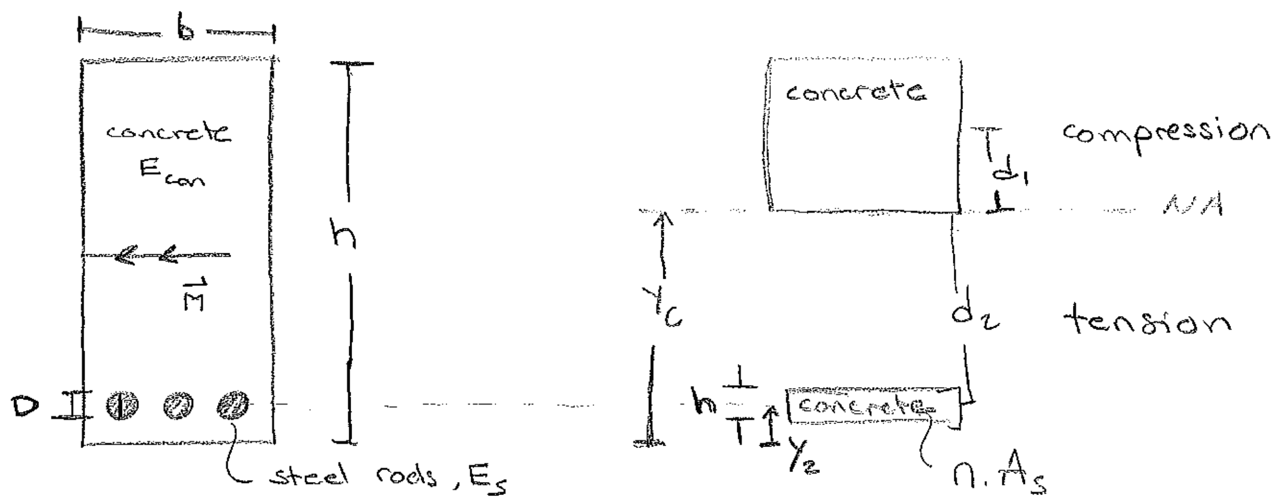
Not to scale

<u>Solution</u>

Example 4.6.2.

Redo the previous problem and switch the transformed area, i.e. if the equivalent area in the previous example is taken to be all wood, take the steel one here (and vice versa)

A common example of a beam made out of two materials and subjected to bending is that of a reinforced concrete beam. While extremely strong in compression, concrete is weak in tension. As a result, reinforcing steel bars are placed along the length of the beam, as far away as possible from the neutral axis, and in the tension part of course. Such a beam can be represented as shown in figure 4.6.3. Note that $n = E_s/E_{con}$. Now since concrete is weak in tension, it is as if it does not even exist. As a result, the concrete area in



a - Actual Beam Area.

b - Equivalent Area

Figure 4.6.3 Reinforced Concrete Beam

the tension part is ignored. What remains is the nA_s area. Since this is the equivalent part of the steel, it cannot be dismissed. The rest of the problem is similar to the examples seen earlier in this section with the exception that when I is computed as $I_{con} + A_{con} d_1^2 + I_{nA_s} + nA_s d_2^2$ the term I_{nA_s} and sometimes $nA_s d_2^2$ are dropped since they are insignificant. I_{nA_s} is the moment of inertia of the thin strip of area nA_s. The height of this strip is needed to find I_{nA_s} $\left(= \frac{1}{12} b h^3\right)$, if needed, h can be taken to be same as the diameter D of the rods. $nA_s d_2^2$ can be obtained without having to compute h. Finding the location of the neutral axis Y_c is a little more involved than for previous areas. If the bottom of the beam is used as a reference, then

$$Y_c = \frac{(h-Y_c)\,b\left(\frac{h-Y_c}{2}\right) + nA_s\left(Y_2\right)}{(h-Y_c)\,b + nA_s} \qquad (4.6.7)$$

↳ w.r.t bottom of beam

This equation needs to be solved for Y_c. Now, another alternative is to take the reference axis, not at the bottom but at the centroid itself. True this position is not known, (that is what is needed), but it yields a Y_c of zero, or

$$Y_c = \frac{A_1Y_1 + A_2Y_2}{A_1 + A_2} = 0 \qquad (4.6.8)$$

↳ w.r.t neutral axis of entire area.

or $$A_1Y_1 + A_2Y_2 = 0 \qquad (4.6.9)$$

→ w.r.t. neutral axis of entire area

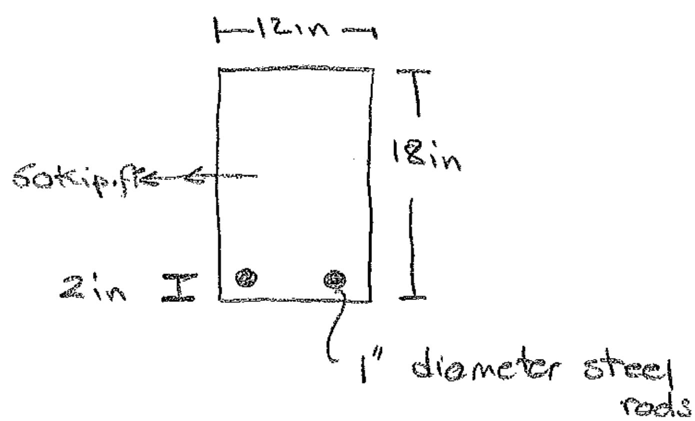
or $$2bd_1d_1 + nA_sd_2 = 0 \qquad (4.6.10)$$

but $$h = 2d_1 + d_2 + Y_2 \qquad (4.6.11)$$

where h and Y_2 are the height of the beam and the location of the centers of the steel bars (from the bottom). (4.6.10) and (4.6.11) can be solved for d_1.

Example 4.6.3

A reinforced concrete beam has the following dimensions. If it is subjected to a moment $M = 60$ Kip.ft, find the maximum stresses in the steel and concrete. Let $E_s = 29 \times 10^3$ Ksi and $E_{con} = 3.6 \times 10^3$ psi. Solve for I by including I_{NA_s} and $nA_s d_1^2$ first then by ignoring them.



4.7 Stress Concentration Factors.

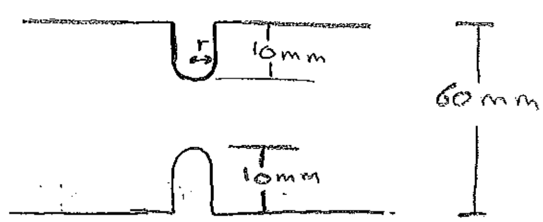
At this point, the concept of the stress concentration factors should be understood. The maximum stress is given by

$$\sigma_{max} = K \cdot \frac{Mc}{I} \qquad (4.7.1)$$

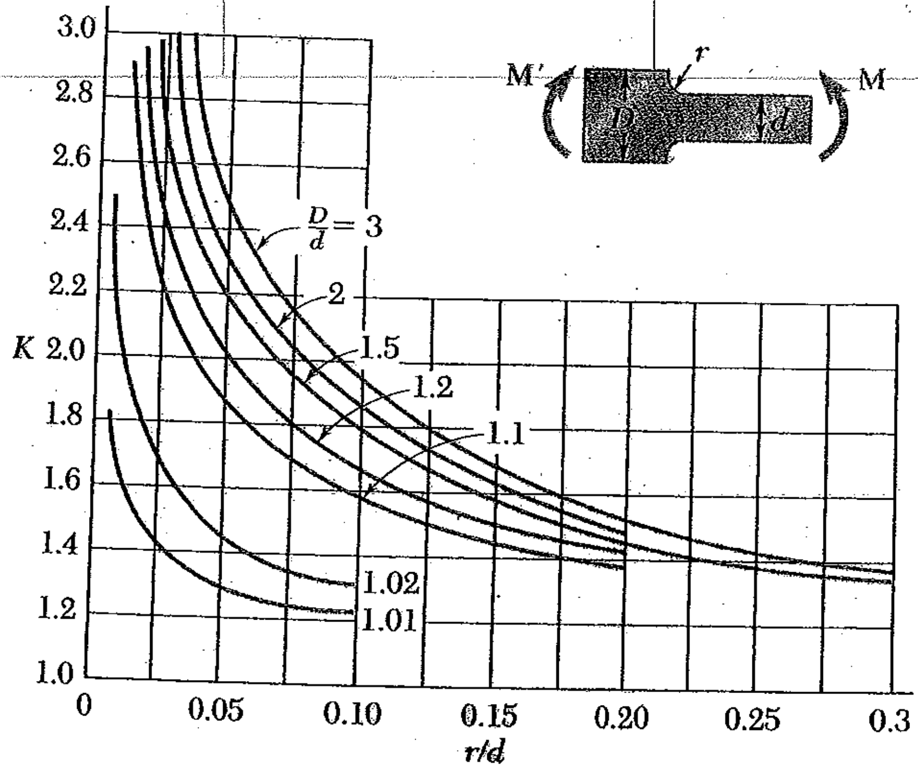
I is that of the beam with diameter d. Figure 4.7.1 shows two graphs for bending moments.

Example 4.7.1.

The stress is not to exceed 150 MPa for a moment $M = 180$ N.m. Find r.



Solution,



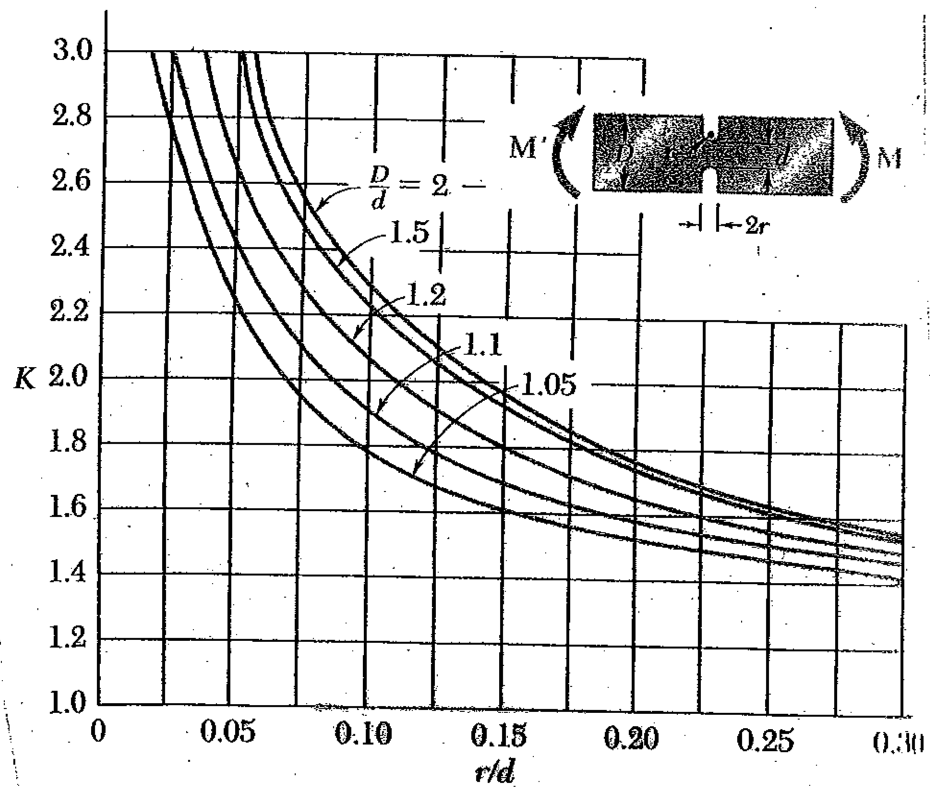


Figure 4.71 Stress Concentrations.