

Due Friday, April 22, at the start of your class

- Neatly write solutions to the following questions on separate paper.
- Write out all steps (correctly) to fully support your answers, even if you can work the problem in your head or on a calculator or computer. This means that all integration needs to be done by hand and work shown to support your steps. If you do not show the integration steps, do not expect to receive full credit.
- All answers should be left in **EXACT** form unless otherwise stated.
- Remember to use proper mathematical notation and vocabulary. Justify your results thoroughly, include explanations of your steps, include graphs where helpful, and use good organization. Part of your score is based on these criteria.
- Work that appears to be copied from any source will receive a score of zero.

(10 pts) 1. Consider the double integral $\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{xy^3}{x^2 + y^2} dy dx$.

- Explain why this integral is inconsistent with the definition of the double integral.
- Explain why Fubini's Theorem does not apply to this integral as written.
- Describe the behavior of $f(x, y) = \frac{xy^3}{x^2 + y^2}$ as $(x, y) \rightarrow (0, 0)$.
- Treat the integral as an improper integral (see section 8.7 if necessary) and use correct notation for improper integrals to evaluate the double integral. Be sure to use correct notation and justify every step of your work.

(6 pts) 2. Evaluate the integral $\iiint_D e^{-\sqrt{x^2+y^2+z^2}} dV$ where D is all of $\mathbb{R}^3$. Note that this is an improper integral, so you should use correct notation and thoroughly justify your steps.

(9 pts) 3. Use an appropriate change of variables to evaluate the integral $\iint_R (2x+2y)e^{x^2-y^2} dA$ over the rectangle with vertices $(0, 0)$, $(\frac{5}{2}, \frac{1}{2})$, $(\frac{3}{2}, \frac{3}{2})$, and $(1, -1)$. (Hint: It may be helpful to notice that $x^2 - y^2 = (x+y)(x-y)$.)