

Quiz #20 – A Complicated Surface Integral Problem

Imagine yourself as the manufacturer of Quaker Oats. Quaker Oats are packaged in cylinders. If the base of the cylinder is on the xy plane, the cost of creating this container is given by the function $f(x, y, z) = 2x + 3y + z$ dollars per square foot. Determine the cost of creating a container with both a lid and a base that is ten feet tall with a radius of 2 feet by completing the questions below.

$$\sin \rightarrow \cos$$

$$\cos \rightarrow -\sin$$

1. How many separate surface integrals do you need to complete? 4
2. First, let's compute the surface integral for the base. Come up with a parameterization for the base.
(Hint: Cylindrical coordinates)



$$\text{Total cost} = \text{cost/square foot} \cdot \text{surface area}$$

$$\text{Cost} = \iint_S (2x + 3y + z) \, dS$$

Parameterize S :

$$x = r \cos(\theta), \quad y = r \sin(\theta), \quad z = 0$$

$$x = 2 \cos(\theta), \quad y = 2 \sin(\theta)$$

3. Find $\|\mathbf{r}_r \times \mathbf{r}_\theta\|$. Show all work, no shortcuts allowed.

$$\vec{r}(r, \theta) = \langle 2 \cos(\theta), 2 \sin(\theta), 0 \rangle$$

$$\vec{r}_\theta = \langle -2 \sin(\theta), 2 \cos(\theta), 0 \rangle$$

$$\vec{r}_r = \langle 2, 2, 0 \rangle$$

4. Find the cost to produce the base.

5. Now, go thorough the same process for the lid. You may use any shortcuts that you learned from finding the cost of the base.
6. Now, we must compute the surface integral for the cylindrical part of the container. Parameterize the cylinder.
7. Compute $\|r_z \times r_\theta\|$. No shortcuts, show all work.
8. Compute the surface integral for the cylinder.
9. Find the total cost.