

1. Use Cauchy's theorem or integral formula to evaluate the integrals:

a) $\oint_C \frac{\sin 2z}{6z - \pi} dz$, where C is the circle $|z| = 3$.

b) $\oint_C \frac{\cosh z}{2 \ln 2 - z} dz$, where C is the circle: (i) $|z| = 1$; (ii) $|z| = 2$.

2. For each of the following functions find the first 4 terms of each of the Laurent series about the origin, furthermore, one series for each annular ring between singular points.

Find the residue of each function at the origin.

a) $\frac{1}{z(z-1)(z-2)}$.

b) $\frac{1}{z^2(z+1)^2}$.

3. Find the Laurent series for the following functions about the indicated point; hence find the residue of the function at the point.

a) $\frac{1}{z(z+1)}$, at $z = 0$.

b) $\frac{\sin z}{z^4}$, at $z = 0$.

4. Find the residues of the function at the indicated points:

$\frac{1}{(3z+2)(2-z)}$, at $z = -\frac{2}{3}$ and at $z = 2$.

5. Evaluate the integrals:

a) $\int_0^{2\pi} \frac{\sin^2 \theta d\theta}{5 + 3 \cos \theta}$.

b) $\int_0^{\infty} \frac{x^2 dx}{x^4 + 16}$.

c) $\int_{-\infty}^{\infty} \frac{\sin x dx}{x^2 + 4x + 5}$.

d) $\int_0^{\infty} \frac{\sqrt{x} dx}{1 + x^2}$.