1. Find the real and imaginary parts, $u(x,y)$ and $v(x,y)$ of the following functions:

a) $f(z) = \sin z$.

b) $f(z) = \dfrac{2z+3}{z+2}$.

2. Show that the derivative of $f(z) = x^2 - y^2 + 2ixy$ exists and is unique by considering $\Delta y = m\Delta x$, that is, Δz goes to zero along a straight line with slope m, thus f(z) is analytic for all z.

3. a) Find out whether the function $\dfrac{y-ix}{x^2+y^2}$ is analytic? Give details to support your results.

b) Using the formal definition of derivative, verify that $d(\ln z)/dz = 1/z$ ($z \neq 0$) holds. Hint: Expand $\ln(1+\Delta z/z)$ in series.

c) Find Cauchy-Riemann conditions in polar coordinates, starting with $z = re^{i\theta}$ and $f(z) = u(r,\theta) + iv(r,\theta)$.

d) Show that $u(x,y) = 3x^2y - y^3$ is a harmonic function and find the function f(z) of which u is the real part. Derive $v(x,y)$ and how that $v(x,y)$ is also harmonic.

4. Evaluate the following integrals in the complex plane by direct integration

a) $\int \dfrac{dz}{z^2+8i}$ along the line $y = x$ from 0 to ∞.

b) $\int_0^{1+2i} |z|^2\, dz$ along the indicated paths:

(i) Along the straight line from 0 to 1+2i.

(ii) First from 0 to 2i, then horizontally from 2i to 1+2i.