(Supplement – part of 13.3)

For each problem, show all work to support your conclusions (even if you can work it in your head or on a calculator), and leave answers in *exact* form unless otherwise indicated. Attach additional paper if necessary. If you need help, get it! Do not turn this in with problems unfinished!

1. Briefly ***compare*** and ***contrast*** the definition of "vector-valued function" and the definition of an ordinary function $y = f(x)$.

2. How do you determine whether a curve C determined by $\mathbf{r}(t)$ is "smooth"? Give an example of a function $\mathbf{r}(t)$ that determines a *non-smooth* curve C, and explain why it fails to be smooth. Your example should be different from the examples we discussed in class.

3. Give the domain of $\mathbf{r}(t) = \ln(t+2)\mathbf{i} + \sin^{-1}(\frac{t}{5})\,\mathbf{j} + \frac{1}{t-1}\mathbf{k}$. ***Justify your answers.***

4. If $\mathbf{r}'(t) = \left\langle t\cos 2t,\ e^{-3t},\ \sqrt{5t+4} \right\rangle$ and $\mathbf{r}(0) = \langle 2,4,6 \rangle$, find $\mathbf{r}(t)$.

5. Let $\mathbf{r}(t) = \langle \sin 2t, 3\cos 2t, e^{t/4} \rangle$, with $-\infty < t < \infty$.

(a) Explain why it is important to consider the domain of $\mathbf{r}(t)$ when sketching the graph of C, the space curve determined by $\mathbf{r}(t)$.

(b) Is C smooth? Explain how you know.

(c) Sketch the space curve C determined by $\mathbf{r}(t)$, and show $\mathbf{r}(\pi)$, $\mathbf{v}(\pi)$, and $\mathbf{a}(\pi)$ on a BIG graph of C. You'll need lots of space for this graph – make sure your scale is appropriate. Be sure your graph of C displays enough of the important characteristics of the curve to be able to infer the shape of the entire graph of C for all values of t in the domain of $\mathbf{r}$. If you wish, you may include a sentence or two to further describe C in addition to your graph.

6. Consider the vector-valued function $\mathbf{r}(t) = at\,\mathbf{i} + b\sin\omega t\,\mathbf{j} + b\cos\omega t\,\mathbf{k}$, where a, b, and ω are positive constants, that describes the position (in units of length from the origin) of a particle t units of time after the object starts moving.

(a) Describe the graph of C, the space curve determined by $\mathbf{r}(t)$.

(b) How does changing the value of a (while leaving b and ω fixed) change the graph of C?

(c) How does changing the value of b (while leaving a and ω fixed) change the graph of C?

(d) How does changing the value of ω (while leaving a and b fixed) change the graph of C?

(e) Find the speed of the particle moving along C at any time t.

(f) Find the arclength of C for one coil. (Note that your answer will involve a, b, and ω.)

(g) Based on your answer to part (e), is the particle speeding up or slowing down as t increases? Explain.