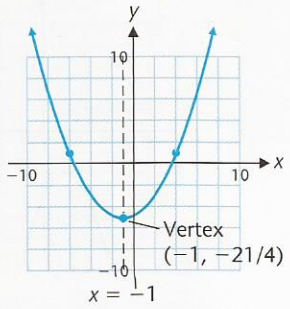
4. (A) Vertex: $(-1, -\frac{21}{4})$; axis of symmetry: $x = -1$

(B)



Min $f(x) = f(-1) = -\frac{21}{4}$; domain: $(-\infty, \infty)$; range: $[-\frac{21}{4}, \infty)$; decreasing on $(-\infty, -1]$; increasing on $[-1, \infty)$

5. $y = \frac{1}{4}(x - 4)^2 - 2 = 0.25x^2 - 2x + 2$

6. (A) $A(x) = (240 - 4x)x$, $0 \le x \le 60$

(B) The maximum combined area of 3,600 ft.^{2} occurs at $x = 30$ feet.

(C) Each pen is 30 feet by 40 feet with area 1,200 ft.^{2}

7. 4.33 seconds

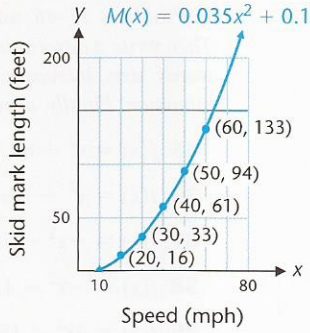
8. $[-3 - \sqrt{15}, -3 + \sqrt{15}]$

9. (A) $P(x) = 46.5x - 0.005x^2 - 58{,}500$, $0 \le x \le 10{,}000$

(B) Profit is positive for sales between 1,500 and 7,800 gallons per week and negative for sales less than 1,500 or for sales between 7,800 and 10,000.

(C) The maximum profit is \$49,612.50 at a sales level of 4,650 gallons.

10. (A)



(B) 52 mph

3-4 Exercises

1. Describe the graph of any quadratic function.
2. How can you tell from a quadratic function whether its graph opens up or down?
3. True or False: Every quadratic function has a maximum. Explain.
4. Using transformations, explain why the vertex of $f(x) = a(x - h)^2 + k$ is (h, k).
5. What information does the constant a provide about the graph of a function of the form $f(x) = ax^2 + bx + c$?
6. Explain how to find the maximum or minimum value of a quadratic function.

In Problems 7–12, find the vertex and axis of the parabola, then draw the graph.

7. $f(x) = (x + 3)^2 - 4$

8. $f(x) = (x + 2)^2 - 2$

9. $f(x) = -\left(x - \frac{3}{2}\right)^2 - 5$

10. $f(x) = -\left(x - \frac{11}{2}\right)^2 + 3$

11. $f(x) = 2(x + 10)^2 + 20$

12. $f(x) = -\frac{1}{2}(x + 8)^2 + 12$

In Problems 13–18, write a brief verbal description of the relationship between the graph of the indicated function and the graph of $y = x^2$.

13. $f(x) = (x - 2)^2 + 1$

14. $g(x) = -(x + 1)^2 - 2$

15. $h(x) = -(x + 1)^2$

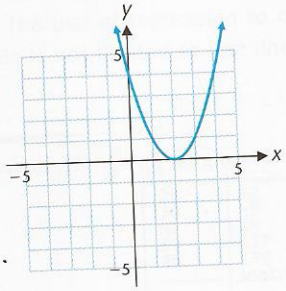
16. $k(x) = (x - 2)^2$

17. $m(x) = (x - 2)^2 - 3$

18. $n(x) = -(x + 1)^2 + 4$

In Problems 19–24, match each graph with one of the functions in Problems 13–18.

19.



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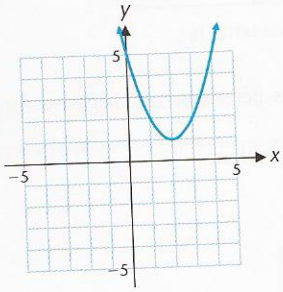

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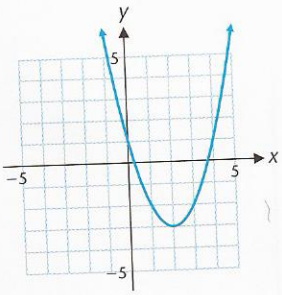

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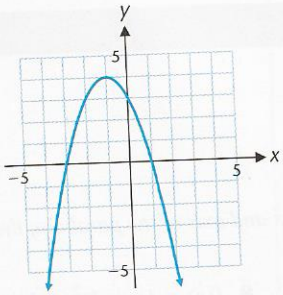

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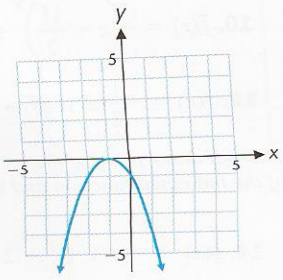

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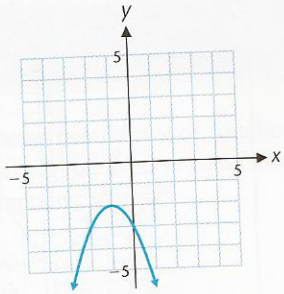


In Problems 25–34, complete the square and find the vertex form of each quadratic function, then write the vertex and the axis and draw the graph.

25. $f(x) = x^2 - 4x + 5$

26. $g(x) = x^2 - 6x + 1$

27. $h(x) = -x^2 - 2x - 3$

28. $k(x) = -x^2 - 10x + 3$

29. $m(x) = 2x^2 - 12x + 22$

30. $n(x) = 3x^2 + 6x - 2$

31. $f(x) = \frac{1}{2}x^2 + 3x - \frac{7}{2}$

32. $g(x) = -\frac{3}{2}x^2 + 9x + \frac{11}{2}$

33. $f(x) = 2x^2 - 24x + 90$

34. $g(x) = 3x^2 + 24x + 30$

In Problems 35–46, use the formula $x = -b/2a$ to find the vertex. Then write a description of the graph using all of the following words: axis, increases, decreases, range, and maximum or minimum. Finally, draw the graph.

35. $f(x) = x^2 + 8x + 8$

36. $f(x) = x^2 + 10x + 10$

37. $f(x) = -x^2 - 7x + 4$

38. $f(x) = -x^2 - 11x + 1$

39. $f(x) = 4x^2 - 18x + 25$

40. $f(x) = 5x^2 + 30x - 17$

41. $f(x) = -10x^2 + 50x + 12$

42. $f(x) = -8x^2 - 24x + 16$

43. $f(x) = x^2 + 3x$

44. $f(x) = 4x - x^2$

45. $f(x) = 0.5x^2 - 2x - 7$

46. $f(x) = 0.4x^2 + 4x + 4$

In Problems 47–60, solve and write the answer using interval notation.

47. $x^2 < 10 - 3x$

48. $x^2 + x < 12$

49. $x^2 + 21 > 10x$

50. $x^2 + 7x + 10 > 0$

51. $x^2 \le 8x$

52. $x^2 + 6x \ge 0$

53. $x^2 + 5x \le 0$

54. $x^2 \le 4$

55. $x^2 + 1 < 2x$

56. $x^2 + 25 < 10x$

57. $x^2 < 3x - 3$

58. $x^2 + 3 > 2x$

59. $x^2 - 1 \ge 4x$

60. $2x + 2 > x^2$

3-5 Exercises

1. Explain how to find the sum of two functions.

2. Explain how to find the product of two functions.

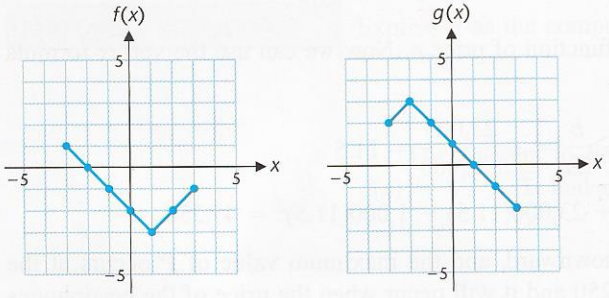
3. Describe in your own words what the composition of two functions means. Don't focus on how to find composition, but rather on what it really means.

4. Is the domain of f/g always the same as the intersection of the domains of f and g? Explain.

5. When composing two functions, why can't you always find the domain by simply looking at the simplified form of the composition?

6. Describe a real-world situation where the composition of two functions would have significance.

Problems 7–18 refer to functions f and g whose graphs are shown below.



In Problems 7–10 use the graphs of f and g to construct a table of values and sketch the graph of the indicated function.

7. $(f+g)(x)$ **8.** $(g-f)(x)$

9. $(fg)(x)$ **10.** $(f-g)(x)$

In Problems 11–18, use the graphs of f and g to find each of the following:

11. $(f \circ g)(-1)$ **12.** $(f \circ g)(2)$

13. $(g \circ f)(-2)$ **14.** $(g \circ f)(3)$

15. $f(g(1))$ **16.** $f(g(0))$

17. $g(f(2))$ **18.** $g(f(-3))$

In Problems 19–26, find the indicated function value, if it exists, given $f(x) = 2 - x$ and $g(x) = \sqrt{3-x}$.

19. $(f+g)(-3)$ **20.** $(g-f)(-5)$

21. $(fg)(-1)$ **22.** $\left(\frac{f}{g}\right)(3)$

23. $(f \circ g)(-2)$ **24.** $(f \circ g)(1)$

25. $(g \circ f)(1)$ **26.** $(g \circ g)(-7)$

27. Functions f and g are defined by Table 5. Find $(f \circ g)(-7)$, $(f \circ g)(0)$, and $(f \circ g)(4)$.

28. Functions h and k are defined by Table 6. Find $(h \circ k)(-15)$, $(h \circ k)(-10)$, and $(h \circ k)(15)$.

Table 5

x	$f(x)$	$g(x)$
-7	5	4
-2	9	10
0	0	-2
4	3	6
6	-10	-3

Table 6

x	$h(x)$	$k(x)$
-20	-100	30
-15	-200	5
-10	-300	15
5	-150	8
15	-90	-10

In Problems 29–42, for the indicated functions f and g, find the functions $f+g$, $f-g$, fg, and f/g, and find their domains.

29. $f(x) = 4x$; $g(x) = x + 1$

30. $f(x) = 3x$; $g(x) = x - 2$

31. $f(x) = 2x^2$; $g(x) = x^2 + 1$

32. $f(x) = 3x$; $g(x) = x^2 + 4$

33. $f(x) = 3x + 5$; $g(x) = x^2 - 1$

34. $f(x) = 2x - 7$; $g(x) = 9 - x^2$

35. $f(x) = \sqrt{2-x}$; $g(x) = \sqrt{x+3}$

36. $f(x) = \sqrt{x+4}$; $g(x) = \sqrt{3-x}$

37. $f(x) = \sqrt{x} + 2$; $g(x) = \sqrt{x} - 4$

38. $f(x) = 1 - \sqrt{x}$; $g(x) = 2 - \sqrt{x}$

39. $f(x) = \sqrt{x^2 + x - 6}$; $g(x) = \sqrt{7 + 6x - x^2}$

40. $f(x) = \sqrt{8 + 2x - x^2}$; $g(x) = \sqrt{x^2 - 7x + 10}$

41. $f(x) = x + \frac{1}{x}$; $g(x) = x - \frac{1}{x}$

42. $f(x) = x - 1$; $g(x) = x - \frac{6}{x-1}$

In Problems 43–60, for the indicated functions f and g, find the functions $f \circ g$, and $g \circ f$, and find their domains.

43. $f(x) = x^3$; $g(x) = x^2 - x + 1$

44. $f(x) = x^2$; $g(x) = x^3 + 2x + 4$

45. $f(x) = |x + 1|$; $g(x) = 2x + 3$

46. $f(x) = |x - 4|$; $g(x) = 3x + 2$

47. $f(x) = x^{1/3}$; $g(x) = 2x^3 + 4$

48. $f(x) = x^{2/3}$; $g(x) = 8 - x^3$

49. $f(x) = \sqrt{x}$; $g(x) = x - 4$

50. $f(x) = \sqrt{x}$; $g(x) = 2x + 5$

51. $f(x) = x + 2$; $g(x) = \dfrac{1}{x}$

52. $f(x) = x - 3$; $g(x) = \dfrac{1}{x^2}$

53. $f(x) = \sqrt{4 - x}$; $g(x) = x^2$

54. $f(x) = \sqrt{x - 1}$; $g(x) = x^2$

55. $f(x) = \dfrac{x + 5}{x}$; $g(x) = \dfrac{x}{x - 2}$

56. $f(x) = \dfrac{x}{x - 1}$; $g(x) = \dfrac{2x - 4}{x}$

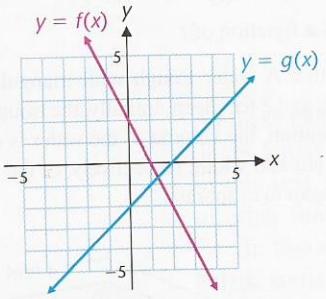
57. $f(x) = \dfrac{2x + 1}{x}$; $g(x) = \dfrac{1}{x - 2}$

58. $f(x) = \dfrac{2}{x + 3}$; $g(x) = \dfrac{2 - 3x}{x}$

59. $f(x) = \sqrt{25 - x^2}$; $g(x) = \sqrt{9 + x^2}$

60. $f(x) = \sqrt{x^2 - 9}$; $g(x) = \sqrt{x^2 + 25}$

Use the graphs of functions f and g shown below to match each function in Problems 61–64 with one of graphs (a)–(d).

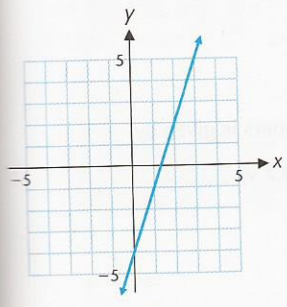


61. $(f + g)(x)$

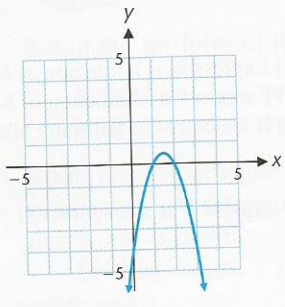
62. $(f - g)(x)$

63. $(g - f)(x)$

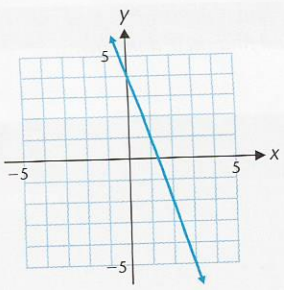
64. $(fg)(x)$



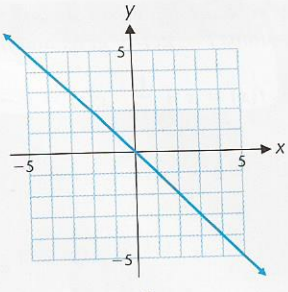
(a)



(b)



(c)



(d)

In Problems 65–72, find $f \circ g$ and $g \circ f$. Graph f, g, $f \circ g$, and $g \circ f$ in the same coordinate system and describe any apparent symmetry between these graphs.

65. $f(x) = \frac{1}{2}x + 1$; $g(x) = 2x - 2$

66. $f(x) = 3x + 2$; $g(x) = \frac{1}{3}x - \frac{2}{3}$

67. $f(x) = -\frac{2}{3}x - \frac{5}{3}$; $g(x) = -\frac{3}{2}x - \frac{5}{2}$

68. $f(x) = -2x + 3$; $g(x) = -\frac{1}{2}x + \frac{3}{2}$

69. $f(x) = \dfrac{x^3}{8}$; $g(x) = 2\sqrt[3]{x}$

70. $f(x) = 3\sqrt[3]{x}$; $g(x) = \dfrac{x^3}{27}$

71. $f(x) = \sqrt[3]{x - 2}$; $g(x) = x^3 + 2$

72. $f(x) = x^3 - 3$; $g(x) = \sqrt[3]{x + 3}$

In Problems 73–80, express h as a composition of two simpler functions f and g.

73. $h(x) = (2x - 7)^4$

74. $h(x) = (3 - 5x)^7$

75. $h(x) = \sqrt{4 + 2x}$

76. $h(x) = \sqrt{3x - 11}$

77. $h(x) = 3x^7 - 5$

78. $h(x) = 5x^6 + 3$

79. $h(x) = \dfrac{4}{\sqrt{x}} + 3$

80. $h(x) = -\dfrac{2}{\sqrt{x}} + 1$

81. Are the functions fg and gf identical? Justify your answer.

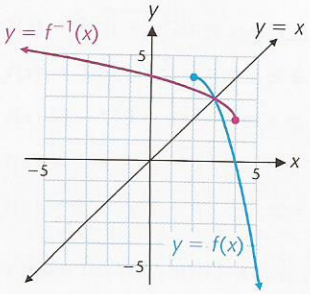
82. Are the functions $f \circ g$ and $g \circ f$ identical? Justify your answer.

83. Is there a function g that satisfies $f \circ g = g \circ f = f$ for all functions f? If so, what is it?

84. Is there a function g that satisfies $fg = gf = f$ for all functions f? If so, what is it?

ANSWERS TO MATCHED PROBLEMS

1. (A) Not one-to-one
   (B) One-to-one
2. They are inverses.
3. $f^{-1}(x) = x^2 - 2,\ x \geq 0$
4. $R(x) = 10x - 0.001x^2$
5. $f^{-1}(x) = 2 + \sqrt{4 - x},\ x \leq 4$



3-6 Exercises

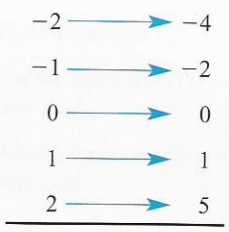
1. When a function is defined by ordered pairs, how can you tell if it is one-to-one?
2. When you have the graph of a function, how can you tell if it is one-to-one?
3. Why does a function fail to have an inverse if it is not one-to-one? Give an example using ordered pairs to illustrate your answer.
4. True or False: Any function whose graph changes direction is not one-to-one. Explain.
5. What is the result of composing a function with its inverse? Why does this make sense?
6. What is the relationship between the graphs of two functions that are inverses?

For each set of ordered pairs in Problems 7–12, determine if the set is a function, a one-to-one function, or neither. Reverse all the ordered pairs in each set and determine if this new set is a function, a one-to-one function, or neither.

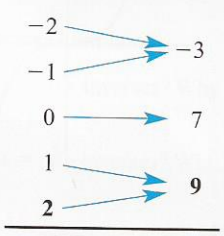
7. $\{(1, 2), (2, 1), (3, 4), (4, 3)\}$ yes
8. $\{(-1, 0), (0, 1), (1, -1), (2, 1)\}$
9. $\{(5, 4), (4, 3), (3, 3), (2, 4)\}$ NO
10. $\{(5, 4), (4, 3), (3, 2), (2, 1)\}$
11. $\{(1, 2), (1, 4), (-3, 2), (-3, 4)\}$
12. $\{(0, 5), (-4, 5), (-4, 2), (0, 2)\}$

In Problems 13–30, determine if the function is one-to-one.

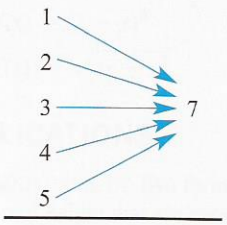
13. Domain yes Range



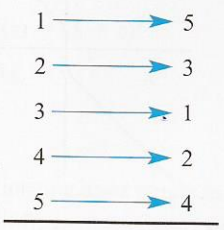
14. Domain NO Range



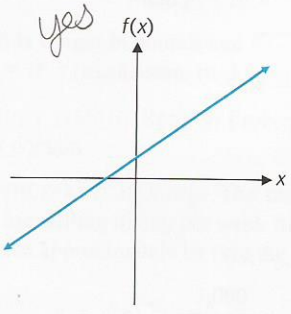
15. Domain NO Range



16. Domain yes Range



17. yes



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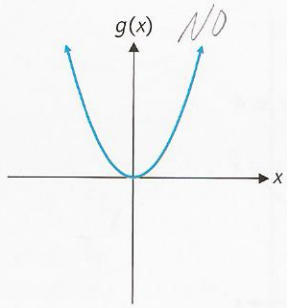

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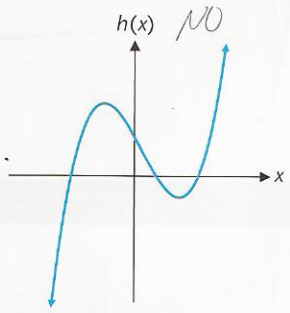

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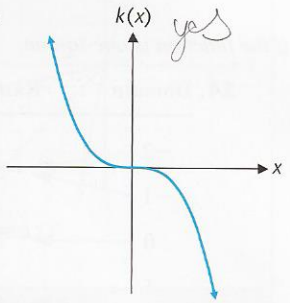

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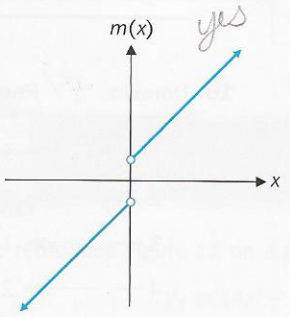

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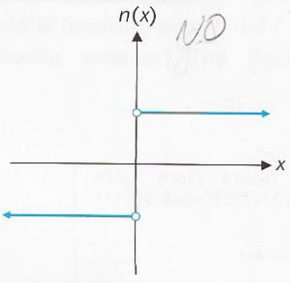

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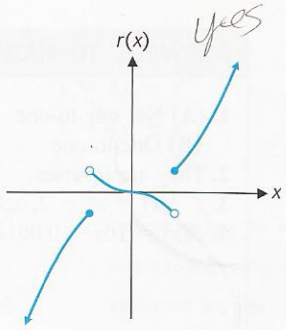

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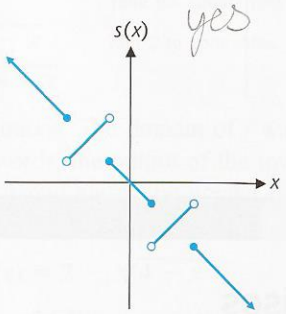

25. $F(x) = \frac{1}{2}x + 2$

26. $G(x) = -\frac{1}{3}x + 1$

27. $H(x) = 4 - x^2$

28. $K(x) = \sqrt{4 - x}$

29. $M(x) = \sqrt{x + 1}$

30. $N(x) = x^2 - 1$

In Problems 31–40, determine if g is the inverse of f.

31. $f(x) = 3x + 5; \quad g(x) = \frac{1}{3}x - \frac{5}{3}$

32. $f(x) = 2x - 4; \quad g(x) = \frac{1}{2}x - 2$

33. $f(x) = 2 - (x + 1)^3; \quad g(x) = \sqrt[3]{3 - x} - 1$

34. $f(x) = (x - 3)^3 + 4; \quad g(x) = \sqrt[3]{x - 4} + 3$

35. $f(x) = \dfrac{2x - 3}{x + 4}; \quad g(x) = \dfrac{3 + 4x}{2 - x}$

36. $f(x) = \dfrac{x + 1}{2x - 3}; \quad g(x) = \dfrac{3x + 1}{2x + 1}$

37. $f(x) = 4 + x^2, x \geq 0; \quad g(x) = \sqrt{x - 4}$

38. $f(x) = \sqrt{x + 2}; \quad g(x) = x^2 - 2, x \geq 0$

39. $f(x) = 1 - x^2, x \geq 0; \quad g(x) = -\sqrt{1 - x}$

40. $f(x) = -\sqrt{x - 2}; \quad g(x) = x^2 + 2, x \leq 0$

In Problems 41–44, find the domain and range of f, sketch the graph of f^{-1}, and find the domain and range of f^{-1}.

41.