TABLE 5-8 HYPOTHETICAL COST-OUTPUT DATA

Y($)	193	226	240	244	257	260	274	297	350	420	Total cost
X	1	2	3	4	5	6	7	8	9	10	Output

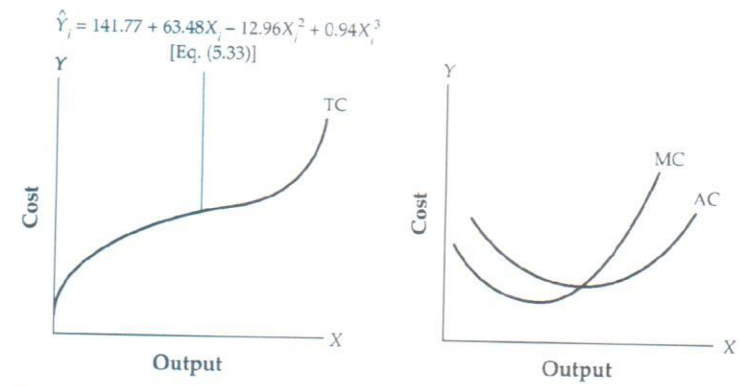

FIGURE 5-8 Cost-output relationship

If cost curves are to have the U-shaped average and marginal cost curves shown in price theory texts, then the theory suggests that the coefficients in model (5.32) should have these a priori values:[24]

1. B_1, B_2, and B_4, each is greater than zero.
2. $B_3 < 0$.
3. $B_3^2 < 3B_2B_4$.

The regression results given in regression (5.33) clearly are in conformity with these expectations.

As a concrete example of polynomial regression models, consider the following example.

Example 5.9. Cigarette Smoking and Lung Cancer

Table 5-9, on the textbook's Web site, gives data on cigarette smoking and various types of cancer for 43 states and Washington, D.C., for 1960.

[24] For the economics of this, see Alpha C. Chiang, *Fundamental Methods of Mathematical Economics*, 3rd ed., McGraw-Hill, New York, 1984, pp. 205–252. The rationale for these restrictions is that to make economic sense the total cost curve must be upward-sloping (the larger the output is, the higher the total cost will be) and the marginal cost of production must be positive.

It should be added that if there is more than one X variable, we can convert each variable into the standardized form. To show this, let us return to the Cobb-Douglas production function data given for real GDP, employment, and real fixed capital for Mexico, 1955–1974, in Table 5-2. The results of fitting the logarithmic function are given in Eq. (5.11). The results of regressing the standardized logs of GDP on standardized employment and standardized fixed capital, using EViews, are as follows:

Dependent Variable: SLGDP
Method: Least Squares
Sample: 1955 1974
Included observations: 20

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
SLE	0.167964	0.089220	1.882590	0.0760
SLK	0.831995	0.089220	9.325223	0.0000
R-squared	0.995080	Mean dependent var		6.29E-06
Adjusted R-squared	0.994807	S.D. dependent var		0.999999
S.E. of regression	0.072063	Sum squared resid		0.093475

where SLGDP = standardized log of GDP
SLE = standardized log of employment
SLK = standardized log of capital

The interpretation of the regression coefficients is as follows: Holding capital constant, a standard deviation increase in employment increases the GDP, on average, by ≈0.17 standard deviation units. Likewise, holding employment constant, a one standard deviation increase in capital, on average, increases GDP by ≈0.83 standard deviation units. (Note that all variables are in the logarithmic form.) Relatively speaking, capital has more impact on GDP than employment. Here you will see the advantage of using standardized variables, for standardization puts all variables on equal footing because all standardized variables have zero means and unit variances.

Incidentally, we have not introduced the intercept term in the regression results. (Why?) If you include intercept in the model, its value will be almost zero.

5.11 SUMMARY OF FUNCTIONAL FORMS

In this chapter we discussed several regression models that, although linear in the parameters, were not necessarily linear in the variables. For each model, we noted its special features and also the circumstances in which it might be

TABLE 5-11 SUMMARY OF FUNCTIONAL FORMS

Model	Form	Slope $= \frac{dY}{dX}$	Elasticity $= \frac{dY}{dX} \cdot \frac{X}{Y}$
Linear	$Y = B_1 + B_2X$	B_2	$B_2\left(\frac{X}{Y}\right)^*$
Log-linear	$\ln Y = B_1 + B_2 \ln X$	$B_2\left(\frac{Y}{X}\right)$	B_2
Log-lin	$\ln Y = B_1 + B_2X$	$B_2(Y)$	$B_2\,(X)^*$
Lin-log	$Y = B_1 + B_2 \ln X$	$B_2\left(\frac{1}{X}\right)$	$B_2\left(\frac{1}{Y}\right)^*$
Reciprocal	$Y = B_1 + B_2\left(\frac{1}{X}\right)$	$-B_2\left(\frac{1}{X^2}\right)$	$-B_2\left(\frac{1}{XY}\right)^*$
Log-inverse	$\ln(Y) = B_1 - B_2\left(\frac{1}{X}\right)$	$B_2\left(\frac{Y}{X^2}\right)$	$B_2\left(\frac{1}{X}\right)$

Note: * Indicates that the elasticity coefficient is variable, depending on the value taken by X or Y or both. When no X and Y are specified, in practice, these elasticities are often measured at the mean values X and Y.

appropriate. In Table 5-11 we summarize the various functional forms that we discussed in terms of a few salient features, such as the slope coefficients and the elasticity coefficients. Although for double-log models the slope and elasticity coefficients are the same, this is not the case for other models. But even for these models, we can compute elasticities from the basic definition given in Eq. (5.7).

As Table 5-11 shows, for the linear-in-variable (LIV) models, the slope coefficient is constant but the elasticity coefficient is variable, whereas for the log-log, or log-linear, model, the elasticity coefficient is constant but the slope coefficient is variable. For other models shown in Table 5-11, both the slope and elasticity coefficients are variable.

5.12 SUMMARY

In this chapter we considered models that are linear in parameters, or that can be rendered as such with suitable transformation, but that are not necessarily linear in variables. There are a variety of such models, each having special applications. We considered five major types of nonlinear-in-variable but linear-in-parameter models, namely:

1. The log-linear model, in which both the dependent variable and the explanatory variable are in logarithmic form.
2. The log-lin or growth model, in which the dependent variable is logarithmic but the independent variable is linear.
3. The lin-log model, in which the dependent variable is linear but the independent variable is logarithmic.
4. The reciprocal model, in which the dependent variable is linear but the independent variable is not.[30]

[30] The dependent variable can also be reciprocal and the independent variable linear, as in Problem 5.15. See also Problem 5.20.