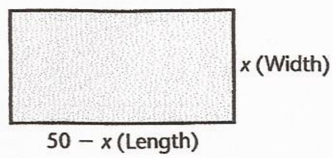
**SOLUTIONS** (A) Draw a figure and label the sides.




Perimeter = 100 meters of fencing.
Half the perimeter = 50.
If $x$ = Width, then $50 - x$ = Length.

$$A = (\text{Width})(\text{Length}) = x(50 - x)$$

(B) To have a pen, $x$ must be positive, but $x$ must also be less than 50 (or the length will not exist). So the domain is

$\{x \mid 0 < x < 50\}$ Inequality notation

$(0, 50)$ Interval notation

**MATCHED PROBLEM 7** Rework Example 7 with the added assumption that a large barn is to be used as one of the sides that run the length of the pen.

**ANSWERS TO MATCHED PROBLEMS**

1. (A) $S$ does not define a function.
   (B) $T$ defines a function with domain $\{-2, -1, 0, 1, 2\}$ and range $\{0, 1, 2\}$.
2. (A) Does not define a function
   (B) Defines a function
3. $\{x \mid x \geq -5\}$ or $[-5, \infty)$
4. (A) $F(4) = -2$, $F(4 + h) = -\dfrac{4}{2 + h}$, $F(4) + F(h) = \dfrac{2h}{2 - h}$
   (B) $G(3) = 22$, $G(h) = h^2 + 5h - 2$, $G(3 + h) = 22 + 11h + h^2$
   (C) $K(4) = 6$, $K(9x) = \dfrac{2}{1 - \sqrt{x}}$, $9K(x) = \dfrac{54}{3 - \sqrt{x}}$
5. (A) $\{x \mid x \neq 2\}$ or $(-\infty, 2) \cup (2, \infty)$ (B) $R$ or $(-\infty, \infty)$
   (C) $\{x \mid x \geq 0, x \neq 9\}$ or $[0, 9) \cup (9, \infty)$
6. (A) $x^2 + 2xh + h^2 + 3x + 3h + 7$
   (B) $2xh + h^2 + 3h$ (C) $2x + h + 3$
7. (A) $A = x(100 - 2x)$
   (B) Domain: $\{x \mid 0 < x < 50\}$ or $(0, 50)$

## 3-1 Exercises

1. Is every correspondence between two sets a function? Why or why not?

2. Describe four different ways that we represented functions in this section.

3. Explain what the domain and range of a function are. Don't just think about functions defined by equations.

4. What do the terms "input" and "output" refer to when working with functions?

5. If $2(x + h) = 2x + 2h$, why doesn't $f(x + h) = f(x) + f(h)$, where $f$ is a function?

6. Describe how to determine if an equation defines a function by looking at the graph of the equation.

*Indicate whether each table in Problems 7–12 defines a function.*

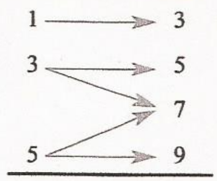
7.

| Domain | Range |
|---|---|
| −1 → 3 | 1 |
| 0 → 2 | 2 |
| 1 → 1 | 3 |

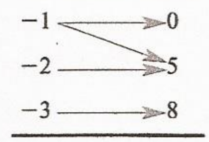
8.

| Domain | Range |
|---|---|
| 2 → 1 | 1 |
| 4 → 3 | 3 |
| 6 → 5 | 5 |

**9.** Domain Range




**10.** Domain Range




**11.**

| Domain | Range |
|---|---|
| English | A |
| Math | B |
| Sociology | A |
| Chemistry | B |

**12.**

| Domain | Range |
|---|---|
| Auburn | Tigers |
| Memphis | Tigers |
| Georgia | Bulldogs |
| Fresno State | Bulldogs |

*Indicate whether each set in Problems 13–18 defines a function. Find the domain and range of each function.*

**13.** $\{(2, 4), (3, 6), (4, 8), (5, 10)\}$

**14.** $\{(-1, 4), (0, 3), (1, 2), (2, 1)\}$

**15.** $\{(10, -10), (5, -5), (0, 0), (5, 5), (10, 10)\}$

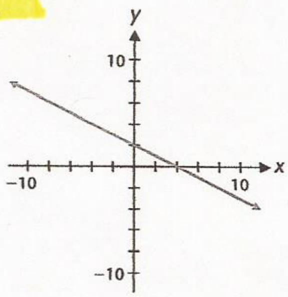
**16.** $\{(0, 1), (1, 1), (2, 1), (3, 2), (4, 2), (5, 2)\}$

**17.** {(Ohio, Obama), (Alabama, McCain), (West Virginia, McCain), (California, Obama)}

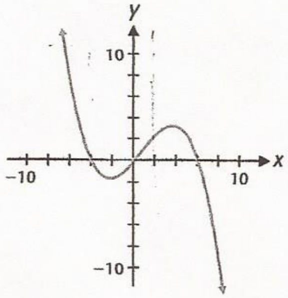
**18.** {(Democrat, Obama), (Republican, Bush), (Democrat, Clinton), (Republican, Reagan)}

*Indicate whether each graph in Problems 19–24 is the graph of a function.*

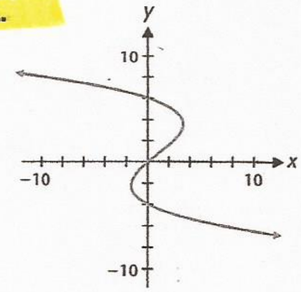
**19.**



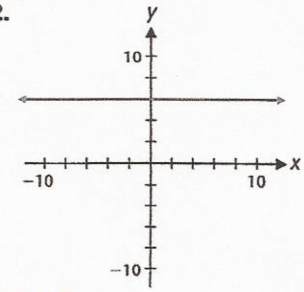

**20.**



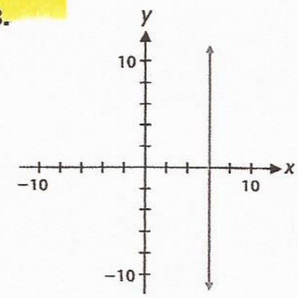

**21.**



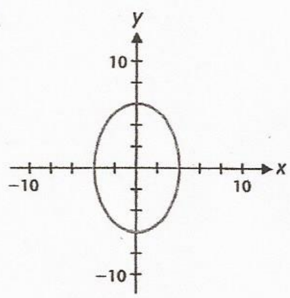

**22.**




**23.**




**24.**




*In Problems 25 and 26, which of the indicated correspondences define functions? Explain.*

**25.** Let $F$ be the set of all faculty teaching Math 125 at Enormous State University, and let $S$ be the set of all students taking that course.

(A) Students from set $S$ correspond to their Math 125 instructors.

(B) Faculty from set $F$ correspond to the students in their Math 125 class.

**26.** Let $A$ be the set of floor advisors in Hoffmann Hall, a dorm at Enormous State. Assume that each floor has one floor advisor. Let $R$ be the set of residents of that dorm.

(A) Floor advisors from set $A$ correspond to the residents on their floor.

(B) Students from set $R$ correspond to their floor advisor.

27. Let $f(x) = 3x - 5$. Find
(A) $f(3)$ (B) $f(h)$
(C) $f(3) + f(h)$ (D) $f(3 + h)$

28. Let $g(y) = 7 - 2y$. Find
(A) $g(4)$ (B) $g(h)$
(C) $g(4) + g(h)$ (D) $g(4 + h)$

29. Let $F(w) = -w^2 + 2w$. Find
(A) $F(4)$ (B) $F(-4)$
(C) $F(4 + a)$ (D) $F(2 - a)$

30. Let $G(t) = 5t - t^2$. Find
(A) $G(8)$ (B) $G(-8)$
(C) $G(-1 + h)$ (D) $G(6 - t)$

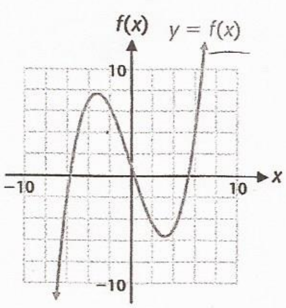
31. Let $f(t) = 2 - 3t^2$. Find
(A) $f(-2)$ (B) $f(-t)$
(C) $-f(t)$ (D) $-f(-t)$

32. Let $k(z) = 40 + 20z^2$. Find
(A) $k(-2)$ (B) $k(-z)$
(C) $-k(z)$ (D) $-k(-z)$

33. Let $F(u) = u^2 - u - 1$. Find
(A) $F(10)$ (B) $F(u^2)$
(C) $F(5u)$ (D) $5F(u)$

34. Let $G(u) = 4 - 3u - u^2$. Find
(A) $G(-8)$ (B) $G(u^2)$
(C) $G(-2u)$ (D) $-2G(u)$

*Problems 35–36 refer to the following graph of a function f.*




35. (A) Find $f(-2)$ to the nearest integer.
(B) Find all values of $x$, to the nearest integer, so that $f(x) = -4$.

36. (A) Find $f(4)$ to the nearest integer.
(B) Find all values of $x$, to the nearest integer, so that $f(x) = 0$.

*Determine which of the equations in Problems 37–46 define a function with independent variable x. For those that do, find the domain. For those that do not, find a value of x to which there corresponds more than one value of y.*

37. $y - x^2 = 1$ 38. $y^2 - x = 1$

39. $2x^3 + y^2 = 4$ 40. $3x^2 + y^3 = 8$

41. $x^3 - y = 2$ 42. $x^3 + |y| = 6$

43. $2x + |y| = 7$ 44. $y - 2|x|^3 = 3$

45. $3y + 2|x| = 12$ 46. $x|y| = x + 1$

*In Problems 47–62, find the domain of the indicated function. Express answers in both interval notation and inequality notation.*

47. $f(x) = 4 - 9x + 3x^2$ 48. $g(t) = 1 + 7t - 2t^2$

49. $L(u) = \sqrt{3u^2 + 4}$ 50. $M(w) = \dfrac{w - 5}{\sqrt{3 + 2w^2}}$

51. $h(z) = \dfrac{2}{4 - z}$ 52. $k(z) = \dfrac{z}{z - 3}$

53. $g(t) = \sqrt{t - 4}$ 54. $h(t) = \sqrt{6 - t}$

55. $k(w) = \sqrt{7 + 3w}$ 56. $j(w) = \sqrt{9 + 4w}$

57. $H(u) = \dfrac{u}{u^2 + 4}$ 58. $G(u) = \dfrac{u}{u^2 - 4}$

59. $M(x) = \dfrac{\sqrt{x + 4}}{x - 1}$ 60. $N(x) = \dfrac{\sqrt{x - 3}}{x + 2}$

61. $s(t) = \dfrac{1}{3 - \sqrt{t}}$ 62. $r(t) = \dfrac{1}{\sqrt{t} - 4}$

*The verbal statement "function f multiplies the square of the domain element by 3 and then subtracts 7 from the result" and the algebraic statement "$f(x) = 3x^2 - 7$" define the same function. In Problems 63–66, translate each verbal definition of a function into an algebraic definition.*

63. Function $g$ subtracts 5 from twice the cube of the domain element.

64. Function $f$ multiplies the square of the domain element by 10 then adds 1,000 to the result.

65. Function $F$ multiplies the square root of the domain element by 8, then subtracts the product of 4 and the sum of the domain element and two.

66. Function $G$ divides the sum of the domain element and 7 by the cube root of the domain element.

*In Problems 67–70, translate each algebraic definition of the function into a verbal definition.*

67. $f(x) = 2x^2 + 5$ 68. $g(x) = -2x + 7$

69. $z(x) = \dfrac{4x + 5}{\sqrt{x}}$ 70. $M(t) = 5t - 2\sqrt{t}$

71. If $F(s) = 3s + 15$, find: $\dfrac{F(2 + h) - F(2)}{h}$

72. If $K(r) = 7 - 4r$, find: $\dfrac{K(1 + h) - K(1)}{h}$

73. If $g(x) = 2 - x^2$, find: $\dfrac{g(3 + h) - g(3)}{h}$

74. If $P(m) = 2m^2 + 3$, find: $\dfrac{P(2 + h) - P(2)}{h}$

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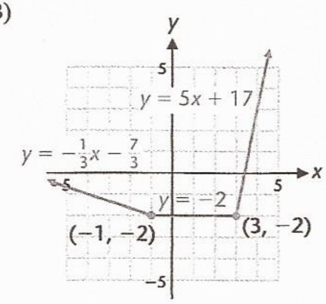
4. (A) $f(-4) = -1$; $f(-1) = -2$; $f(3) = -2$; $f(4) = 3$

(B)




(C) Domain: $(-\infty, \infty)$; range: $[-2, \infty)$; increasing: $[3, \infty)$; decreasing: $(-\infty, -1]$; constant: $(-1, 3)$

5. $T(x) = \begin{cases} 0.05x & x \le 6{,}100 \\ 0.07x - 122 & 6{,}100 < x \le 15{,}200 \\ 0.09x - 426 & x > 15{,}200 \end{cases}$

$T(4{,}000) = \$200$; $T(10{,}000) = \$578$; $T(18{,}000) = \$1{,}194$

$T(4{,}000) = \$200$; $T(10{,}000) = \$594$; $T(18{,}000) = \$1{,}248$

6. (A) 6 (B) 2 (C) 3 (D) −4 (E) −3; $f$ rounds decimal fractions to the nearest integer.

## 3-2 Exercises

1. Describe in your own words what the graph of a function is.

2. Explain how to find the domain and range of a function from its graph.

3. How many $y$ intercepts can a function have? What about $x$ intercepts? Explain.

4. True or false: On any interval in its domain, every function is either increasing or decreasing. Explain.

5. Explain in your own words what it means to say that a function is increasing on an interval.

6. Explain in your own words what it means to say that a function is decreasing on an interval.

7. What does it mean for a function to be defined piecewise?

8. Explain how the output of the greatest integer function is calculated for any real number input.

*Problems 9–20 refer to functions f, g, h, k, p, and q given by the following graphs.*

9. For the function $f$, find:
   (A) Domain (B) Range
   (C) $x$ intercepts (D) $y$ intercept
   (E) Intervals over which $f$ is increasing
   (F) Intervals over which $f$ is decreasing
   (G) Intervals over which $f$ is constant
   (H) Any points of discontinuity

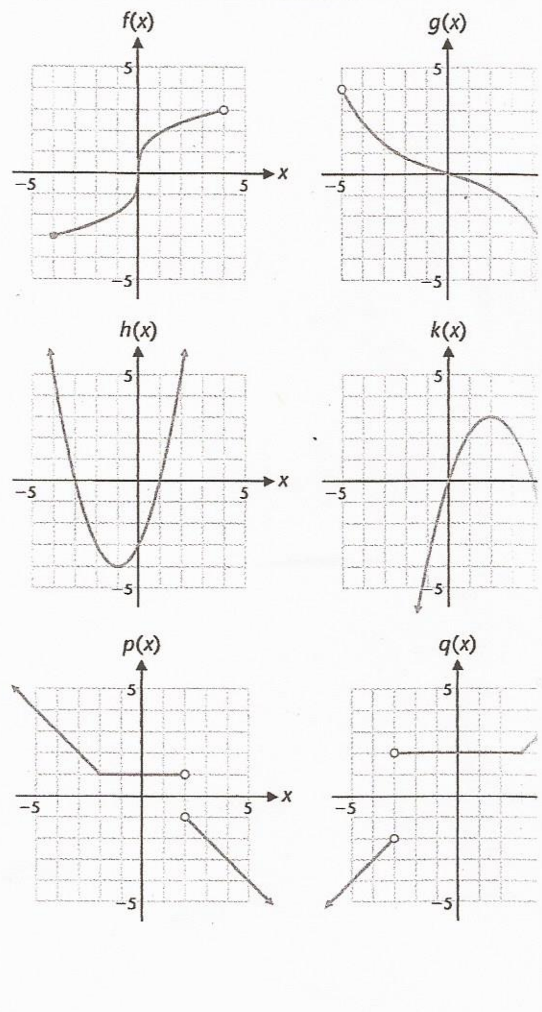
10. Repeat Problem 9 for the function $g$.

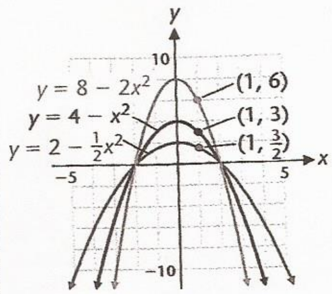
11. Repeat Problem 9 for the function $h$.

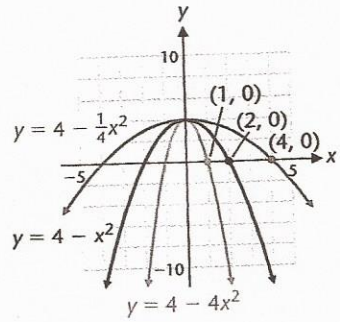
12. Repeat Problem 9 for the function $k$.

13. Repeat Problem 9 for the function $p$.

14. Repeat Problem 9 for the function $q$.






5. The graph of function $h$ is a reflection through the $x$ axis and a horizontal translation of three units to the left of the graph of $y = x^3$. An equation for $h$ is $h(x) = -(x + 3)^3$.
6. (A) Odd (B) Even (C) Neither

## 3-3 Exercises

1. Explain why the graph of $y = f(x) + k$ is the same as the graph of $y = f(x)$ moved upward $k$ units when $k$ is positive.

2. Explain why the graph of $y = Af(x)$ is a vertical stretch of the graph of $y = f(x)$ when $A > 1$, and a vertical shrink when $A < 1$.

3. Explain why the graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ about the $x$ axis, and why the graph of $y = f(-x)$ is a reflection about the $y$ axis.

4. Is every function either even or odd? Explain your answer.

*In Problems 5–10, without looking back in the text, indicate the domain and range of each of the following functions. (Making rough sketches on scratch paper may help.)*

5. $h(x) = -\sqrt{x}$ 6. $m(x) = -\sqrt[3]{x}$

7. $g(x) = -2x^2$ 8. $f(x) = -0.5|x|$

9. $F(x) = -0.5x^3$ 10. $G(x) = 4x^3$

*Problems 11–26 refer to the functions f and g given by the graphs below. The domain of each function is [−2, 2], the range of f is [−2, 2], and the range of g is [−1, 1]. Use the graph of f or g, as required, to graph the function h and state the domain and range of h.*

11. $h(x) = f(x) + 2$

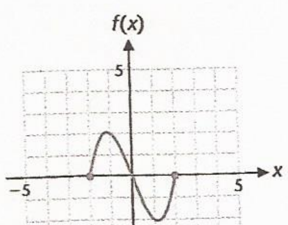
12. $h(x) = g(x) - 1$

13. $h(x) = g(x) + 2$

14. $h(x) = f(x) - 1$

15. $h(x) = f(x - 2)$

16. $h(x) = g(x - 1)$




18. $h(x) = f(x - 1)$

19. $h(x) = -f(x)$

20. $h(x) = -g(x)$

21. $h(x) = 2g(x)$

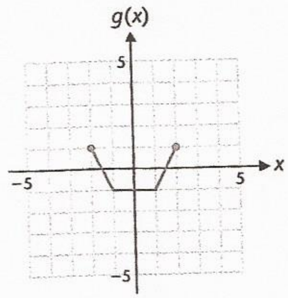
22. $h(x) = \frac{1}{2}f(x)$

23. $h(x) = g(2x)$

24. $h(x) = f\left(\frac{1}{2}x\right)$

25. $h(x) = f(-x)$

26. $h(x) = -g(-x)$




*Indicate whether each function in Problems 27–36 is even, odd, or neither.*

27. $g(x) = x^3 + x$ 28. $f(x) = x^5 - x$

29. $m(x) = x^4 + 3x^2$ 30. $h(x) = x^4 - x^2$

31. $F(x) = x^5 + 1$ 32. $f(x) = x^5 - 3$

33. $G(x) = x^4 + 2$ 34. $P(x) = x^4 - 4$

35. $q(x) = x^2 + x - 3$ 36. $n(x) = 2x - 3$

*In Problems 37–44, the graph of the function g is formed by applying the indicated sequence of transformations to the given function f. Find an equation for the function g. Check your work by graphing f and g in a standard viewing window.*

37. The graph of $f(x) = \sqrt[3]{x}$ is shifted four units to the left and five units down.

**39.** The graph of $f(x) = \sqrt{x}$ is shifted six units up, reflected in the $x$ axis, and vertically shrunk by a factor of 0.5.

**40.** The graph of $f(x) = \sqrt{x}$ is shifted two units down, reflected in the $x$ axis, and vertically stretched by a factor of 4.

**41.** The graph of $f(x) = x^2$ is reflected in the $x$ axis, vertically stretched by a factor of 2, shifted four units to the left, and shifted two units down.

**42.** The graph of $f(x) = |x|$ is reflected in the $x$ axis, vertically shrunk by a factor of 0.5, shifted three units to the right, and shifted four units up.

**43.** The graph of $f(x) = \sqrt{x}$ is horizontally stretched by a factor of 0.5, reflected in the $y$ axis, and shifted two units to the left.

**44.** The graph of $f(x) = \sqrt[3]{x}$ is horizontally shrunk by a factor of 2, shifted three units up, and reflected in the $y$ axis.

*Use graph transformations to sketch the graph of each function in Problems 45–62.*

**45.** $f(x) = 4x^2$

**46.** $g(x) = \frac{1}{3}\sqrt{x}$

**47.** $h(x) = |x + 2|$

**48.** $k(x) = |x - 4|$

**49.** $m(x) = -|4x - 8|$

**50.** $n(x) = -|9 + 3x|$

**51.** $p(x) = 3 - \sqrt{x}$

**52.** $q(x) = -2 + \sqrt{x + 3}$

**53.** $r(x) = 3\sqrt{x - 1} + 2$

**54.** $s(x) = \sqrt{x - 1} + 2$

**55.** $h(x) = x^2 + 3$

**56.** $h(x) = 4 - x^2$ $-x^2+4$

**57.** $k(x) = 2x^3 + 1$

**58.** $h(x) = 3x^3 - 1$

**59.** $n(x) = (x + 2)^2$

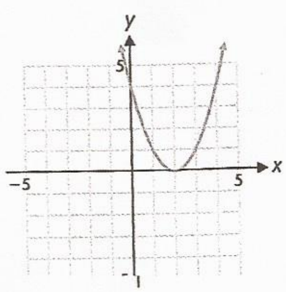
**60.** $m(x) = (x - 4)^2$

**61.** $q(x) = 4 - \frac{1}{2}(x - 2)^2$

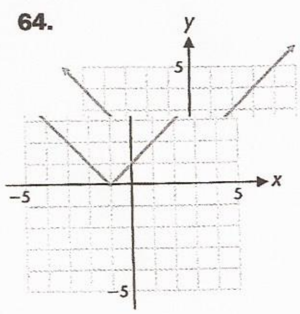
**62.** $p(x) = 5 - \frac{2}{3}(x + 3)^2$

*Each graph in Problems 63–78 is a transformation of one of the six basic functions in Figure 1. Find an equation for the given graph.*

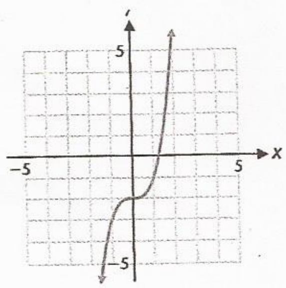
**63.**



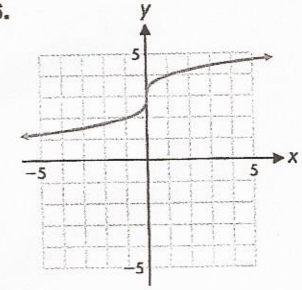

**64.**



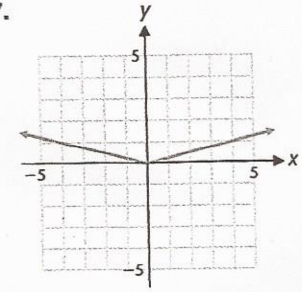

**65.**



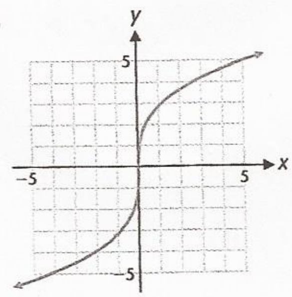

**66.**



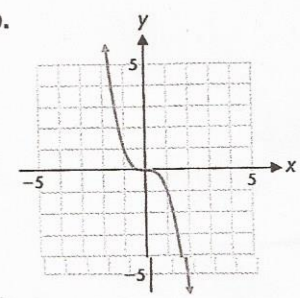

**67.**




**68.**




**69.**




**70.**