

Given the following design matrix, the $\mathbf{y}$ vector used in a least squares estimation is arranged as $\mathbf{y} = [12 \ 10 \ 6 \ 15 \ 17 \ 11 \ 3 \ 22 \ 19 \ 8 \ 9 \ 15]^T$.

	Factors		y		
Run	A	B	I	II	III
1	+	+	12	17	19
2	+	-	10	11	8
3	-	+	6	3	9
4	-	-	15	22	15

We want to estimate all the main effects as well as the two factor interaction. Write down the appropriate $\mathbf{X}$ matrix [Hint: You need to find $\mathbf{X}$ in the regression model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$. DO NOT SOLVE for $\boldsymbol{\beta}$, just write the matrix $\mathbf{X}$].

Consider the viscosity example discussed in class. Recall that we have two factors reactant concentration (factor A, or x_1) and feed rate (factor B, or x_2), and the response is viscosity. Suppose we want to try a model adequacy test where

$$\text{Model I: } y = \beta_0 + \beta_1 x_1 + \epsilon$$

$$\text{Model II: } y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \epsilon$$

5.1 Write the null (H_0) and alternative (H_1) hypotheses for this test.

5.2 If number of experiments $n = 10$, what are the degrees of freedom for the associated F test of model adequacy?