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0 Step 1 of 5 Done

We perform this hypothesis test by following the six-step hypothesis testing procedure outlined in Section 9.3.

Step 1 Develop the null and alternative hypotheses.

If we define hotel rooms in Atlanta as population 1 and hotel rooms in Houston as population 2, then

μ_1 = mean price for the population of hotel rooms in Atlanta

μ_2 = mean price for the population of hotel rooms in Houston

To determine whether the mean price of a hotel room in Atlanta is lower than one in Houston, a lower tail test with $H_a: \mu_1 - \mu_2 < 0$ is appropriate. The null and alternative hypotheses for the hotel room hypothesis test are as follows:

$$H_0: \mu_1 - \mu_2 \geq 0$$

$$H_a: \mu_1 - \mu_2 < 0$$

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Done

Step 2 Specify the level of significance.

We use $\alpha = .05$ as the level of significance for this test.

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Step 3 Select samples and compute the value of the test statistic.

Using the file Hotel, a random sample of $n_1 = 35$ hotel rooms in Atlanta has a sample mean price of $\bar{x}_1 = \$91.7143$, and a random sample of $n_2 = 40$ hotel rooms in Houston has a sample mean price of $\bar{x}_2 = \$101.125$. Based on historical data, the two population standard deviations can be assumed known with $\sigma_1 = \$20$ and $\sigma_2 = \$25$. Because the two sample sizes are large, by the central limit theorem, the sampling distribution of $\bar{x}_1 - \bar{x}_2$ can be approximated by a normal distribution. For the σ_1 and σ_2 known case, the test statistic for hypothesis tests about $\mu_1 - \mu_2$ is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

where

$\bar{x}_1$ = sample mean from population 1

$\bar{x}_2$ = sample mean from population 2

D_0 = hypothesized difference between μ_1 and μ_2

σ_1 = standard deviation of population 1

σ_2 = standard deviation of population 2

n_1 = sample size from population 1

n_2 = sample size from population 2

With $\bar{x}_1 = \$91.7143$, $\bar{x}_2 = \$101.125$, $D_0 = 0$, $\sigma_1 = \$20$, $\sigma_2 = \$25$, $n_1 = 35$, and $n_2 = 40$, the value of the test statistic is

$$\begin{aligned} z &= \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \\ &= \frac{(91.7143 - 101.125) - 0}{\sqrt{\frac{20^2}{35} + \frac{25^2}{40}}} \\ &= -1.81 \end{aligned}$$

Step 4 Compute the *p* -value.

For a lower tail test, the *p*-value is the probability of obtaining a value for the test statistic at least as small as that provided by the sample. Thus, for the hotel room hypothesis test, the *p*-value is the probability that z is less than or equal to -1.81. Using the standard normal probability table, we find

$$P(Z \le -1.81) = \boxed{.0351}$$

Step 5 Use the rejection rule to determine whether to reject H_0.

For the hotel room hypothesis test, the chosen level of significance is $\alpha = .05$. Thus, the rejection rule using the *p*-value approach is

Reject H_0 if $p\text{-value} \le .05$

Since $p\text{-value} = .0351 < .05$, we reject the null hypothesis $\mu_1 - \mu_2 \ge 0$ in favor of the alternative hypothesis $H_a: \mu_1 - \mu_2 < 0$ at the .05 level of significance.

Step 6 Interpret the statistical conclusion in the context of the application.

Based on the sample data, we can conclude that the mean price of a hotel room in Atlanta is significantly lower than one in Houston.