

CHAPTER TWO: BASIC IDEAS OF LINEAR REGRESSION: THE TWO-VARIABLE MODEL 45

- Efficient*
- d. In the linear regression model the explanatory variable is the cause and the dependent variable is the effect.
 - e. The conditional and unconditional mean of a random variable are the same thing.
 - f. In Eq. (2.2) the regression coefficients, the B 's, are random variables, whereas the b 's in Eq. (2.4) are the parameters.
 - g. In Eq. (2.1) the slope coefficient B_2 measures the slope of Y per unit change in X .
 - h. In practice, the two-variable regression model is useless because the behavior of a dependent variable can never be explained by a single explanatory variable.
 - i. The sum of the deviation of a random variable from its mean value is *always* equal to zero.
- 2.5. What is the relationship between
 a. B_1 and b_1 ; b. B_2 and b_2 ; and c. u_i and e_i ? Which of these entities can be observed and how?
- 2.6. Can you rewrite Eq. (2.22) to express X as a function of Y ? How would you interpret the converted equation?
- 2.7. The following table gives pairs of dependent and independent variables. In each case state whether you would expect the relationship between the two variables to be positive, negative, or uncertain. In other words, tell whether the slope coefficient will be positive, negative, or neither. Give a brief justification in each case.

Dependent variable	Independent variable
(a) GDP	Rate of interest
(b) Personal savings	Rate of interest
(c) Yield of crop	Rainfall
(d) U.S. defense expenditure	Soviet Union's defense expenditure
(e) Number of home runs hit by a star baseball player	Annual salary
(f) A president's popularity	Length of stay in office
(g) A student's first-year grade-point average	S.A.T. score
(h) A student's grade in econometrics	Grade in statistics
(i) Imports of Japanese cars	U.S. per capita income

PROBLEMS

- 2.8. State whether the following models are linear regression models:

- a. $Y_i = B_1 + B_2(1/X_i)$
- b. $Y_i = B_1 + B_2 \ln X_i + u_i$
- c. $\ln Y_i = B_1 + B_2 X_i + u_i$
- d. $\ln Y_i = B_1 + B_2 \ln X_i + u_i$
- e. $Y_i = B_1 + B_2 B_3 X_i + u_i$
- f. $Y_i = B_1 + B_2^3 X_i + u_i$

Note: \ln stands for the natural log, that is, log to the base e . (More on this in Chapter 4.)

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- 2.9. Table 2-8 gives data on weekly family consumption expenditure (Y) (in dollars) and weekly family income (X) (in dollars).

TABLE 2-8 HYPOTHETICAL DATA ON WEEKLY CONSUMPTION EXPENDITURE AND WEEKLY INCOME

Weekly income (\$)(X)	Weekly consumption expenditure (\$)(Y)
80	55, 60, 65, 70, 75
100	65, 70, 74, 80, 85, 88
120	79, 84, 90, 94, 98
140	80, 93, 95, 103, 108, 113, 115
160	102, 107, 110, 116, 118, 125
180	110, 115, 120, 130, 135, 140
200	120, 136, 140, 144, 145
220	135, 137, 140, 152, 157, 160, 162
240	137, 145, 155, 165, 175, 189
260	150, 152, 175, 178, 180, 185, 191

- For each income level, compute the mean consumption expenditure, $E(Y | X_i)$, that is, the conditional expected value.
 - Plot these data in a scattergram with income on the horizontal axis and consumption expenditure on the vertical axis.
 - Plot the conditional means derived in part (a) in the same scattergram created in part (b).
 - What can you say about the relationship between Y and X and between mean Y and X ?
 - Write down the PRF and the SRF for this example.
 - Is the PRF linear or nonlinear?
- 2.10. From the data given in the preceding problem, a random sample of Y was drawn against each X . The result was as follows:

Y	70	65	90	95	110	115	120	140	155	150
X	80	100	120	140	160	180	200	220	240	260

- Draw the scattergram with Y on the vertical axis and X on the horizontal axis.
 - What can you say about the relationship between Y and X ?
 - What is the SRF for this example? Show all your calculations in the manner of Table 2-4.
 - On the same diagram, show the SRF and PRF.
 - Are the PRF and SRF identical? Why or why not?
- 2.11. Suppose someone has presented the following regression results for your consideration:

$$\hat{Y}_t = 2.6911 - 0.4795X_t$$

where Y = coffee consumption in the United States (cups per person per day)
 X = retail price of coffee (\$ per pound)
 t = time period

- Is this a time series regression or a cross-sectional regression?
- Sketch the regression line.

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- c. What is the interpretation of the intercept in this example? Does it make economic sense?
- d. How would you interpret the slope coefficient?
- e. Is it possible to tell what the true PRF is in this example?
- f. The *price elasticity* of demand is defined as the percentage change in the quantity demanded for a percentage change in the price. Mathematically, it is expressed as

$$\text{Elasticity} = \text{Slope} \left(\frac{X}{Y} \right)$$

That is, elasticity is equal to the product of the slope and the ratio of X to Y , where X = the price and Y = the quantity. From the regression results presented earlier, can you tell what the price elasticity of demand for coffee is? If not, what additional information would you need to compute the price elasticity?

- 2.12. Table 2-9 gives data on the Consumer Price Index (CPI) for all items (1982–1984 = 100) and the Standard & Poor's (S&P) index of 500 common stock prices (base of index: 1941–1943 = 10).

TABLE 2-9 CONSUMER PRICE INDEX (CPI) AND S&P 500 INDEX (S&P), UNITED STATES, 1978–1989

Year	CPI	S&P
1978	65.2	96.02
1979	72.6	103.01
1980	82.4	118.78
1981	90.9	128.05
1982	96.5	119.71
1983	99.6	160.41
1984	103.9	160.46
1985	107.6	186.84
1986	109.6	236.34
1987	113.6	286.83
1988	118.3	265.79
1989	124.0	322.84

Source: *Economic Report of the President*, 1990, Table C-58, for CPI and Table C-93 for the S&P index.

- a. Plot the data on a scattergram with the S&P index on the vertical axis and CPI on the horizontal axis.
- b. What can you say about the relationship between the two indexes? What does economic theory have to say about this relationship?
- c. Consider the following regression model:

$$(\text{S\&P})_t = B_1 + B_2 \text{CPI}_t + u_t$$

Use the method of least squares to estimate this equation from the preceding data and interpret your results.

- d. Do the results obtained in part (c) make economic sense?
- e. Do you know why the S&P index dropped in 1988?