Instructions: The problems below are due by the end of class Wednesday!

$T_k(x) = \sum_{n=0}^{k} \frac{f^{(n)}(a)}{n!}(x-a)^n$

1. Let $f(x) = \dfrac{1}{1+x}$.
   (a) Determine the Taylor series, $T(x)$ for $f(x)$ centered at 1.
   (b) Using Geometric Series, determine $T(0)$ and show that it is equal to $f(0)$.
   (c) Determine the interval of convergence for the Taylor series.
   (d) Based on your previous answer, for what values of a, b can we conclude
   $$\int_a^b T(x)dx = \int_a^b \frac{1}{1+x}dx?$$
   (e) Write down $T_5(x)$
   (f) Use $T_5(x)$ to approximate $\displaystyle\int_1^2 \frac{1}{1+x}dx$
   (g) Determine the actual value for $\displaystyle\int_1^2 \frac{1}{1+x}dx$

2. Use known Taylor series to determine the sum of each series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n}{n!}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{-2^{n+1}}{3 \cdot n!}$ (c) $\displaystyle\sum_{n=0}^{\infty} (-1)^{n+1} \frac{\pi^{2n}}{(2n)!}$

3. Sketch the vector that is the sum of $\langle 2,0\rangle$ and $\langle 3,4\rangle$ and determine what the vector is.

4. Sketch the vector that is the difference $\langle 2,0\rangle - \langle 3,4\rangle$.

5. Determine the angle (in radians) between the vectos,

   (a) $\langle 1,2\rangle$ and $\langle 5,-2\rangle$
   (b) $c\langle a,0\rangle$ and $d\langle 0,b\rangle$ where $a,b,c,d \neq 0$
   (c) $\langle 2,2,-1\rangle$ and $\langle 5,-3,2\rangle$
   (d) $\langle 4,1,2\rangle$ and $\langle -2,1,1\rangle$

6. Compute the cross product of the listed vectors:

   (a) $\langle 1,2,0\rangle$ and $\langle 5,-2,0\rangle$
   (b) $c\langle a,0,0\rangle$ and $d\langle 0,b,0\rangle$ where $a,b,c,d \neq 0$
   (c) $\langle 2,2,-1\rangle$ and $\langle 5,-3,2\rangle$
   (d) $\langle 4,1,2\rangle$ and $\langle -2,1,1\rangle$

7. For problems 6a and 6d, show that the cross product is orthogonal to each vector in the cross product. (Th
   will be 4 angles to check.)

8. Determine the projection of the first vector onto the second.

   (a) $\langle 2,2,-1\rangle$ and $\langle 5,-3,2\rangle$
   (b) $\langle 4,1,2\rangle$ and $\langle -2,1,1\rangle$

9. Explain why if two vectors $\mathbf{v}, \mathbf{w}$ are orthogonal then $proj_{\mathbf{v}}(\mathbf{w}) = \mathbf{0}$.