

A material subjected to a stretching force F will tend to elongate a distance Δs . If the force is not excessive, the elongation will be proportional to the force in accordance with equation (1).

$$F = k \Delta s \quad (1)$$

k is a constant of proportionality known as the spring constant. The value of k depends upon the composition of the material and the size and shape of the body formed from it. If a suitable material is formed into a helical spring, equation (1) may be valid up to a relatively large elongation of the spring.

Sometimes it is convenient to define Δs as the increase in elongation that occurs when the stretching force is increased from F to $F + \Delta F$. In this case equations (2) and (3) apply.

$$\Delta F = k \Delta s \quad (2)$$

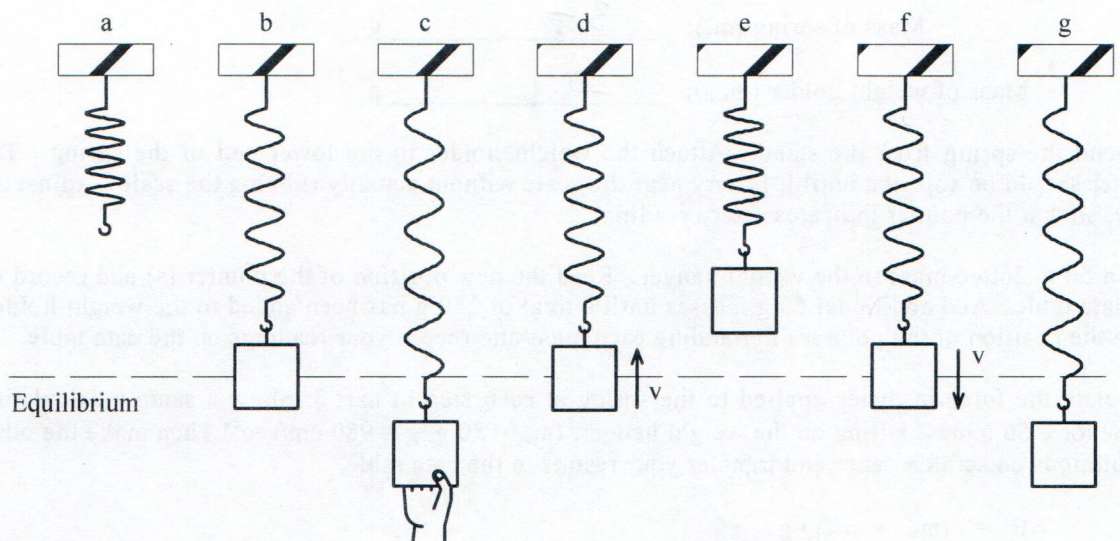
$$k = \frac{\Delta F}{\Delta s} \quad (3)$$

The value of k in equations (2) and (3) is the same as its value in equation (1), if the stretching force and the resulting elongation are not too great.

Suppose that a mass is suspended from a spring as in Figure 1b. The spring stretches due to the weight of the mass. There is an equilibrium position at which the spring exerts a force F on the mass that is equal and opposite to the gravitational force on the mass. Now suppose that the mass is pulled down an amount Δs from its equilibrium position (Figure 1c). The force in the spring increases from F to $F + \Delta F$, while the gravitational force remains unchanged. Now suppose that the mass is released. An unbalanced force of ΔF accelerates the mass in an upward direction. As the mass passes its equilibrium position (Figure 1d), Δs is zero and there is no net force acting on the mass. But the mass has a velocity and continues to move upward because of its inertia. When the mass is above its equilibrium position, the spring is not stretched as much and exerts a smaller upward force on the mass. But the gravitational force remains the same, so that there is a net downward force on the mass. The downward force eventually brings the mass to a momentary stop (Figure 1e) and then causes the mass to move downward. As the mass passes the equilibrium position (Figure 1f), the net force acting upon the mass is zero. But the mass has a downward velocity and continues to move downward because of inertia. Below the equilibrium position a net upward force acts on the mass and brings it to momentary stop (Figure 1g). The cycle of Figure 1c to 1g then repeats itself, and the mass continues to oscillate up and down. The amplitude (maximum displacement from equilibrium) of the oscillation gradually decreases as the energy associated with the oscillation is gradually converted to heat.

The mass and spring oscillate after an initial disturbance because (1) the spring provides a restoring force that tends to accelerate the mass toward the equilibrium position and (2) the mass provides inertia that keeps the mass moving through and beyond the equilibrium position. There are numerous other examples of mechanical oscillations made possible by a restoring force and inertia, e.g. a swinging pendulum or a vibrating guitar string. Rotational oscillation can occur when a rotatable body with a moment of inertia is subject to a restoring torque. In many of these examples (including that of the mass and spring), the restoring force is proportional to the displacement of the mass. In such cases the displacement of the vibrating mass is a sinusoidal function of time, and the mass is said to execute simple harmonic motion.

Oscillation of a Mass and Spring



The period of vibration of the mass and spring – that is, the time required for the mass to move from its maximum displacement in one direction, the maximum displacement in the other direction, and back again – may be predicted by the following relationship:

$$T_p = 2\pi \sqrt{\frac{M}{k}} \quad (4)$$

T_p is the predicted period in seconds, k is the spring constant in dynes per centimeter, and M is the mass in grams. Equation (5) gives the correct value of M for this experiment.

$$M = m_w + m_{wh} + \frac{1}{3} m_s \quad (5)$$

where: m_w = the mass of the slotted weights

m_{wh} = the mass of the weight holder

m_s = the mass of the spring

Apparatus

Hooke's law stand and scale

Helical spring

Weight holder

Slotted weights, 50 g (5)

Stopwatch

Triple-beam balance

Procedure, Data, and Calculations

1. Use a triple-beam balance and measure the mass of the spring and the mass of the weight holder. Record these values below and on the data table.

Mass of spring (m_s): 3.4 g

Mass of weight holder (m_{wh}): 26.1 g

2. Suspend the spring from the stand. Attach the weight holder to the lower end of the spring. The pointer should be adjusted until it is very near the scale without actually rubbing the scale. Adjust the scale, so that the pointer indicates a zero reading.
3. Add a 50 g slotted mass to the weight hanger. Read the new position of the pointer (s) and record on the data table. Add additional 50 g masses until a total of 250 g has been added to the weight holder. Read the position of the pointer after adding each mass and record your readings on the data table.
4. Calculate the force in dynes applied to the spring at each step in part 3. Show a sample calculation below for a 50 g mass sitting on the weight hanger. ($m_w = 50$ g, $g = 980$ cm/sec²) Then make the other calculations on scratch paper and transfer your results to the data table.

$$\begin{aligned}
 F &= (m_w + m_{wh}) g \\
 &= (50 + 26.1) \times 980 \quad \text{(Substitute)} \\
 &= \underline{74578} \text{ dynes} \quad \text{(Final answer)}
 \end{aligned}$$

5. Calculate the values of ΔF and Δs corresponding to the addition of each 50 g mass. Record the results of these calculations on the data table. ΔF is calculated by subtracting successive values of F on the data table. Δs is calculated from s in a similar manner. Include sample calculations below for the case where m_w changes from 50 g to 100 g.

$$\begin{aligned}
 \Delta F &= \underline{123578} \text{ dynes} - \underline{74578} \text{ dynes} \quad \text{(Substitute)} \\
 &= \underline{49000} \text{ dynes} \quad \text{(Final answer)} \\
 \Delta s &= \underline{5.4} \text{ cm} - \underline{2.6} \text{ cm} \quad \text{(Substitute)} \\
 &= \underline{2.8} \text{ cm} \quad \text{(Final answer)}
 \end{aligned}$$

6. Calculate the spring constant k below for the interval in which m_w changes from 50 g to 100 g.

$$\begin{aligned}
 k &= \frac{\Delta F}{\Delta s} = \frac{49000}{2.8} \quad \text{(Substitute)} \\
 &= \underline{17500} \text{ dynes/cm} \quad \text{(Final answer)}
 \end{aligned}$$

7. Calculate the values of k for the other intervals on scratch paper. However, omit this calculation for the interval in which m_w changes from 0 to 50 g. Some springs cannot contract as much as they would "like to" at small applied forces, because the turns of the helix may interfere with each other. Consequently, measurements of the spring constant may be misleading when the applied force is too small. Record all your values of k in the data table.
8. Calculate below the average value of k . Also record the result of the calculation in the data table.

$$\text{Average } k = \frac{\text{Sum of values of } k}{\text{Number of values added}}$$

$$= \frac{68793.2}{4}$$

(Substitute)

$$= 17198.3 \text{ dynes/cm}$$

(Final answer)

9. Calculate the predicted period of oscillation (T_p) when a 100 g mass is placed on the weight holder. ($m_w = 100$ g, $\pi = 3.142^*$, k from step 8)

$$M = m_w + m_{wh} + \frac{1}{3} m_s$$

$$= 100 + 26.1 + 33.34$$

(Substitute)

$$= 127.8 \text{ g}$$

(Answer)

$$T_p = 2\pi \sqrt{\frac{M}{k}} = (2 \times 3.142) \sqrt{\frac{127.8}{17198.3}}$$

(Substitute)

$$= 0.542 \text{ s}$$

(Final answer)

10. Actually measure the period of oscillation with the 100 g mass sitting on the weight holder. Set the mass in vertical oscillation and use a stopwatch to measure the time required for 50 complete cycles of oscillation. (The mass has undergone one complete cycle of oscillation when it has moved from the lowest position, to the highest position, and back to the lowest position.) Then divide by 50 to get the measured value of the period of oscillation.

$$\text{Time for 50 cycles} = 25.9 \text{ s}$$

$$T_m = \frac{\text{Time for 50 cycles}}{50} =$$

(Substitute)

$$= 0.518 \text{ s}$$

(Final answer)

*Some calculators have a π button.

11. Compare your measured and predicted values of the period and calculate the percent difference.

$$\% \text{ Dif.} = \frac{|T_p - T_m|}{T_p + T_m} \times 200$$

$$= \frac{|0.542 - 0.518|}{0.542 + 0.518} \times 200$$

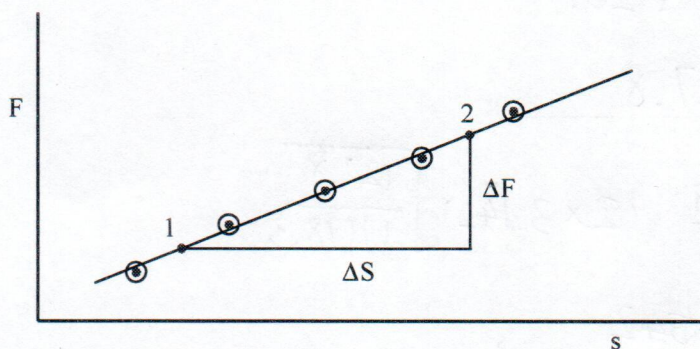
(Substitute)

$$= \underline{4.5} \%$$

(Final answer)

12. Repeat steps 9, 10, and 11 for two additional values of mass greater than 100 g on the hanger. Record all measured and calculated data on the data table.

13. Ask your laboratory instructor if he wants you to perform steps 13 and 14. If so specified by your instructor, plot a graph of F vs. s . Plot the data points on the graph paper provided. Do not assume that every data point falls on an intersection of grid lines. Estimate between grid lines for better accuracy in the graph. Draw small circles around the data points and draw the straight line that passes closest to most of the data points. The spring constant is the slope of this line. Draw a large triangle having a horizontal side and a vertical side and having a part of the straight line of your graph as the hypotenuse. Points 1 and 2 must not be any of your original data points.



13. Calculate the spring constant k , using readings from your graph at the triangle. Don't read the coordinates to the nearest grid line; estimate readings between the lines for greater accuracy.

$$k = \text{slope} = \frac{\Delta F}{\Delta s} = \frac{F_2 - F_1}{s_2 - s_1}$$

$$= \frac{49000 \text{ dynes}}{2.8 \text{ cm}} = \frac{123578 - 74578}{5.4 - 2.6}$$

(Substitute)

$$= \underline{17500} \text{ dynes/cm}$$

(Final answer)

Data Table

Mass of spring: $m_s = 3.4$ g

Mass of weight holder: $m_{wh} = 26.1$ g

Determination of spring constant k:

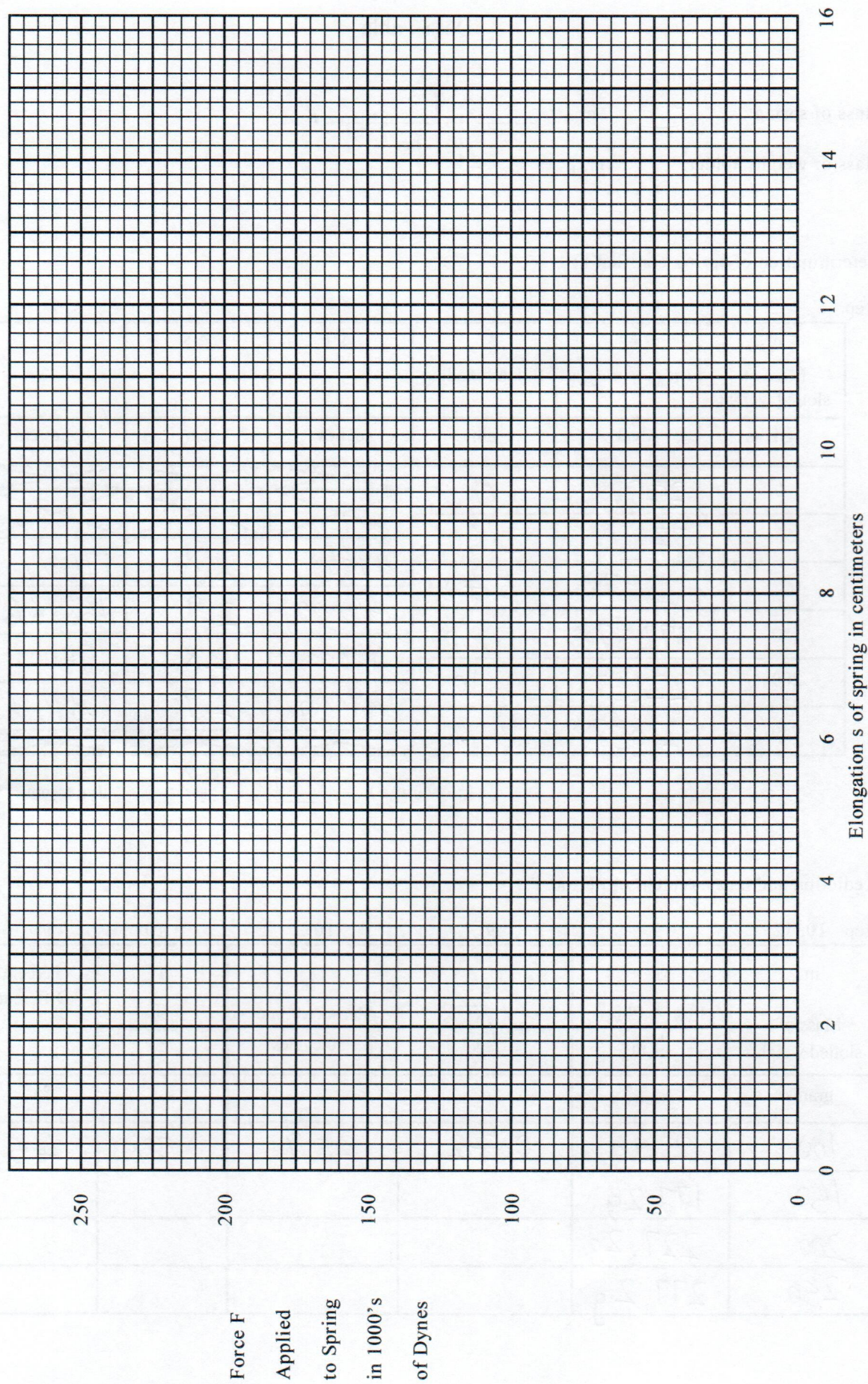
Step	3	4	3	5	5	6, 7
	m_w	F	s	ΔF	Δs	k
	Mass of slotted weights	$(m_w + m_{wh})g$	Pointer reading			
	grams	dynes	cm	dynes	cm	dynes/cm
	0	25578	0			
	50	74578	2.60			
	100	123578	5.40	49000	2.8	17500
	150	172578	8.30	49000	2.9	16896.6
	200	221578	11.10	49000	2.8	17500
	250	270578	14.00	49000	2.9	16896.6

Average k = 17198.3 dynes/cm

Prediction and measurement of period T:

Step	10, 12	9	9	10	10	11
	m_w	M	T_p	T_{50}	T_m	Percent Difference
	Mass of slotted weights	$m_w + m_{wh}$ $+ \frac{1}{3} m_s$	$2\pi\sqrt{\frac{M}{k}}$	Measured time for 50 cycles	$= \frac{T_{50}}{50}$	
	grams	grams	s	s	s	%
	100	127.8g	0.542	25.9	0.518	4.5
	150	177.2g				
	200	227.2g				
	250	277.2g				

Elasticity of Spring



Questions

1. Were your measured values of the period of oscillation approximately equal to the predicted values? What specific defects in the apparatus and/or procedure might have contributed to the percent difference?
2. Were your values of $k = \Delta F / \Delta s$ nearly equal for each interval? If you plotted a graph of F vs. s , were the data points nearly along a straight line? (If so, your spring could be said to obey Hooke's law.)
3. Suppose a 350 g mass were suspended from your spring and allowed to reach the equilibrium position. Then suppose that the mass were pulled down 3 cm further and released. What would be the resultant force on the mass at the moment of release?
4. What would be the instantaneous acceleration at the moment of release?