Ch 6 Normal Probability Distributions

Questions we will be able to answer by studying this chapter:

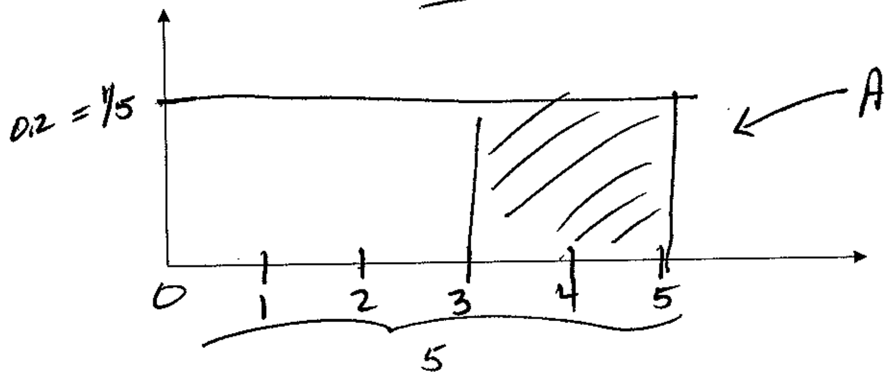
- What percentages of men and women can easily navigate in an area with the height clearance of 80 inches that is stipulated in the Americans with Disabilities Act?
- What percentages of men satisfy the flight crew requirements of having a height between 5 ft 2 in and 6 ft 1 in?
- What percentage of women are eligible for membership in Tall Clubs International because that are at least 5 ft 10 in tall?
- Current doorways are typically 6 ft 8 in tall, but if we were to redesign doorways to accommodate 99% of the population, what should the height be?

Requirements for Density Curve

1. The total area under the curve must equal 1.
2. Every point on the curve must have a vertical height that is 0 or greater (and less than or equal to 1).

Uniform Distribution

Ex: Waiting times for a certain sub-way train can be any value between 0 and 5 minutes, so it is possible to have a waiting time of 3.23547 minutes. Note also that all of the different possible waiting times are equally likely. $\Rightarrow$ uniform Distribution (same height)

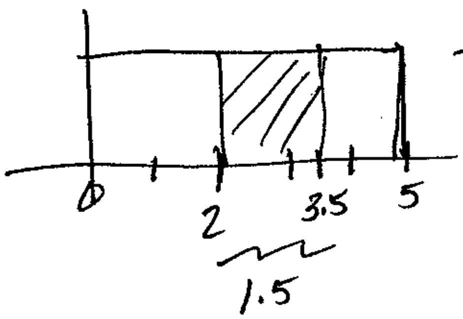


Area = 1
∴ height = 1/5 = .2

Find the probability that you will need to wait more than 3 minutes.

$$P = A = L \cdot W = 2 \cdot \left(\tfrac{1}{5}\right) = \tfrac{2}{5} = \boxed{.4}$$

P(~~weight~~ wait time between 2 min + 3.5 min)



$$P = A = (1.5)(.2) = \boxed{0.30}$$

Standard Normal Distribution

1) $\mu = 0$
2) $\sigma = 1$

$\sum P = 1 \Rightarrow$ total area under curve $= 1$

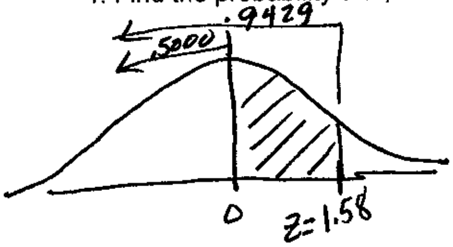
$0 \le P \le 1 \Rightarrow$ height between 0 + 1

Recall: z-score is # st. dev a value is away from the mean.

$\mu = 0$
$\sigma = 1$
$\therefore$ St. Normal

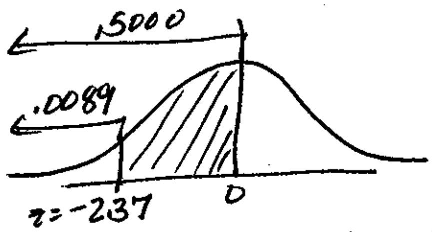
Example: A manufacture of themometers states that its thermometer read 0° C at the freezing point of water. Some read below, some above, but the mean reading is 0° C and the standard deviation is 1° C. Also assume that the distribution resembles a normal distribution.

1. Find the probability that, at the freezing point of water, the reading is between 0° and 1.58°.



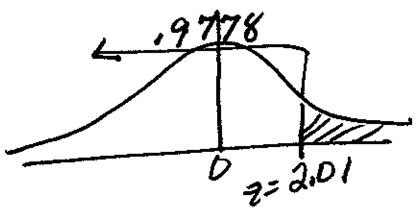
$P(0 < z < 1.58) = .9429 - .5000 = \boxed{.4429}$

2. Find the probability that the reading is between -2.37° and 0°.



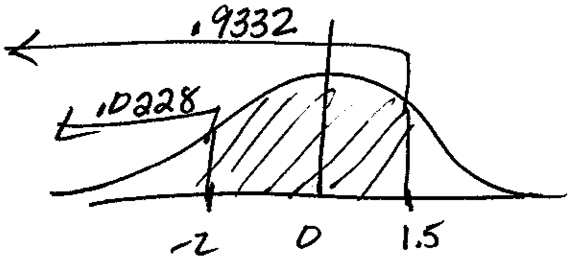
$P(-2.37 < z < 0) = .5000 - .0089 = .4911$

3. Find the probability that the reading is more than 2.01°.



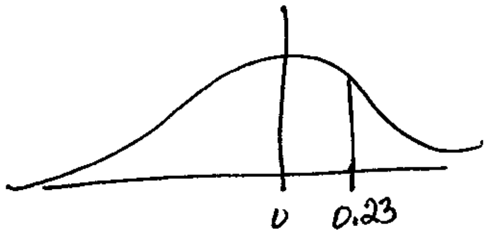
$P(z > 2.01) = 1 - .9778 = \boxed{.0222}$

4. Find the probability that the reading is between -2° and 1.5°.



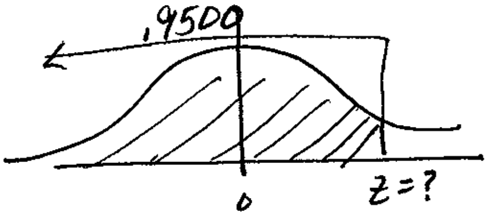
$P(-2 < z < 1.5) = .9332 - .0228 = .9104$

5. Find the probability that the reading is exactly 0.23°.



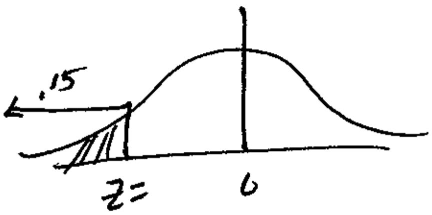
$P(x=0.23)=0$

6. Find the temperature corresponding to the P_{95}, the 95th percentile.



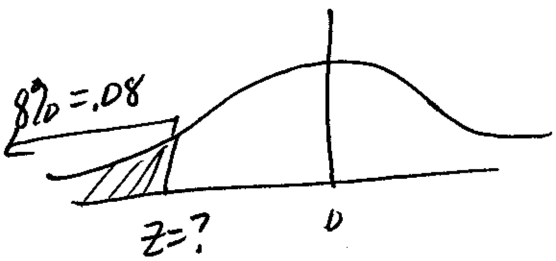
$z = \text{temp} = 1.645°C = P_{95}$

7. Find P_{15}.



$z = -1.04°C = P_{15}$

8. 8% of the thermometers are rejected because the readings are too low. Find that cut-off temperature.



$P_8 = z = -1.41°C$

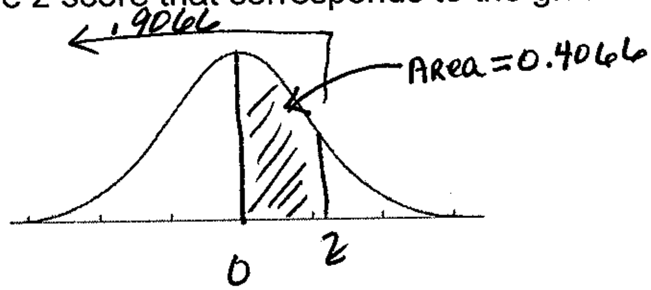
What would the corresponding Normal curves look like for:

$\mu = 25$
$\sigma = 8$

SAME shape – just moved

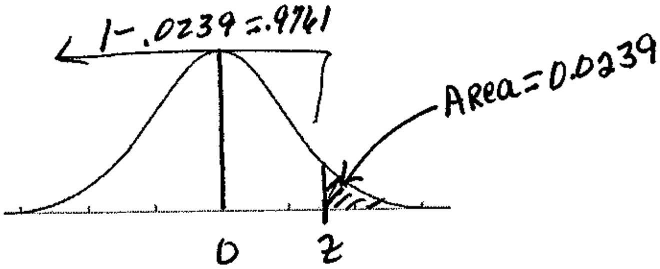
$\mu = 100$
$\sigma = 8$

Find the z-score that corresponds to the given area.

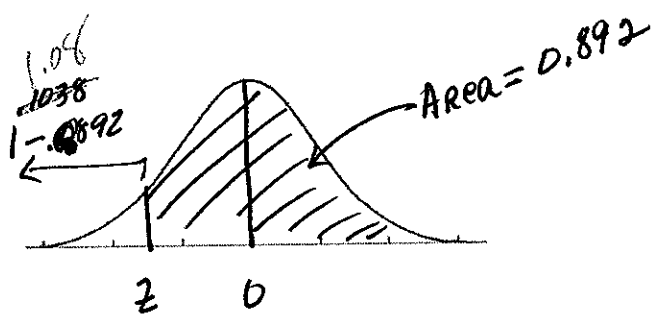
1. 

Look for z-score for 0.9066

$\boxed{z = 1.32}$

2. 

$\boxed{z = 1.98}$

3. 

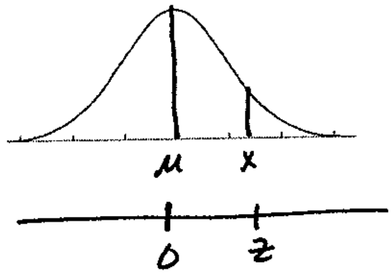
$z = \cancel{-1.26}$ -1.24

1.00
.892
.108

6.3 Applications of Normal Distributions

$\mu \neq 0$ or/and $\sigma \neq 1$

Need to Convert or Standardize



$z = \frac{x - \bar{x}}{s}$ or $\boxed{z = \frac{x - \mu}{\sigma}}$

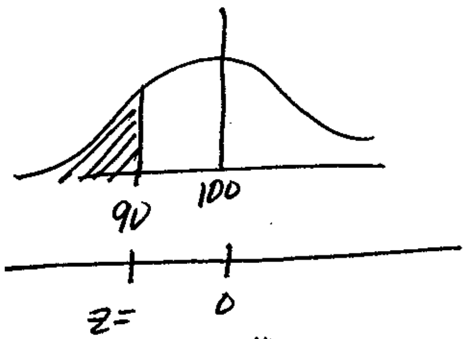
DRAW GRAPHS

Round z to 2 digits

Ex: Assume that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15 (Wechsler test)

$\mu = 100$
$\sigma = 15$

a. Find the probaility that a randomly selected adult has an IQ less than 90.

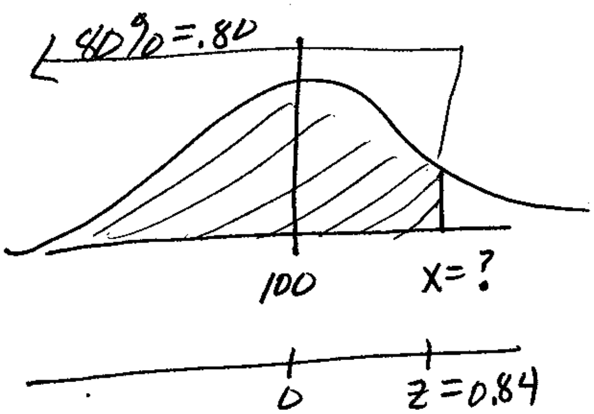


$z = \frac{90-100}{15} = \frac{-10}{15} = -.67$

$P(x < 90) = P(z < -0.67)$

$= \boxed{.2514}$

b. Find the P80, which is the IQ seperating the bottom 80% from the top 20%.



$z = \frac{x - \mu}{\sigma}$

$0.84 = \frac{x - 100}{15}$

$(0.84)(15) + 100 = x$

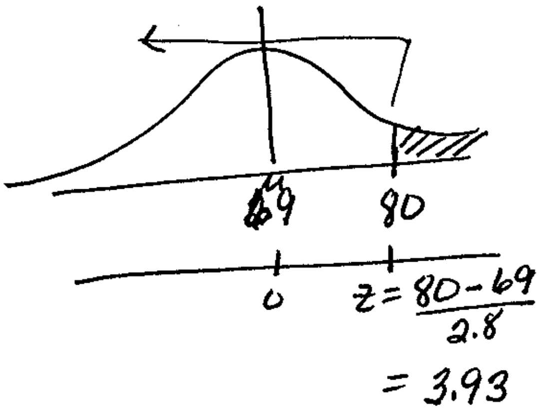
$\boxed{112.6 = x}$

Ex: Men's heights are normally distributed with a mean 69.0" and a stdev of 2.8".

height = 80"

$\mu = 69$
$\sigma = 2.8$

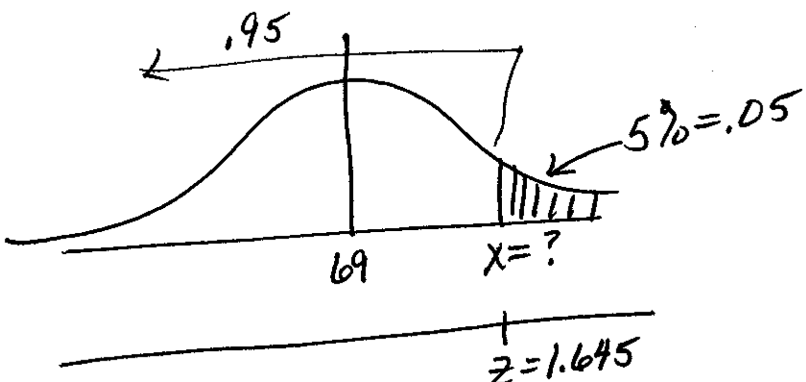
a. What percentage of men are too tall to fit through a standard door without bending. Based on your results, does it appear that the current doorway design is adequate?



$P(x > 80) = P(z > 3.93)$
$= 1 - .9999$
$= .0001$

← very small prob
< 5%
∴ not likely
∴ door design OK.

b. If a statistician designs a house so that all of the doorway have heights that are sufficient for all men except the tallest 5%, what doorway height would be used?



$z = \frac{x - \mu}{\sigma}$

$1.645 = \frac{x - 69}{2.8}$

$(1.645)(2.8) + 69 = x$

$\boxed{73.61 = x}$

Door height 73.61 inches
≈ 6 ft. 1 in

over

Central Limit Theorem - Section 6.4 in 4th edition, Sec 6.5 in 5th edition

(Class with look at dice problem instructor has in Excel)

Given:

- The random variable x has a distribution (which may or may not be normal) with a mean μ and stdev σ.
- Samples of sizes n are randomly selected.

Conclusions:

- The distribution will approach a normal distribution as the sample sizes increase.
- The mean of the sample means will be the population mean. $\mu_{\bar{x}} = \mu$
- The st dev of the sample means will be: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

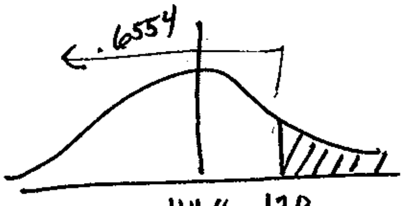
Ex: For women aged 18-24, their systolic blood pressures are normally distributed with a mean of 114.8 and a stdev of 13.1.

a. If a woman is randomly selected, find the prob that her systolic blood pressure is above 120.

b. If 30 women are randomly selected, find the prob that their mean systolic blood pressure is above 120.

$\mu = 114.8$
$\sigma = 13.1$

a)

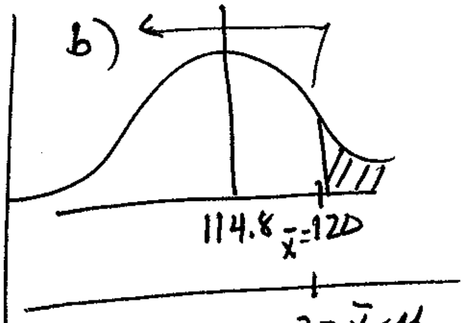


$$z = \frac{120 - 114.8}{13.1} = 0.3969 = 0.40$$

$$P(x > 120) = P(z > 0.40) = 1 - .6554 = \boxed{.3446}$$

$\approx 34\%$

b)



$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{120 - 114.8}{13.1/\sqrt{30}} = \frac{5.2}{2.39} = 2.17$$

$$P(\bar{x} > 120) = P(z > 2.17) = 1 - .9850 = \boxed{0.015}$$

$\approx 1.5\%$

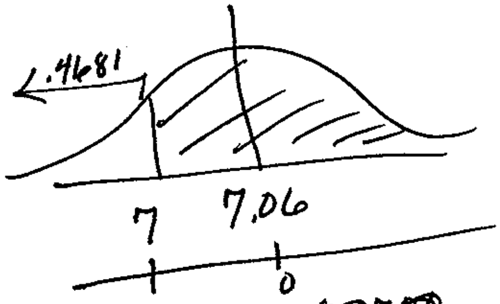
Ex: A study of amounts of time (in hours) college freshman study each week found that the mean was 7.06 hours and the stdev was 5.32 hours. If 55 freshman are randomly selected, find the prob that their mean weekly study time exceeds 7 hours.

$\mu = 7.06$

$\sigma = 5.32$

$n = 55$

$P(\bar{x} > 7)$



$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{7 - 7.06}{5.32/\sqrt{55}}$$

$$= \frac{-0.06}{0.717}$$

$$= -0.08$$

$$P(\bar{x} > 7) = P(z > -0.08)$$

$$= 1 - .4681$$

$$= \boxed{.5319}$$

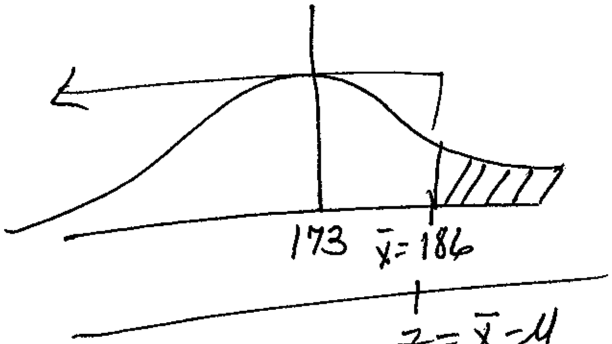
Ex: The population of weigths of men is normally distributed with a mean of 173 pounds and a st dev of 30 pounds. An elevator in the Dallas Men's Club is limited to 32 occupants, but it will be overloaded if the 32 occupants have a mean weight in excess of 186 lbs. If 32 randomly selected men enter the elevator, find the probability that their mean weight will exceed 186 lbs, causing the elevator to be overloaded. Based on your answer, is there reason for concern?

$\mu = 173$

$\sigma = 30$

$n = 32$

$P(\bar{x} > 186)$



$z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \dfrac{186 - 173}{30/\sqrt{32}} = \dfrac{13}{5.303} = 2.45$

$= P(z > 2.45)$

$= 1 - .9929$

$= .0071$

$P = .71\%$ Not likely

$\Rightarrow$ No Reason for concern